Adaptive LES of Immersed-Body Flows Based on Variable Wavelet Threshold Filtering

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1 Introduction

In the wavelet-based adaptive large-eddy simulation (LES) approach to turbulent flows, the multi-resolution wavelet threshold filtering (WTF) procedure is exploited to separate coherent energetic eddies, which are resolved, from residual background flow, which is modeled. The governing equations for incompressible LES are formally obtained by applying the WTF operator to the Navier–Stokes equations.

The wavelet transform is also used for the automatic grid adaptation since a hierarchical high-order finite difference scheme, which takes advantage of the wavelet multilevel decomposition, is used for the numerical differentiation. Due to the one-to-one correspondence between wavelets and collocation points, some grid points are omitted from the numerical mesh when the associated wavelets are omitted from

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the WTF representation, because the corresponding coefficients are below a given thresholding level, say, ε. This way, the method allows the numerical grid to dynamically adapt to the evolution of the resolved flow structures, in both location and scale. Higher resolution calculations are actually carried out only in the regions where sharp gradients in the flow-field exist, which leads to the high grid compression property of wavelet-based methods.

Until very recently, the wavelet-based adaptive LES method has employed a prescribed constant thresholding level to separate resolved from unresolved turbulent velocity fields, say, $u_i = \overline{u}_i^{>\varepsilon} + u_i'$. A new fully adaptive methodology that makes use of either a spatially-uniform time-dependent [1] or a spatio-temporally varying [2] threshold have been recently proposed for homogeneous isotropic turbulence.

The present work aims at extending the wavelet-based adaptive LES approach with space-time variable thresholding for the simulation of flows with immersed obstacles. The numerical methodology combines the Brinkman volume penalization technique with the parallel Adaptive Wavelet Collocation Method [3]. The former is used for imposing the flow geometry while the latter guarantees the efficient flow resolution on a continuously adaptive computational grid.

2 Space-Time Variable Thresholding

The space-time variability of the wavelet threshold is introduced to employ a physics-based coupling mechanism between computational grid and turbulence modeling that ensures the desired uniform fidelity of the numerical solution, all over the time of simulation. The main idea is to continuously adjust the threshold variable by controlling a suitable measure of the local turbulence resolution. At the same time, a feedback mechanism governing the evolution of the threshold field is provided. To achieve this aim, the ratio between modeled subgrid-scale (SGS) and resolved viscous dissipations, say $\mathscr{R} = \Pi / D$, is assumed here as the measure of the turbulence resolution of the LES solution. This ratio was demonstrated in past research to increase with the square of the thresholding level. Moreover, the threshold variation should be sufficiently smooth in order not to lead to unphysical small scales in the resolved wavelet-filtered velocity field. In order to avoid this undesirable behavior, the wavelet threshold field is prescribed to depend upon the local flow evolution on the corresponding time scale.

All that is accomplished by solving the following Lagrangian transport equation to simulate the evolution of the threshold field

$$\frac{\partial \varepsilon}{\partial t} + \left(\overline{u}_i^{>\varepsilon} + U_i\right) \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i}\left[(\nu + \nu_\varepsilon)\frac{\partial \varepsilon}{\partial x_i}\right] + f_\varepsilon \,, \tag{1}$$

where U_i represents the known freestream velocity. The diffusion term with artificial viscosity coefficient ν_ε is introduced to prevent the creation of undesired high-frequency modes in the threshold field. Due to a Smagorinsky-like scaling, this coefficient is expressed in terms of the local strain-rate magnitude as follows

$$\nu_\varepsilon = C_{\nu_\varepsilon} \Delta^2 |\overline{S}^{>\varepsilon}| \,, \tag{2}$$

where Δ stands for the local characteristic WTF width, while the constant parameter C_{ν_ε} is set to 0.1 in this study. This way, the variable ε can be interpreted as the path-line diffusive averaged threshold, according to the averaging procedure proposed in [4].

The forcing term at the right hand side of Eq. (1) plays the key role in the current approach as it expresses the two-way feedback mechanism between the physical resolution of the LES field and the numerical resolution of the computational grid. In this work, the threshold field is forced so that the adaptive LES solution approaches a prescribed value $\mathscr{R}_0$ for the dissipations ratio. Namely, the following forcing definition is assumed

$$f_\varepsilon = \frac{\mathscr{H}(\Pi)}{\sqrt{\nu \mathscr{R}_0}} \left(\sqrt{\mathscr{R}_0 D} - \sqrt{\Pi} \right) \varepsilon \,, \tag{3}$$

where the presence of the Heaviside function $\mathscr{H}(\cdot)$ ensures that the threshold evolution is forced only in regions of energy forward scatter. According to this approach, in turbulent flow regions with low turbulence resolution ($\mathscr{R} > \mathscr{R}_0$), the thresholding variable is forced to decrease, which leads to local mesh refining. That leads to the subsequent growth of resolved viscous dissipation and the associate reduction of SGS dissipation. On the other hand, in regions of high resolution ($0 < \mathscr{R} < \mathscr{R}_0$), the threshold is forced to increase, which leads to mesh coarsening with the subsequent decrease of the resolved dissipation and the increase of the modeled one. It is worth noting that, in a practical calculation, the threshold must be explicitly bounded from both below and above. On the one side, the use of very low thresholds would lead to the no-model coherent vortex simulation regime while, on the other side, using very high thresholds would deteriorate the numerical accuracy of the adaptive LES solution.

The above forcing scheme inherently works in turbulent flow regions. It was found that in laminar flow regions, where both the resolved and the modeled dissipations vanish, the forcing (3) does not exhibit the right limiting behavior and needs to be switched to a different mechanism that, on the convective time scale, forces the wavelet threshold to take the maximum allowable value, which is prescribed by the user. The right limiting behavior is accomplished by following the modified forcing scheme

$$f_\varepsilon = \frac{\mathscr{H}(\Pi)}{\sqrt{\nu \mathscr{R}_0}} \left(\sqrt{\mathscr{R}_0 D} - \sqrt{\Pi} \right) \varepsilon + \max\left(\frac{U}{L} - A, 0 \right) \varepsilon \,, \tag{4}$$

where A stands for the magnitude of the velocity gradient tensor and L represents the characteristic length of the obstacle. The additional forcing term acts on the convective time scale only in regions of relatively low velocity gradients ($A < U/L$), while being absent in both the near wall and the wake regions, where the velocity gradients are generally much higher.

3 Results

The incompressible turbulent flow around a square cylinder is considered as a benchmark for the proposed numerical method. The cylinder, which has the cross-section with side length L in the (x, y) plane, is immersed in a uniform stream with velocity U along the x direction. The shear layers that separate from the sides of the cylinder become unstable and the transition to turbulence occurs at moderately high Reynolds-number [5]. The present numerical experiment is conducted at $Re = UL/\nu = 2 \times 10^3$. Eight levels of numerical resolution are used, which means that eight nested wavelet collocation grids are involved in the calcualtion. The hexahedral grid elements have a square cross section in the transverse plane with minimum size $L/128$, while the highest spanwise resolution corresponds to $L/64$. The threshold evolution equation (1) is solved along with the volume-penalized wavelet-based LES governing equations, supplied with the localized dynamic one-equation eddy-viscosity model proposed in [6]. The turbulence modeling approach that is adopted allows for negative eddy-viscosity coefficients, which corresponds to mimic the energy backscatter from unresolved to resolved motions. The threshold field may take values in the bounded interval $0.01 < \varepsilon < 0.2$.

The results of the present simulation have been successfully validated against both numerical non-adaptive solutions [7] and experimental findings [8]. For brevity, the

Fig. 1 Main vortical structures visualized through the iso-surfaces of $Q = 0.3U^2/L^2$

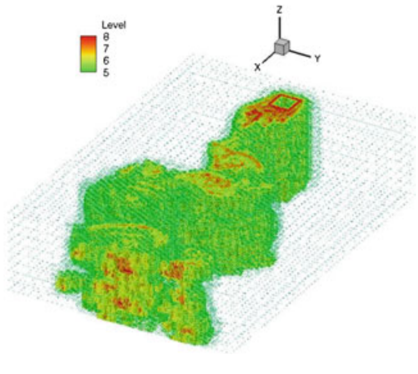

Fig. 2 Scatter plot of the retained collocation points at the highest levels of resolution ($5 \le j \le 8$)

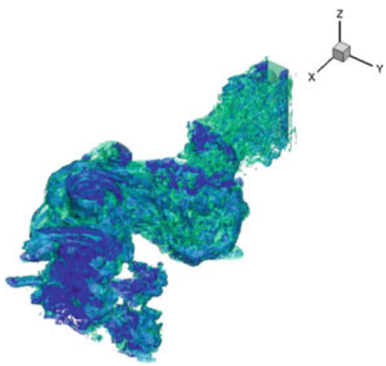

Fig. 3 Instantaneous threshold field visualized through the iso-surfaces of $\varepsilon = 0.08$ (*blue*) and 0.1 (*green*)

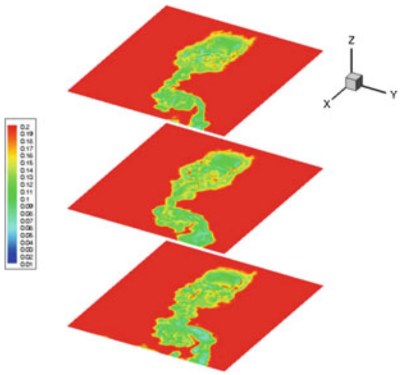

Fig. 4 Threshold contours in three volume *slice planes* along the spanwise direction

pertinent discussion is not reported in this paper. Here, it is preferred to elaborate on the space-time variability of the threshold that is achieved. The turbulent wake of the cylinder is visualized in Fig. 1, where the iso-surfaces of the second invariant of the velocity gradient tensor corresponding to the level $Q = 0.3U^2/L^2$ are depicted at a given time instant. The ability of the method to adapt to the local flow conditions is clearly demonstrated by making a comparison with the spatial distribution of the retained collocation points, which is reported in Fig. 2. For the sake of clarity, only the four finest levels of resolution are considered. Due to the two-way feedback mechanism that is employed, the evolution of the resolved flow field and the threshold distribution mutually influence each other. This is also apparent by inspection of Fig. 3, where the threshold field is visualized through the iso-surfaces corresponding to two different values that are $\varepsilon = 0.08$ (blue) and 0.1 (green). On Fig. 4, the threshold contours in three different volume slice planes along the spanwise direction are depicted, while the contours in volume slice planes along the streamwise and lateral directions are illustrated on Figs. 5 and 6, respectively. On the one hand, the structure of the threshold field closely resembles the wake structure. On the other hand, the pointwise value of the threshold dictates the local numerical resolution. In the flow regions where the threshold is relatively low, more collocation points are retained at

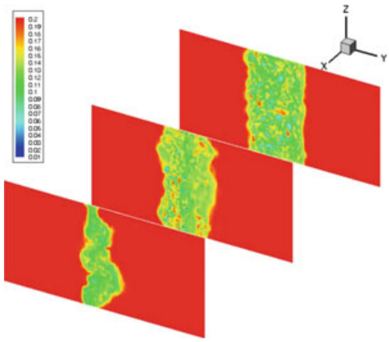

Fig. 5 Threshold contours in three volume *slice planes* along the streamwise direction

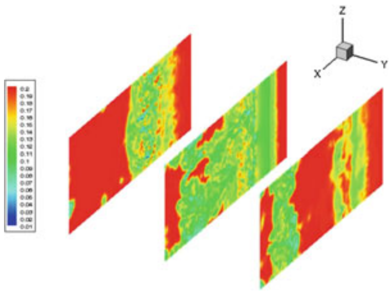

Fig. 6 Threshold contours in three volume *slice planes* along the lateral direction

the highest levels of resolution as happens, for instance, at the fluid-body interface. As expected, away from the body, the threshold takes the maximum value that is allowed, whereas close to the wall and in the turbulent wake it is adjusted so that the LES solution locally tends to achieve the prescribed resolution.

4 Conclusions

The coupled wavelet-collocation/volume-penalization method with variable thresholding is developed for the adaptive large-eddy simulation of turbulent flows around obstacles. The use of a spatio-temporally varying threshold allows numerical simulations with the prescribed turbulence resolution, on a near optimal computational mesh, to be carried out. The desired uniform fidelity is achieved thanks to the solution of a Lagrangian evolution equation for the threshold field in conjunction with the wavelet-filtered Navier–Stokes equations supplied with an energy-based eddy-viscosity model. The results obtained for a classical benchmark, which is the turbulent shedding flow past a square cylinder at supercritical Reynolds-number, demonstrate the accuracy and the efficiency of the proposed adaptive method.

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