Stochastic coherent adaptive LES of forced isotropic turbulence

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1 Introduction

The stochastic coherent adaptive large eddy simulation (SCALES) method [1] exploits a wavelet thresholding filter-based dynamic grid adaptation strategy to solve for the energetic "coherent" eddies in a turbulent flow field. The effect of the residual less energetic flow structures is modeled by supplying the simulation with a suitable subgrid-scale (SGS) model. The SCALES approach was successfully applied to the simulation of decaying homogeneous turbulence (*e.g.* see [2, 3]). Here, the wavelet-based approach is applied to statistically steady turbulence by considering linearly forced homogeneous turbulence at moderate Reynolds-number.

Due to the adaptive nature of the present approach, it is preferable to introduce forcing directly in physical space. For this reason, we adopt the linear forcing scheme proposed by Lundgren [4] and extensively studied by Rosales and Meneveau [5]. The continuity and Navier-Stokes equations for incompressible flow can be written in the forced case as

$$\frac{\partial u_i}{\partial x_i} = 0 \, , \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + Q u_i \, , \tag{2}$$

where ρ and ν are the constant density and kinematic viscosity of the fluid, while Q stands for the linear forcing coefficient assumed constant. The latter can be easily expressed in terms of the turbulence parameters by considering the volume-averaged energy equation in the homogeneous case

$$\frac{dK}{dt} = -\langle \varepsilon \rangle + 2QK \, , \tag{3}$$

where K stands for the mean kinetic-energy, ε for the turbulent dissipation, and angular brackets denote volume-averaging. Thus, in the equilibrium hypothesis for statistically steady turbulence, $Q = \frac{\langle\varepsilon\rangle}{2K}$, and the characteristic time scale of the solution directly links to the forcing parameter that is $\tau_{\text{eddy}} = (3Q)^{-1}$.

2 Adaptive LES

By applying the wavelet thresholding filter [1] to the continuity (1) and Navier-Stokes (2) equations, one gets the following governing equations for the adaptive LES of linearly forced isotropic turbulence

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \, , \tag{4}$$

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + Q\overline{u}_i \, , \tag{5}$$

where $\overline{u}_i$ stands for the wavelet-filtered velocity field, and $\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \, \overline{u}_j$ represents the unknown SGS stress tensor. The corresponding evolution equation for the volume-averaged resolved energy becomes

$$\frac{dK_{\text{res}}}{dt} = -\langle \varepsilon_{\text{res}} \rangle - \langle \Pi \rangle + 2QK_{\text{res}} \, , \tag{6}$$

where K_{res} is the mean resolved kinetic-energy and ε_{res} the resolved turbulent dissipation. In the above equation the term Π stands for the SGS dissipation, which represents the energy transferred from the energetic resolved scales towards the unresolved background flow.

For a given forcing coefficient, as the resolved kinetic-energy practically coincides with that one of the corresponding unfiltered solution ($K_{\text{res}} \cong K$), the SGS model should provide the right amount of energy dissipation in order to have $\langle \varepsilon_{\text{res}} \rangle + \langle \Pi \rangle \cong \langle \varepsilon \rangle$ in order to match results from a direct numerical simulation (DNS).

In this study the localized dynamic kinetic-energy model (LDKM), proposed in [3], is exploited to close the filtered momentum equations (5). The modeling procedure involves the numerical solution of an additional evolution equation for the SGS kinetic-energy. The corresponding dynamic procedure involves the definition of two parameters, one for the eddy-viscosity model, the other one for the SGS energy dissipation model. A Bardina-like approach is exploited here for the dynamical evaluation of both the parameters [3].

The filtered momentum and the SGS energy equation are solved by means of the adaptive wavelet collocation methodology [8, 9, 10].

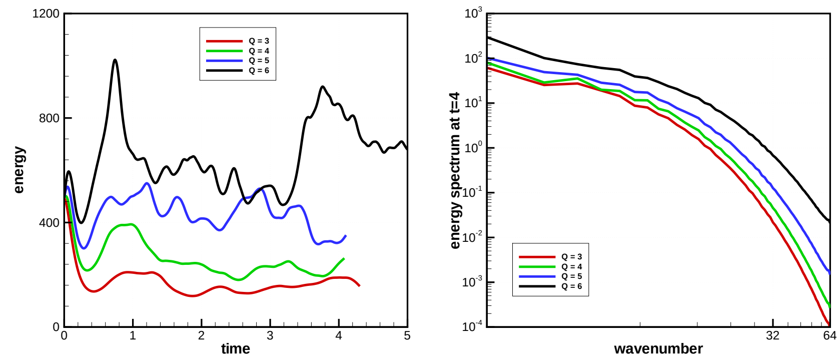

Fig. 1. DNS solution: energy evolution and averaged energy spectra for different forcing coefficients

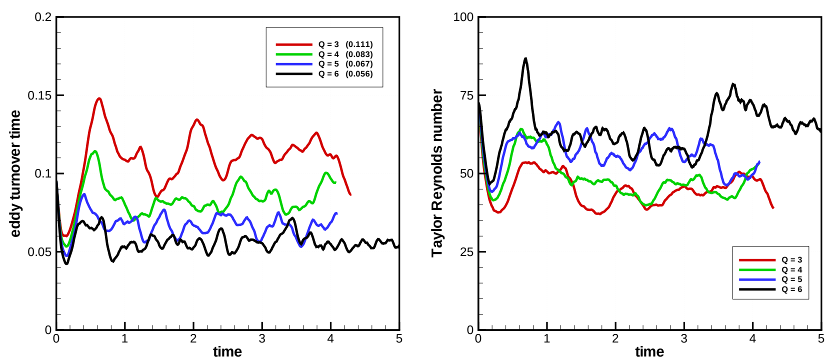

Fig. 2. DNS solution: eddy turnover time and Taylor Reynolds-number for different forcing coefficients

3 Numerical experiments

The numerical experiments were conducted starting with the initial velocity field obtained by wavelet-filtering of a pseudo-spectral solution at moderate Reynolds-number $Re_\lambda \cong 72$ [6]. In order to build a reference DNS solution, a fully de-aliased pseudo-spectral simulation with 192^3 Fourier modes was conducted with the same spectral code. In Fig. 1 the DNS kinetic-energy evolution for different values of the forcing coefficient is reported, along with the corresponding time-averaged energy spectra. The DNS solution is further analyzed by plotting in Fig. 2 the time evolutions of the eddy turnover time and the Taylor Reynolds-number. The different numerical solutions show a characteristic time-scale that is in agreement with the theoretical values corresponding

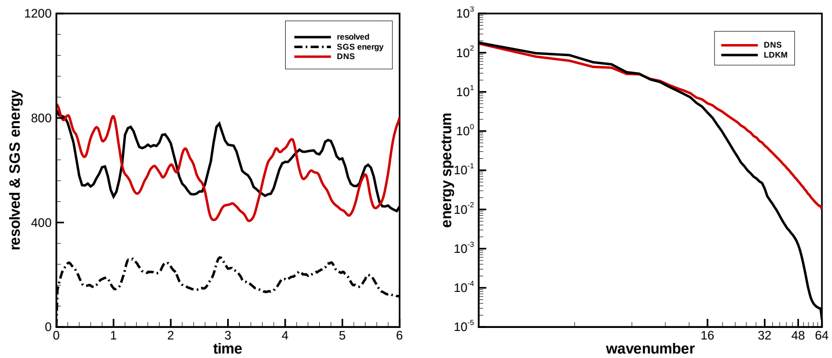

Fig. 3. Adaptive LES solution: energy evolution and averaged energy spectrum

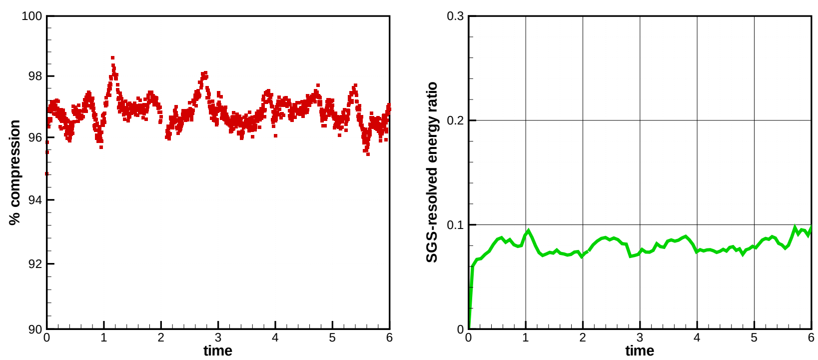

Fig. 4. Adaptive LES solution: grid compression (percentage) and SGS to resolved energy ratio

to the prescribed forcing coefficients. For instance, it holds $\tau_{\text{eddy}} \cong 0.056$ for $Q = 6$.

For the adaptive LES experiments the value $Q = 6$ was prescribed for the forcing parameter. The simulation was conducted for more than one hundred eddy-turnover times. The SCALES solution was obtained using a maximum resolution of 256^3 grid points. However, owing to the choice of an appropriate threshold for the wavelet-filtering procedure, only a very low fraction of these points was actually used during the simulation. The wavelet-based solution is able to reproduce the DNS energy level, as demonstrated in Fig. 3 (left side). More importantly, the energetic small-scale motions are resolved to some extent as it clearly appears by inspection of time-averaged energy spectra illustrated on the right side of the same figure. The energy content of the flow is mostly captured by a limited number of wavelets. In fact, the

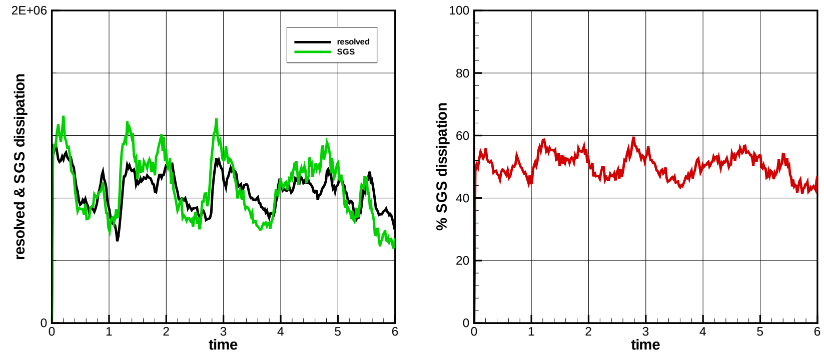

Fig. 5. Adaptive LES solution: SGS dissipation and percentage of SGS dissipation

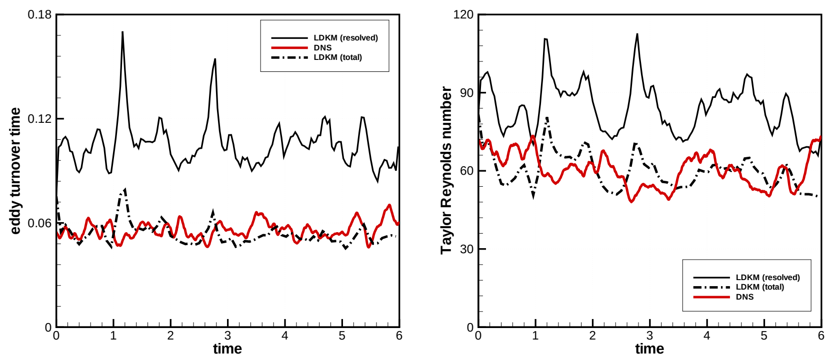

Fig. 6. Adaptive LES solution: eddy turnover time and Taylor Reynolds-number

SCALES solution uses in average only 3% of the available 256^3 wavelets, as demonstrated in Fig. 4, where the grid compression (that is the number of filtered-out wavelets with respect to total ones) is reported.

On the right side of Fig. 4, the ratio between SGS energy and resolved energy is plotted. This quantity represents a measure of the turbulent resolution achieved by the LES solution. The results show that the turbulence resolution is maintained during the simulation. Moreover, the SGS dissipation provided by the LDKM procedure is 50% of the total energy dissipation for the present moderate Reynolds-number flow, as reported in Fig. 5.

Finally, let us stress the distinctive feature of the SCALES method, namely the ability to locally refine the computational mesh in flow regions where the SGS model does not provide adequate dissipation, thus, resulting in a partial resolution of small dissipative scales. As a result, a direct comparison with DNS solution is more meaningful than for classical non-adaptive LES. However, the statistics that involve the level of energy dissipation must be "corrected" by expressing the definitions in terms of total dissipation rather than resolved one. For instance, since K_{res} practically coincides with K, the eddy turnover time is rewritten as

$$\tau_{\mathrm{eddy}} = \frac{(2/3)K}{\langle \varepsilon_{\mathrm{res}} \rangle + \langle \Pi \rangle}, \tag{7}$$

while, for the Taylor microscale definition (see, for instance, [7]), it holds

$$\lambda^2 = \frac{10\nu K}{\langle \varepsilon_{\mathrm{res}} \rangle + \langle \Pi \rangle}. \tag{8}$$

The evolutions of these two quantities are illustrated in Fig. 6. Note how the LES results, once corrected, match almost perfectly DNS data.

In conclusion, the SCALES method supplied with the LDKM modeling procedure is able to reproduce DNS low-order statistics for statistically steady turbulence, with a high grid compression over long-time integrations. The present approach appears very promising for the simulation of high Reynolds-number turbulent flows with reduced computational cost with respect to classical non-adaptive methods.

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Acknowledgments

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