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A-priori dynamic test for deterministic/stochastic modeling in large-eddy simulation of turbulent flow

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Abstract

The coherent/incoherent decomposition of the subgrid-scale stresses based on the wavelet de-noising procedure is exploited in the framework of large-eddy simulation of turbulence. Dynamic *a-priori* tests based on the *perfect* modeling approach are performed for decaying isotropic turbulence. The theoretical performances of deterministic/stochastic subgrid-scale models are evaluated during the simulation. The main result is that in large-eddy simulations low order statistics can be almost exactly reproduced when only the effect of the coherent subgrid-scale modes is accounted for, while the incoherent subgrid-scale stresses not affecting the energy transfer.

Keywords: Turbulence; Large-eddy simulation; Deterministic/stochastic modeling

1. Introduction

In numerical simulation of turbulent flows the computing requirements increase so rapidly with the Reynolds number that the applicability of direct numerical simulation (DNS), i.e. the fully resolved numerical solution of Navier–Stokes equations, is limited to regimes of low practical interest [1]. Thus, in order to overcome this difficulty, other approaches, such as Reynolds-averaged Navier–Stokes equations (RANS) [2], large-eddy simulation (LES) [3–5] or lattice Boltzmann method (LBM) [6] must be used. In the RANS approach the statistical mean velocity is solved for, while in LES the filtered NS equations are solved for the large-scale velocity field. In both RANS and LES approaches suitable closure models must be introduced to take into account the effect of unresolved turbulent motions. Alternatively, turbulent flow can also be modeled by exploiting the Boltzmann kinetic equation instead of the Navier–Stokes ones with the lattice Boltzmann methodology.

In this work we focus on the LES approach. In particular, we study the effect of deterministic/stochastic models for the unresolved subgrid-scale (SGS) stresses. The nearly universal approach in LES is to use

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deterministic models, by defining the SGS force as a given function of the resolved field. An alternative approach is to use stochastic modeling, by also considering a noise contribution to the model. In fact, a stochastic force can be added to a classical eddy viscosity term to develop improved models [7]. The motivation behind stochastic modeling is the recognition that large-scale motions are randomly forced through non-linear interaction with small-scale eddies, resulting in bi-directional energy transfer, e.g. [8]. Stochastic models more realistically mimic the behavior of filtered direct numerical simulation (DNS), since different flow realizations can be observed for the same initial large-scale field. However, flow realizations for stochastic LES are different from the filtered DNS solution. Actually, it is impossible to construct an LES model providing large-scale velocity that matches the filtered DNS field realization by realization. Turbulence is characterized by energetic eddies that are localized in space and contain significant energy at all length-scales, from the characteristic length-scale of the physical domain down to the Kolmogorov one. In fact, visualization of three-dimensional turbulence from laboratory as well as numerical experiments has shown the presence of basic structures, namely vorticity tubes, at all the flow scales, e.g., [9]. When a low-pass filter is used with LES, the small-scale coherent energetic eddies are filtered out. Therefore, the effect of these small-scale energetic structures, hereafter referred to as coherent SGS modes, must be modeled. It has been hypothesized from preliminary results that the coherent SGS modes have a disproportionately strong effect on the total SGS dissipation and thus on the evolution of the resolved field. Here, some numerical experiments are performed to assess the theoretical effect of coherent/incoherent SGS modes.

The present analysis is conducted for decaying isotropic turbulence at $Re_\lambda = 72$, by means of the *perfect* modeling approach [10,11]. The procedure consists in building the DNS database and exploiting it to evaluate the SGS force on the LES space–time grid, then performing LES runs supplied with these known stresses. In fact, though unfeasible for flows of practical interest, nevertheless DNS remains an invaluable tool when studying turbulence models [1].

Also, the concept of coherent/incoherent turbulence decomposition based on wavelet de-noising is exploited, by adopting the simple idea according to which the coherent eddies correspond to the de-noised flow field [12]. De-noising is obtained through wavelet filtering, which is performed in wavelet space by wavelet coefficient thresholding. Namely, given a turbulent velocity field, a forward wavelet transform is first performed and wavelet coefficients below a given threshold are set to zero, then an inverse transform is performed, so obtaining the coherent (de-noised) velocity field.

Previous studies have clearly demonstrated that such a filter is able to decompose an instantaneous turbulent field into a non-Gaussian coherent part (corresponding to the energetic coherent eddies) and an incoherent one, which is close to Gaussian white noise [13].

2. Results and discussion

When considering the filtered incompressible Navier–Stokes equations for the filtered velocity $\bar{u}_j$,

$$\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial \overline{\bar{u}_j \bar{u}_k}}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_j} + \nu \frac{\partial^2 \bar{u}_j}{\partial x_k \partial x_k} - \frac{\partial \bar{\tau}_{jk}}{\partial x_k},$$

the SGS stresses are defined as $\bar{\tau}_{jk} = \overline{u_j u_k} - \overline{\bar{u}_j \bar{u}_k} \equiv \overline{\bar{u}_j u'_k} + \overline{u'_j \bar{u}_k} + \overline{u'_j u'_k}$, while $u'_j = u_j - \bar{u}_j$ stands for the SGS velocity field. After introducing the coherent/incoherent decomposition of the SGS field by using the wavelet de-noising procedure, $u'_j = u'_{j>} + u'_{j<}$, the SGS stresses can be consistently decomposed. In fact, the effect of SGS coherent modes is taken into account by the *coherent* SGS stresses, $\bar{\tau}^{(\mathrm{coh})}_{jk} = \overline{\bar{u}_j u'_{k>}} + \overline{u'_{j>} \bar{u}_k} + \overline{u'_{j>} u'_{k>}}$, while the effect of SGS incoherent modes by the *incoherent* SGS stresses, $\bar{\tau}^{(\mathrm{inc})}_{jk} = \overline{\bar{u}_j u'_{k<}} + \overline{u'_{j<} \bar{u}_k} + \overline{u'_{j>} u'_{k<}} + \overline{u'_{j<} u'_{k>}} + \overline{u'_{j<} u'_{k<}}$. This way, the SGS stresses split according to $\bar{\tau}_{jk} = \bar{\tau}^{(\mathrm{coh})}_{jk} + \bar{\tau}^{(\mathrm{inc})}_{jk}$ and the same decomposition holds for the SGS dissipation, $-\langle \bar{\tau}_{jk} \bar{S}_{jk} \rangle$, the contributions from coherent and incoherent SGS fields being defined as *coherent* SGS dissipation and *incoherent* SGS dissipation.

In a real LES the SGS stresses are unknown quantities, which must be somehow modeled in terms of the resolved field. In this theoretical analysis, the effect of coherent and incoherent SGS stresses is considered using the *perfect* modeling approach [11]. Namely, 16^3

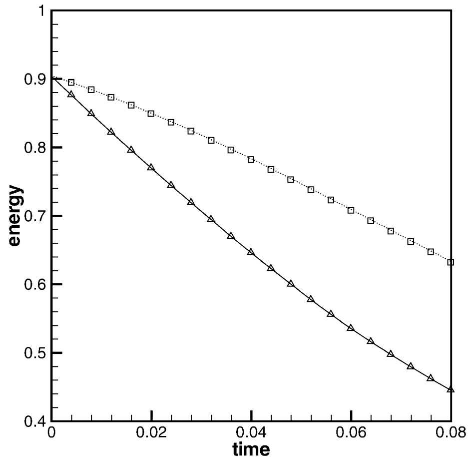

Fig. 1. Energy decay for LES supplied with coherent (△) and incoherent (□) SGS stresses, compared to truncated DNS (solid) and no-modeled solutions (dotted).

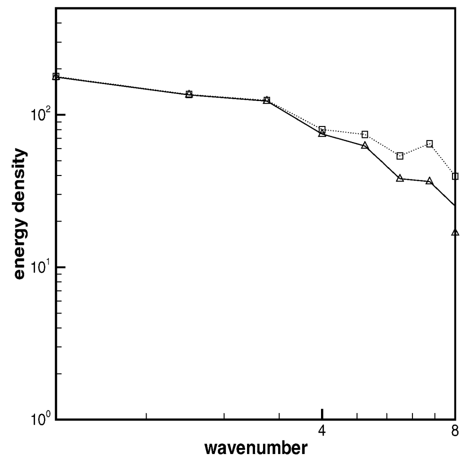

Fig. 2. Energy spectrum for LES supplied with coherent (△) and incoherent (□) SGS stresses, compared to truncated DNS (solid) and no-modeled solutions (dotted).

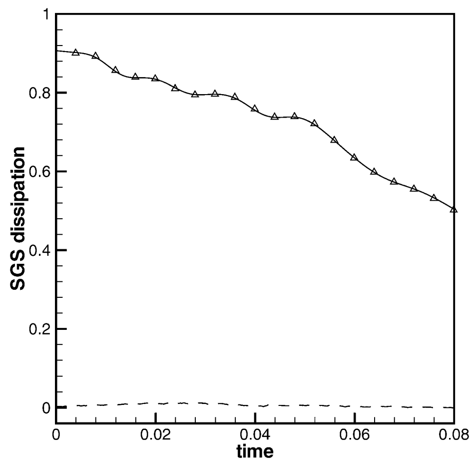

Fig. 3. SGS dissipation for LES with only coherent (△), only incoherent (dashed) and total (solid) stresses.

LES is supplied with the known SGS stresses evaluated by definition upon a reference 128^3 DNS solution.

For the purpose of the discussion, we assume that the fixed thresholding level decomposes the SGS field into coherent and incoherent components. For the actual value, $\epsilon = 0.015$ a time average compression of about 94% is obtained. It has been verified that the residual incoherent field corresponds to a nearly Gaussian white noise. In addition, it has been shown that both coherent and incoherent SGS modes contribute to local bi-directional energy transfer with the net energy transfer from large to small scales almost entirely corresponding to the coherent modes [13].

The study is conducted by performing different LES, namely supplied with only the perfect *coherent*, only the perfect *incoherent*, and the perfect total SGS stresses. Energy evolution in time for the different solutions is shown in Fig. 1. For comparison, the truncated DNS and the no-model LES solutions are also reported. It turns out that when supplying the simulation with only the coherent SGS force, one obtains very good results while, on the contrary, supplying only the incoherent SGS modes gives no substantial contribution to the model, the solution practically coinciding with the no-modeled one. As a trivial but necessary verification, it has been verified that LES supplied with the total stresses provides results that are indistinguishable from the filtered DNS ones. The same performances have been verified over the time in terms of spectral energy distribution. For example, in Fig. 2, the energy density spectrum after 10^3 time integration steps is illustrated. By supplying only the incoherent SGS modes, the solution shows the same energy pile-up characteristic of the no-model solution.

Fig. 3 shows the time evolution for the SGS dissipation for the different solutions. It appears evident how few coherent SGS modes are responsible for almost all the SGS dissipation. The issue is further clarified by looking at the different contributions to the SGS dissipation for LES supplied with the total perfect stresses

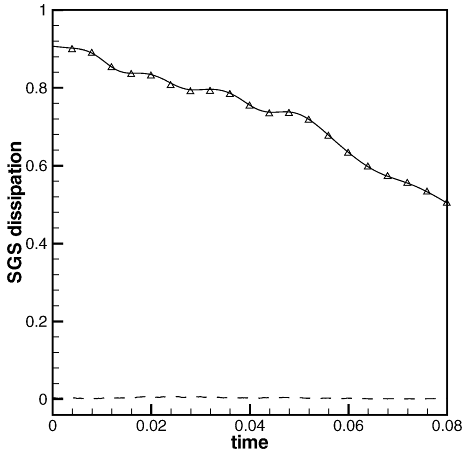

Fig. 4. Coherent (△), incoherent (dashed) and total (solid) SGS dissipation for LES with total stresses.

in Fig. 4. The SGS dissipation mostly comes from the coherent component, with the effect of the incoherent subgrid scales being essentially non-dissipative. In fact, a very large fraction of the SGS dissipation results from only the 6% (on average) of the SGS modes that are coherent.

Thus, it is demonstrated that modeling incoherent SGS modes has a negligible effect upon the large-scale energy dynamics. Supplying pure coherent SGS stresses provides results that are very close to truncated DNS. In fact, the low order statistics are relatively insensitive to the effect of the incoherent SGS stresses, the energy transfer between resolved and residual scales being almost exclusively governed by interactions with coherent SGS modes.

The strong message is that the effect of the coherent SGS modes needs to be modeled, while modeling the incoherent stochastic component is less important for recovering low order statistics. This fact partially explains why deterministic SGS models work in LES, since they reproduce the effect of the coherent SGS modes. At the same time, the lack of success in showing significant statistical correlation in *a-priori* tests may be attributed to the dominant role of the incoherent SGS modes on any stochastic estimation. Thus, one can also argue that different SGS models should be adopted for the corresponding contributions to the LES solution. Namely, deterministic models should be used to mimic the effect of the unresolved coherent modes and stochastic models for the incoherent ones.

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