Progress in the Development of Stochastic Coherent Adaptive LES Methodology

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1 Introduction

Stochastic Coherent Adaptive Large Eddy Simulation (SCALES) methodology, originally proposed by Goldstein and Vasilyev [1], is an extension of the Large Eddy Simulation (LES) approach that uses a wavelet filter-based dynamic grid adaptation strategy to solve for the most *energetic* coherent structures in a turbulent flow field, while modeling the effect of the less energetic eddies. Despite some similarities, SCALES is drastically different from classical LES, mainly in its ability to couple grid evolution and flow physics. This direct coupling is achieved by resolving and "tracking" on a space-time adaptive mesh physically important flow structures, whose evolution is influenced by a subgrid scale (SGS) model. This coupling provides a unique feedback mechanism that allows numerics to compensate for the inadequacies of the SGS model: mesh is automatically refined in flow regions, where the SGS model does not provide adequate dissipation, and coarsened in regions, where the model is over-dissipative.

2 SCALES Methodology

Let us very briefly outline the main features of the wavelet thresholding filter used for the coherent vortex identification and grid adaptation. More details can be found, for instance, in [2]. A velocity field $u_i(\mathbf{x})$ can be represented in terms of wavelet basis functions as

$$u_i(\mathbf{x}) = \sum_{\mathbf{l}\in\mathcal{L}^0} c^0_{\mathbf{l}}\phi^0_{\mathbf{l}}(\mathbf{x}) + \sum_{j=0}^{+\infty}\sum_{\mu=1}^{2^n-1}\sum_{\mathbf{k}\in\mathcal{K}^{\mu,j}} d^{\mu,j}_{\mathbf{k}}\psi^{\mu,j}_{\mathbf{k}}(\mathbf{x}), \tag{1}$$

where $\phi^0_{\mathbf{l}}$ and $\psi^{\mu,j}_{\mathbf{k}}$ are n-dimensional scaling functions and wavelets of different families and levels of resolution, indexed with μ and j, respectively. One way to think of a wavelet decomposition is as a multi-level or multi-resolution representation of u_i, where each level of resolution j (except the coarsest one) consists of families of wavelets $\psi^{\mu,j}_{\mathbf{k}}$ having the same scale, but located at different positions. Scaling function coefficients represent the averaged values of the field, while the wavelet coefficients represent the details of the field at different scales.

Wavelet filtering is performed in the wavelet coefficient space by discarding coefficients below a given threshold

$$\overline{u_i}^{\geq\epsilon}(\mathbf{x}) = \sum_{\mathbf{l}\in\mathcal{L}^0} c_{\mathbf{l}}^0 \phi_{\mathbf{l}}^0(\mathbf{x}) + \sum_{j=0}^{+\infty} \sum_{\mu=1}^{2^n-1} \sum_{\substack{\mathbf{k}\in\mathcal{K}^{\mu,j} \\ |d_{\mathbf{k}}^{\mu,j}|>\epsilon\|\mathbf{u}\|}} d_{\mathbf{k}}^{\mu,j} \psi_{\mathbf{k}}^{\mu,j}(\mathbf{x}), \tag{2}$$

where $\epsilon > 0$ stands for the non-dimensional (relative) threshold parameter, $\|\mathbf{u}\|$ is the (absolute) dimensional velocity scale. Note that the wavelet thresholding filter (2) is *uniquely* defined by the nondimensional threshold parameter ϵ and the velocity scale $\|\mathbf{u}\|$.

Depending on the threshold level, the effect of the discarded (unresolved or subgrid scale) modes on the evolution of energetic (resolved) flow structures can be insignificant or substantial. In the latter case, this effect must be modeled. When the threshold is chosen simply to satisfy numerical accuracy and the effect of the subgrid scales is negligible, we call this method Wavelet based Direct Numerical Simulation, or WDNS [3]. For larger values of the threshold parameter, it was shown that the unresolved SGS field is nearly Gaussian white noise by [1, 4], which, due to its decorrelation with the resolved modes, results in practically no SGS dissipation. Therefore, simulations with no SGS model capture turbulent energy cascade and were shown to recover low order and some high order DNS statistics. This regime is called Coherent Vortex Simulation [5]. Further increase in the wavelet threshold parameter results in discarding of too many modes so that the energy cascade is no longer captured, which necessitates the use of a SGS model. This regime corresponds to the SCALES approach [1].

The first step towards the construction of SGS models for SCALES was undertaken in [6], where a global dynamic Smagorinsky eddy viscosity model (GDM) based on the classical Germano procedure redefined in terms of two wavelet thresholding filters was developed. The main drawback of this formulation is the use of a global (spatially non-variable) model coefficient. In order to realize the full benefit of SCALES approach for highly non-homogenous flows in complex geometries two different families of local dynamic SGS models have been developed:

1. Lagrangian Dynamic Model (LDM) with path-line/tube averaging [7] and
2. Localized Dynamic Kinetic Energy based Models (LDKM) [8].

The LDM consists of Smagorinsky eddy-viscosity model with the dynamic procedure based on statistical moving filtered averages over the trajectory of a fluid particle, $\mathbf{x} = \mathbf{x}(t)$:

$$\overline{\mathcal{I}}_i(\mathbf{x},t) = \frac{1}{T}\int_{-\infty}^{t}\int_D e^{\frac{\tau-t}{T}} G\left(\mathbf{y}-\mathbf{x}(\tau)\right)\mathcal{I}_i\left(\mathbf{x}(\tau),\tau\right) d\tau d\mathbf{y},$$

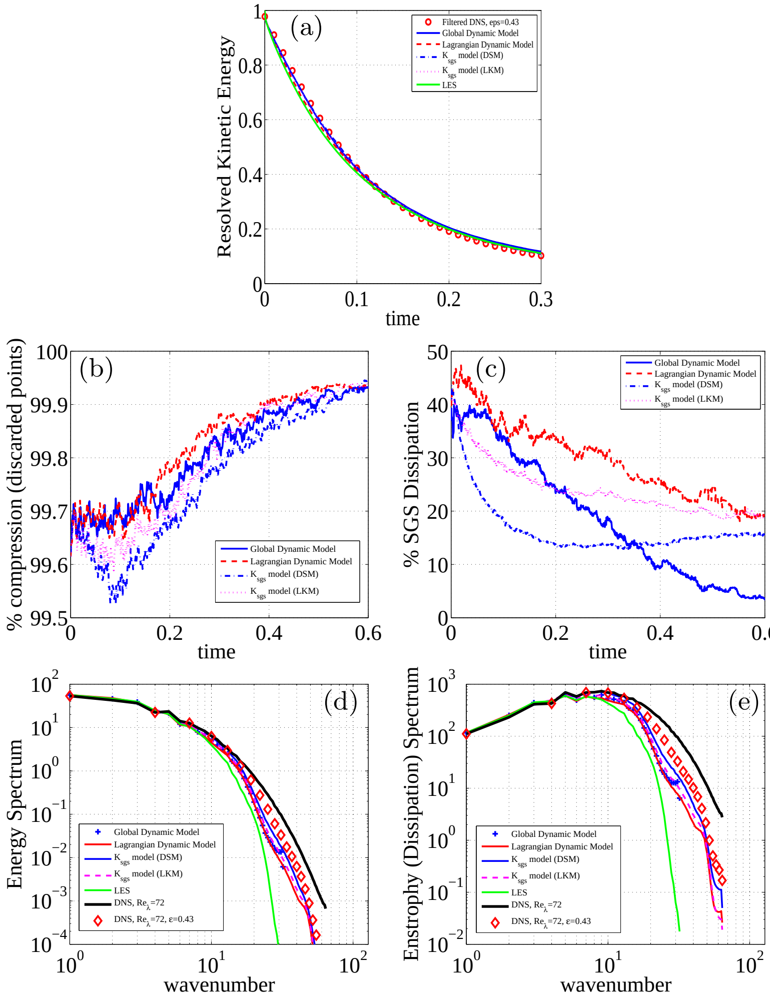

Fig. 1. Energy decay (a); Field compression (b); Percentage of SGS (modeled) dissipation (c); Energy (d) and Enstrophy (e) density spectra at $t = 0.08$ for SCALES of decaying homogeneous turbulence at $\mathrm{Re}_\lambda = 72$ with Global Dynamic (———), Local Lagrangian Dynamic (– – – –), Kinetic Energy Dynamic Structure (—·—), and Localized Dynamic Kinetic Energy (·········) models, reference LES with global dynamic model (———), DNS (———), and wavelet filtered DNS (∘).

where $G(\boldsymbol{\xi},\ \mathbf{x})$ is the local low-pass filter moving together with fluid particle and $\mathcal{I}_i$ $(i = 1, 2)$ are instantaneous quantities used by the local dynamic model. The *Localized Dynamic Kinetic Energy Models*, of both eddy-viscosity [9] and non eddy-viscosity [10] types, involve the solution of an evolution equation for the additional field variable that represents the kinetic energy associated with the unresolved motions. This way, the energy transfer between resolved and residual flow structures is explicitly taken into account by the modeling procedure without an equilibrium assumption of the classical dynamic Smagorinsky approach.

To demonstrate the efficiency and accuracy of the SCALES approach, the results of the simulation of incompressible isotropic freely decaying turbulence using different SGS models are shown in Fig. 1. The initial velocity field is a realization of a statistically stationary turbulent flow at $Re_\lambda = 72$ (λ being the Taylor microscale) as provided by a pseudo-spectral DNS [11]. The unique feature of the SCALES approach, namely the coupling of modeled SGS dissipation and the computational mesh, is illustrated in Fig. 1(b-c): more grid points are used for models with lower levels of SGS dissipation. In other words, the SCALES approach compensates for inadequate SGS dissipation by increasing the local resolution and, hence, the level of resolved viscous dissipation. For example, the decrease of SGS dissipation in the Dynamic Structure Model (DSM) results in the decrease of grid compression and the increase of resolved energy dissipation.

Another crucial strength of the SCALES approach is its ability to match the DNS energy and enstrophy density spectra (illustrated in Fig. 1(d-e)) up to the dissipative wavenumber range using very few degrees of freedom. It is important to emphasize that for all localized models the close match is achieved using less than 0.4% of the total non-adaptive nodes required for a DNS with the same wavelet solver (Fig. 1(b)). To highlight the significance of such a close match, it is interesting to compare these results with those of an LES with the global dynamic Smagorinsky model. Despite the fact that LES uses almost four times the number of modes (1.56%), it fails to capture the small-scale features of the spectrum.

References

1. Goldstein DE, Vasilyev OV (2004) Phys Fluids 16:2497–2513
2. Daubechies I (1992) Ten Lectures on Wavelets, SIAM, Philadelphia
3. Vasilyev OV (2003) Int J Comp Fluid Dyn 17:151–168
4. Farge M, Pellegrino G, Schneider K (2001) Phys Rev Lett 87:054501-1–054501-4
5. Farge M, Schneider K, Kevlahan N-KR (1999) Phys Fluids 11:2187–2201
6. Goldstein D, Vasilyev O, Kevlahan, N-KR (2005) J of Turbulence 6:1–20
7. Vasilyev OV, De Stefano G, Goldstein, DE, Kevlahan, N-KR (2008) J of Turbulence 9:1–14
8. De Stefano G, Vasilyev OV, Goldstein DE (2008) Phys Fluids 20:045102.1–045102.14
9. Ghosal S, Lund TS, Moin P, Akselvoll K (1995) J Fluid Mech 286:229–255
10. Pomraning E, Rutland CJ (2002) AIAA Journal 40:689–701
11. De Stefano G, Goldstein DE, Vasilyev OV (2005) J Fluid Mech 525:263–274

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