CONTINUOUSLY VARIABLE FIDELITY ADAPTIVE LARGE EDDY SIMULATION

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Since the inception of Computational Fluid Dynamics, turbulence modeling and numerical methods evolved as two separate fields of research with the perception that once a turbulence model is developed, any suitable computational approach can be used for the numerical simulations of the model. Over the last decade, our group has pursued research with cardinally different philosophy in its belief that in order to increase the computational efficiency of turbulent flow simulations and substantially improve the accuracy of predictions of flow characteristics, both the numerics and physics-based modeling need to be tightly integrated to ensure better capturing of the flow physics on a near optimal adaptive computational grid, ultimately leading to substantial reduction in the computational cost, while resolving dynamically dominant flow structures.

Turbulence is difficult to approximate mathematically, and to calculate numerically, because it is active over a large and continuous range of length scales (e.g. from less than a millimeter to hundreds of kilometers in the atmosphere). The range of active scales increases with Reynolds number (like $Re^{3/4}$ for three-dimensional turbulence), which means flows are increasingly difficult to calculate at the large Reynolds numbers of practical interest. Although the active flow regions extend over many scales, they are distributed inhomogeneously in both space and time. This inhomogeneity is called intermittency. This talk will provide an overview of a novel framework for continuously variable fidelity adaptive large eddy simulation that tightly integrates numerics and physics-based modeling and fully exploits the spatial and temporal intermittency of turbulent flows by constructing reduced models of turbulence (e.g. in terms of coherent vortices) and by optimally using a finite number of computational elements (e.g. using adaptive mesh refinement).

Latest advancements in wavelet-based numerical methodologies for the solution of partial differential equations [1-4], combined with the unique properties of wavelet analysis to unambiguously identify and isolate localized dynamically dominant flow structures [5-6], and to track them on adaptive computational meshes [7-9], make it feasible to develop intelligent methods for turbulent flow simulation that tightly integrate numerics and physics-based modeling.

The integration of turbulence modeling with adaptive wavelet methods results in a hierarchical approach in which coherent flow structures are either totally or partially resolved on self-adaptive computational grids, while modeling the effect of unresolved motions. The separation between resolved (more energetic) eddies and residual (less energetic) flow is achieved by means of nonlinear wavelet thresholding filter. For details of wavelet filtering we refer to the recent review [11]. Briefly, the filtering operation is accomplished by applying the wavelet-transform to the unfiltered field, discarding the wavelet coefficients below a given relative wavelet threshold parameter ϵ and transforming back to the physical space. The application of the wavelet filter to the turbulent velocity field results in the decomposition of the field into two different parts: a coherent more energetic velocity field $\bar{\mathbf{u}}^{>\epsilon}$ and a residual less energetic coherent/incoherent one $\mathbf{u}'$, i.e., $\mathbf{u} = \bar{\mathbf{u}}^{>\epsilon} + \mathbf{u}'$. The value of wavelet threshold ϵ explicitly defines the relative energy level of the turbulent eddies that are filtered out and, consequently, controls the relative importance of resolved field and residual background flow and, thus, the fidelity of turbulence simulations. By increasing the wavelet threshold a unified hierarchy of wavelet-based turbulence models of different fidelity can be obtained.

The first approach in this hierarchy is wavelet-based direct numerical simulation (WDNS) [1,3,9], which uses wavelet-based discretization of the Navier-Stokes equations to adapt dynamically the local resolution to intermittent flow structures. The choice of a sufficiently small threshold for wavelet filtering eliminates the need to do any modeling because the eliminated flow part is insignificant for the flow dynamics and the resulting simulations could be interpreted as adaptive DNS [11].

The next method in the wavelet-based turbulence modeling hierarchy is the Coherent Vortex Simulation approach (CVS), which was introduced by Farge et al. [12]. The underlying idea is the decomposition of the flow into coherent and incoherent contributions by means of wavelet filtering of the velocity or vorticity fields. The evolution of the coherent flow is then computed deterministically, while the influence of the incoherent background flow is neglected since they provide no turbulent-dissipation. CVS achieves a significant compression compared with WDNS. However, the number of retained active modes remains large and the process of calculating the optimal threshold for denoising at each time-step is quite expensive, since it requires the variance of the incoherent modes. Moreover, the wavelet-based coherent vortex extraction for inhomogeneous turbulence is still an open question [10].

In order to further reduce the computational cost, the Stochastic Coherent Adaptive Large Eddy Simulation (SCALES) [6,7] has been recently proposed. SCALES inherits the ability of the CVS to dynamically “track” the most energetic part of the coherent eddies in a turbulent flow field, while, similarly to classical Large Eddy Simulation (LES), model the effect of the less energetic (unresolved) motions. The SCALES approach inherits the advantages of both CVS and LES, while overcoming the shortcomings of both methods. Unlike coherent/incoherent and large/small structures decomposition in CVS and LES, respectively, in SCALES the separation is between more and less energetic structures. Also, differently from CVS, the effect of the flow structures that are filtered out can not be ignored but, similarly to conventional non-adaptive LES, needs to be modeled. Furthermore, the filtering process and, consequently, the subgrid scale (SGS) modeling procedure are benefited from wavelet nonlinear threshold filter, which depends on the instantaneous flow realizations. The use of SGS models results in the further reduction of the number of degrees of freedom compared to CVS and, thus, a higher grid-compression is achieved.

In order to consistently switch from one computational approach to another, it is essential that the different methods share the same structure of governing equations. Goldstein and Vasilyev [6] showed that the turbulent velocity-field, similarly to vorticity field [12] can be directly decomposed into deterministic coherent and stochastic incoherent (with Gaussian PDF) modes by applying the wavelet filter with sufficiently low values of the threshold. Therefore, CVS can also be based on velocity-pressure formulation. This way, both CVS and SCALES solve the wavelet-filtered Navier-Stokes equations, without and with the aid of SGS models, respectively, though at different threshold levels. Finally, WDNS can be also viewed as the solution of the same no-modeled equations with an even smaller value of the threshold parameter.

The fidelity for wavelet-based methods is controlled by the value of the thresholding factor. Very small thresholds correspond to WDNS, moderately small ones to CVS, larger values along with the use of SGS models result in SCALES. In the continuously variable fidelity adaptive large eddy simulation framework, the back and forth transition between WDNS and CVS is straightforward. Since both approached do not utilize any SGS model, the only difference stands in the different level of threshold. On the contrary, the transition from CVS to SCALES (and vice versa) is apparently more difficult. An additional effort for switching on (and off) the SGS modeling procedure, by monitoring/controlling the numerical dissipation induced by the wavelet truncation, would be required. More practically, the modeled wavelet-filtered governing equations can be resolved in both CVS and SCALES regimes, while continuously adjusting the level of SGS dissipation through spatially and temporally varying threshold level. The actual value of the threshold explicitly controls the level of SGS dissipation and, thus, the fidelity of the simulation.

Until recently, almost all wavelet-based turbulence modeling approaches, as well as all wavelet-based methods for numerical solution of PDEs, have employed an a priori defined threshold. The robustness of the wavelet approach was recently enhanced [15] by exploring the variable threshold strategy. The new spatio-temporally varying threshold methodology is the key-element of a

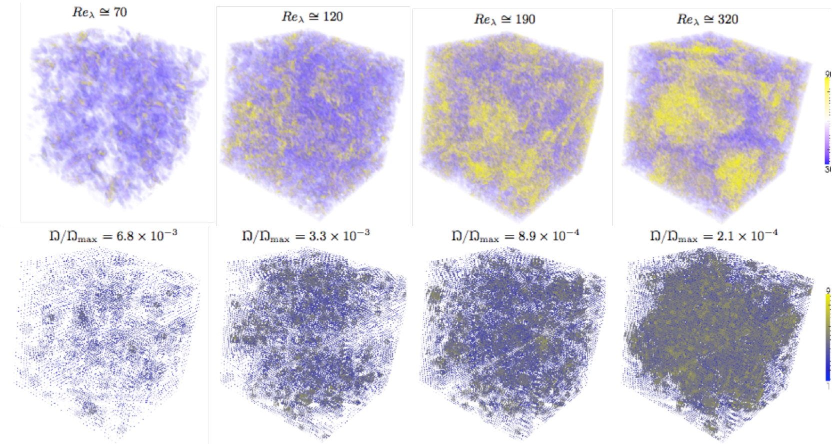

Fig. 1 Volume rendered vorticity magnitude (top row) and the adaptive computational mesh with grid compression ratios, $\mathcal{N}/\mathcal{N}_{\max}$ (bottom row) for adaptive large eddy simulations of linearly forced homogeneous turbulence at Re_λ of 70, 120, 190, 320 [14]. The scale for vorticity magnitude and the levels of resolution is the same for all four cases.

more general wavelet-based hybrid turbulence modeling framework, which fully utilizes spatial/temporal turbulent flow intermittency.

The variable threshold strategy proposed in [13] is based on the quadratic proportionality of the SGS dissipation to the threshold level [6], which implies that the rate of local energy transfer from energetic resolved eddies to unresolved less energetic structures (and vice versa) can be controlled by varying the threshold factor. A decrease of the threshold ϵ results in the local grid refinement with the subsequent rise of the resolved viscous dissipation, while an increase of ϵ leads to mesh coarsening that results in the growth of the local SGS dissipation. Therefore, the variable fidelity adaptive large eddy simulation methodology can be improved by exploiting a spatially varying threshold as a way to control the SGS dissipation. The basic idea is to locally vary ϵ wherever the level of modeled dissipation deviates from an a priori defined magnitude.

In order to vary ϵ in a physically consistent fashion, it should follow the local flow structures as they evolve in space and time, thus, necessitating the Lagrangian representation of the WTF parameter. For the details of the mathematical formulation of variable threshold strategy we refer to Ref. 12. Briefly, the space/time variable threshold strategy consist in tracking the statistical average of ϵ along the trajectory of a fluid particles using the Lagrangian path-line diffusive averaging approach [14], which can be written in terms of partial differential equation for ϵ-field evolution. The adjustment of the wavelet threshold

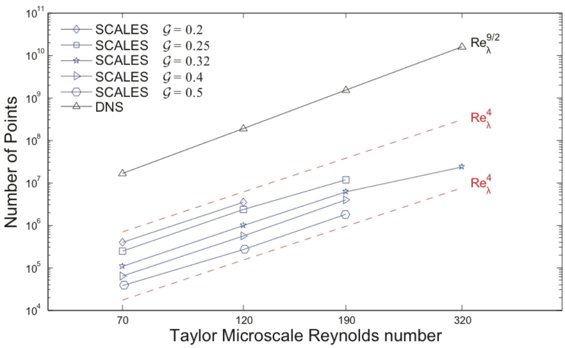

Fig. 2 Reynolds number scaling of constant-dissipation adaptive large eddy simulation at various levels of fidelity.

$\epsilon(\mathbf{x},t)$ is achieved within material framework through spatially and temporarily varying forcing.

The described spatio-temporarily varying thresholding strategy ensures that the wavelet threshold is determined on the fly according to the desired level of *turbulence resolution*. The latter is measured by the local fraction of SGS dissipation (FSGSD) that is defined as

$$\mathcal{F}(\boldsymbol{x},t) = \frac{\Pi}{\varepsilon_{\text{res}} + \Pi},$$

where ε_{res} and Π are respectively the resolved and SGS dissipations. A prescribed level for the quantity $\mathcal{F}$ determines the level at which the most energetic structures are resolved and the effect of SGS residual motions is modeled. The forcing mechanism for the evolution of the wavelet threshold is designed to maintain the FSGSD variable $\mathcal{F}$ at a priori defined goal value G. To summarize, the present Lagrangian variable thresholding is a new strategy for the SCALES method that provides a two-way feedback mechanism between the modeled dissipation and the computational mesh. This allows to maintain an a priori defined level of SGS dissipation, namely, a prescribed degree of turbulence resolution for the ongoing simulation.

The continuously variable fidelity adaptive large eddy simulation provides a unique framework for performing a dynamic computational complexity study, where the dependence of the active degrees of freedom on Reynolds number for different levels of turbulent resolution can be explored. To construct the Reynolds number scaling statistics, a series of simulations of linearly forced homogeneous turbulence, where the Reynolds number is progressively increased, were recently performed [15] on a dynamically adaptive grids with effective non-adaptive resolutions of $256^3, 512^3, 1024^3, 2048^3$, Fig. 1. These correspond to Taylor micro-scale Reynolds number of $Re_\lambda = 70, 120, 190, 320$. The effective resolutions are chosen to maintain the ratio of Kolmogorov length-scale to the smallest grid-spacing constant, $\eta/\Delta_{\min} = 2$. In order to study the

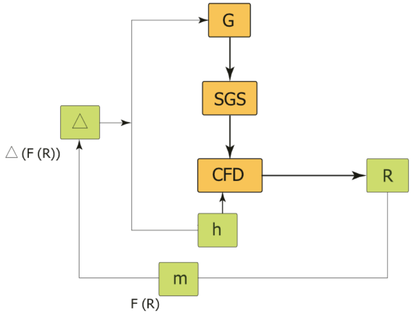

Fig. 3 Dependency diagram for adaptive variable fidelity large eddy simulation. Notation: G - filter, R - results, m - model feedback, Δ - adjusted spatially variable filter width, ϵ - variable wavelet threshold for model adaptation, $\mathcal{F}$- arbitrary dynamically important physical quantity to be controlled.

influence of the fidelity of simulation on the Reynolds number scaling, a series of simulations of different turbulence resolutions is conducted. The different fidelity is achieved by using spatially variable thresholding approach with different goal values of $\mathcal{F}$, namely $G = 0.2, 0.25, 0.32, 0.4, 0.5$. It is observed that the number of active spatial modes of constant-dissipation adaptive large eddy simulations scale as ${Re_\lambda}^4$ regardless of the level of turbulence resolution, Fig. 2.

It is important to mention that the concept of variable fidelity adaptive large eddy simulation is not specific to the wavelet-based computational framework. The adaptive large eddy simulation can be based on the classical formulation integrated with adaptive mesh refinement. One possible implementation is illustrated in Fig. 3. A variable fidelity adaptive LES would include an additional feedback mechanism from the results (any physical quantity) in order to incorporate a filter-width/model adaptation preferably coupled with adaptation of the numerical resolution as well. Hence, both filtering-mechanism/model (via the filter-width) and CFD-engine/numerics (through the resolution) should be dynamically coupled based on any objective physics-based fidelity measure.

Currently the variable fidelity adaptive large eddy simulation methodology has been investigated for the generalization to wall bounded flows, where the flow geometry is enforced through Brinkman volume-penalization [16]. The preliminary results for the wavelet-based variable fidelity adaptive large eddy simulation of vortex shedding flow behind an isolated stationary prism with square cross-section is successfully carried out for $Re = 2000$, Fig. 4. The results are in good agreement with adaptive large eddy simulations based on constant wavelet threshold [17].

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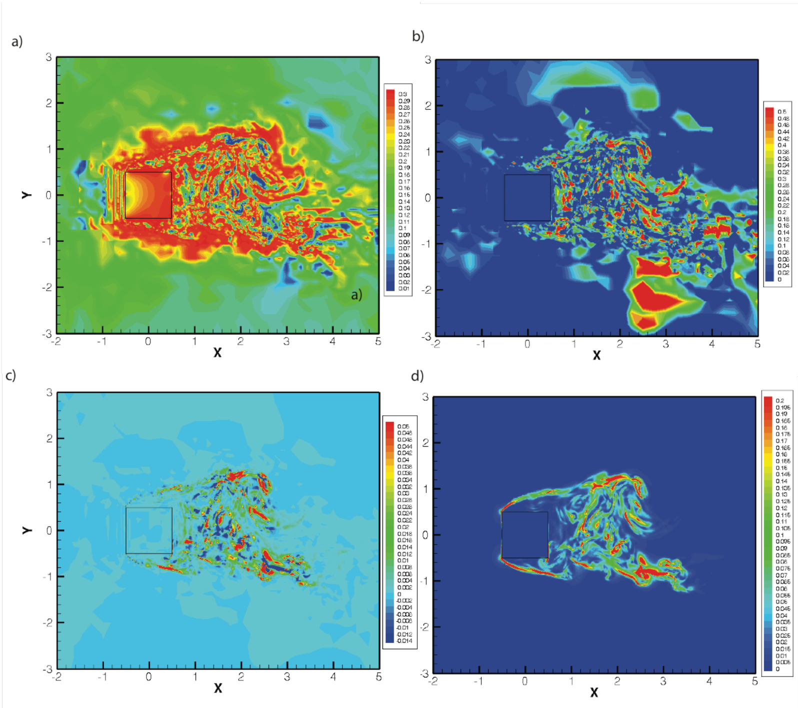

Fig. 4 Dependency Space-time variable wavelet thresholding: a) wavelet threshold level, b) fraction of SGS dissipation, c) modeled SGS dissipation, d) resolved viscous dissipation.

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References

1. Vasilyev, O. V., 2003. Solving multi-dimensional evolution problems with localized structures using second generation wavelets. Int. J. Comp. Fluid Dyn., 17, 2, 151–168.
2. Vasilyev, O. V. and Bowman, C., 2000. Second generation wavelet collocation method for the solution of partial differential equations. J. Comp. Phys. 165, 660–693.
3. Vasilyev, O. V. and Kevlahan, N. K. R., 2005. An adaptive multilevel wavelet collocation method for elliptic problems. J. Comp. Phys. 206, 2, 412–431.

4. Nejadmalayeri, A., 2012. Hierarchical multiscale adaptive variable fidelity wavelet-based turbulence modeling with Lagrangian spatially variable thresholding. PhD thesis, University of Colorado Boulder, Boulder, CO.
5. Farge, M., Schneider, K., Pellegrino, G., Wray, A. A. and Rogallo, R. S., 2003. Coherent vortex extraction in three-dimensional homogeneous turbulence: Comparison between CVS-wavelet and POD-Fourier decompositions. Phys. Fluids 15, 2886–2896.
6. Goldstein, D. E. and Vasilyev, O. V., 2004. Stochastic coherent adaptive large eddy simulation method. Phys. Fluids 16, 7, 2497–2513.
7. Goldstein, D. E., Vasilyev, O. V. and Kevlahan, N. K. R., 2005. CVS and SCALES simulation of 3D isotropic turbulences. J. of Turb. 6, 37, 1–20.
8. Kevlahan, N. K. R. and Vasilyev, O. V., 2005. An adaptive wavelet collocation method for fluid-structure interaction at high Reynolds numbers. SIAM J. Sc. Comp. 26, 6, 1894–1915.
9. Vasilyev, O. V. and Kevlahan, N. K. R., 2002. Hybrid wavelet collocation - Brinkman penalization method for complex geometry flows. Int. J. Numerical Methods in Fluids 40, 531–538.
10. Schneider, K. & Vasilyev, O.V., 2010 Wavelet methods in computational fluid dynamics. Ann. Rev. Fluid Mech. 42, 473–503.
11. Sagaut, P., Deck, S. and Terracol, M., 2006. Multiscale and Multiresolution Approaches in Turbulence. Imperial College Press.
12. Farge, M., Schneider, K. and Kevlahan, N., 1999. Non-Gaussianity and coherent vortex simulation for two-dimensional turbulence using an adaptive orthogonal wavelet basis. Phys. Fluids. 11, 8, 2187–2201.
13. Nejadmalayeri, A., Vezolainen, A., De Stefano, G., and Vasilyev, O.V., 2014. Fully adaptive turbulence simulations based on Lagrangian spatio-temporally varying wavelet thresholding, *Journal of Fluid Mechanics*, **749**, pp. 794-817.
14. Vasilyev, O.V., De Stefano, G., Goldstein, D.E. & Kevlahan, N.K.-R., 2008 Lagrangian dynamic SGS model for SCALES of isotropic turbulence. Journal of Turbulence 9 (11), 1–14.
15. Nejadmalayeri, A., Vezolainen, A., and Vasilyev, O.V., 2013. Reynolds number scaling of coherent vortex simulation and stochastic coherent adaptive large eddy simulation, *Physics of Fluids*, **25**, 110823.
16. Liu, Q. and Vasilyev, O.V., 2007. Brinkman Penalization Method for Compressible Flows in Complex Geometries," *Journal of Computational Physics*, **227**(2), pp. 946–966.
17. De Stefano, G. and Vasilyev, O.V., 2014. Wavelet-Based Adaptive Simulations of Three-Dimensional Flow Past a Square Cylinder, *Journal of Fluid Mechanics*, **748**, pp. 433 – 456.