Stochastic coherent adaptive LES with time-dependent thresholding

Giuliano De Stefano and Oleg V. Vasilyev

1 Introduction

With the recent development of wavelet-based techniques for computational fluid dynamics, adaptive numerical simulations of turbulent flows have become feasible [1]. Adaptive wavelet methods are based on wavelet threshold filtering that makes it possible to separate coherent energetic eddies, which are numerically simulated, from residual background flow structures that are filtered out. By varying the filter thresholding level different approaches with different fidelity are obtained: from the highly accurate wavelet-based direct numerical simulation (WDNS) that do not involve any model to the stochastic coherent adaptive large eddy simulation (SCALES) that needs a closure modeling procedure, e.g. [2].

The prescription of a given threshold for SCALES filtering directly links to the desired turbulence resolution. By decreasing the thresholding level more and more eddies with smaller energy are directly simulated so that the effect of unresolved background flow becomes less and less important and, correspondingly, the influence of the modeling procedure as well. To date, the SCALES method has been applied for both decaying and forced turbulence with a specified thresholding level that is based on *a-priori* studies, e.g. [3].

In this work, a new original strategy is presented for which the wavelet filtering threshold is not prescribed but determined on the fly for a given level of turbulence resolution. A completely adaptive eddy capturing approach that allows to perform variable fidelity numerical simulations of turbulence is proposed. The new method is based on wavelet filtering with time-dependent thresholding that automatically

Giuliano De Stefano
Dipartimento di Ingegneria Aerospaziale e Meccanica, Seconda Università di Napoli, I-81031 Aversa (Italy), e-mail: giuliano.destefano@unina2.it

Oleg V. Vasilyev
Department of Mechanical Engineering, University of Colorado, Boulder, CO 80309 (USA), e-mail: oleg.vasilyev@colorado.edu

adapts to the desired level or turbulence resolution. The SCALES governing equations supplemented by a localized dynamic energy-based closure model are solved by means of the adaptive wavelet collocation numerical method.

2 Time-dependent thresholding

The SCALES governing equations for incompressible turbulent flow are represented by the following wavelet-filtered Navier-Stokes equations

$$\frac{\partial \overline{u_i}^{>\varepsilon}}{\partial t} + \overline{u_j}^{>\varepsilon} \frac{\partial \overline{u_i}^{>\varepsilon}}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \overline{p}^{>\varepsilon}}{\partial x_i} + \nu \frac{\partial^2 \overline{u_i}^{>\varepsilon}}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \,, \tag{1}$$

where $\overline{u_i}^{>\varepsilon}$ stands for the wavelet-filtered velocity field while τ_{ij} are the subgrid-scale (SGS) stresses to be modeled [2]. As usual for large eddy simulations, the SGS model is mainly required to provide the right energy dissipation in order to approximate the net effect of unresolved background flow upon the dynamics of resolved eddies. The amount of energy dissipation to be modeled clearly depends upon the wavelet filtering threshold that is used in (1) and, thus, the ratio of SGS to total (resolved plus modeled) dissipation can be practically used as a measure of the turbulence resolution for the SCALES solution. The above ratio is defined as

$$\mathscr{R}(t) = \frac{\mathscr{D}_{\mathrm{sgs}}}{\mathscr{D}_{\mathrm{res}} + \mathscr{D}_{\mathrm{sgs}}} \,, \tag{2}$$

where $\mathscr{D}_{\mathrm{res}} = 2\nu\langle \overline{S}_{ij}^{>\varepsilon} \overline{S}_{ij}^{>\varepsilon} \rangle$ represents the volume-averaged resolved viscous dissipation and $\mathscr{D}_{\mathrm{sgs}} = \langle -\tau_{ij}^{*} \overline{S}_{ij}^{>\varepsilon} \rangle$ stands for the volume-averaged SGS dissipation, with $0 < \mathscr{R} < 1$. This way, instead of using a prescribed wavelet threshold based upon subjective considerations, a goal value $\mathscr{R}_0$ for the resolved flow parameter (2) can be actually assigned. Correspondingly, the thresholding level ε is evaluated as a time-dependent function according to the simple evolution equation

$$\frac{d\varepsilon}{dt} = -\left(\mathscr{R} - \mathscr{R}_0\right) \frac{\varepsilon}{\tau_\varepsilon}, \tag{3}$$

where τ_ε is a time constant that can be linked to the eddy turnover time of the turbulence. This equation is explicitly discretized and solved step-by-step in time along with the SCALES governing equations. When the grid is too coarse and the SGS dissipation is higher than the goal ($\mathscr{R} > \mathscr{R}_0$), the threshold value is decreased, which leads to the mesh refining. On the contrary, when the turbulence is over-resolved and the SGS dissipation is lower than the goal ($\mathscr{R} < \mathscr{R}_0$), the threshold value is increased, which leads to the mesh coarsening.

With the adoption of the above time-dependent wavelet-filtering threshold, the dynamically adaptive nature of the SCALES method is fully exploited. In fact, the

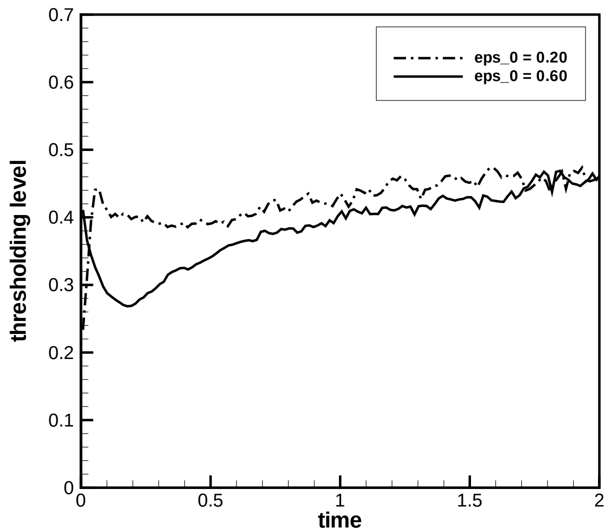

Fig. 1 Thresholding level evolution for two different initial conditions

proposed methodology can be referred to as a “complete” LES approach to the numerical simulation of homogeneous turbulence [4].

3 Results

In this paper, some preliminary results are presented for SCALES of linearly forced homogeneous turbulence at $Re_\lambda = 60$. The filtered governing equations (1) are solved by adding at the right-hand-side a forcing term proportional to the velocity field, viz. $Q\overline{u_i}^{>\varepsilon}$ [5]. The numerical experiments are carried out with $Q = 5.2$ for the sake of comparison with [3].

Given the desired turbulence resolution that corresponds to the prescribed goal value $\mathscr{R}_0 = 0.40$ for the ratio of SGS to total dissipation (2), the solution is initialized with two very different initial thresholding levels that are $\varepsilon_0 = 0.60$ and 0.20, respectively. When starting with a too high threshold like $\varepsilon_0 = 0.60$ the resolution is initially too low and the SGS dissipation provided by the model is too high so that the threshold automatically tends to decrease. The solution with a very low initial threshold like $\varepsilon_0 = 0.20$ shows the opposite behavior because the initial resolution is too high with respect to the goal. In Figure 1 the evolution of the wavelet thresholding level is reported for both solutions for a time corresponding to just two non-dimensional time units to make it possible to examine the short initial transient

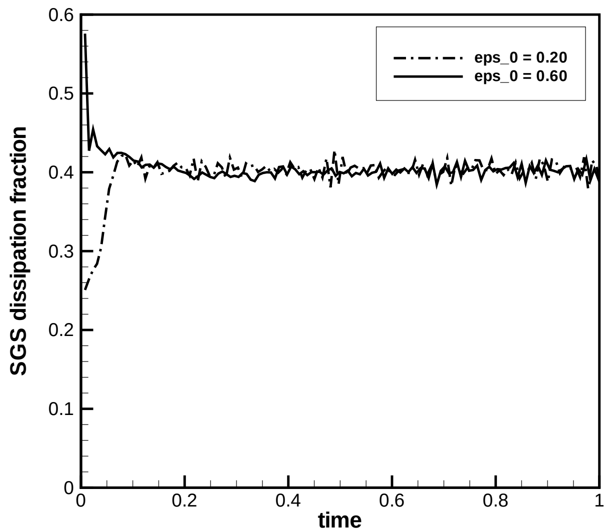

Fig. 2 Fraction of modeled dissipation evolution for two different initial conditions

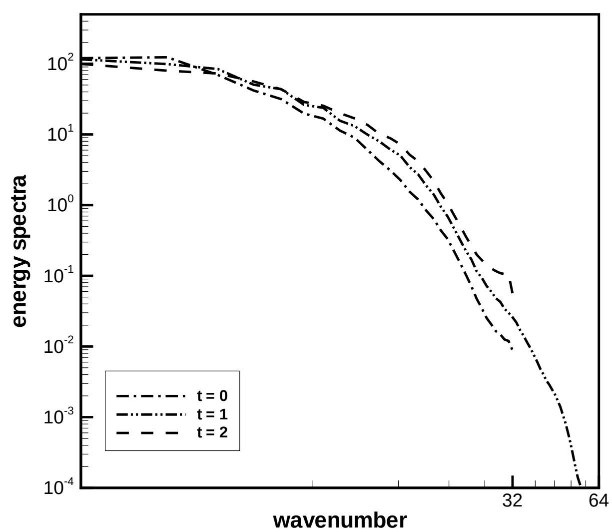

Fig. 3 Energy spectra evolution for $\varepsilon_0 = 0.60$

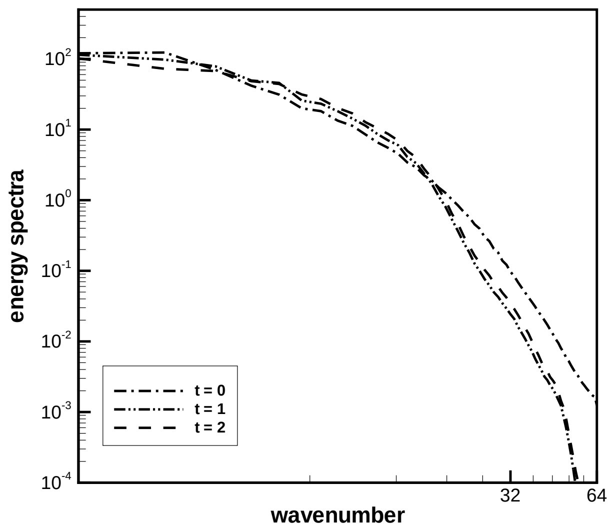

Fig. 4 Energy spectra evolution for $\varepsilon_0 = 0.20$

in some detail. The SCALES solution tends to achieve the prescribed resolution, regardless of the initial thresholding level, as illustrated on Figure 2, where the time history of the SGS to total dissipation ratio is given for one non-dimensional time unit. For both initial conditions the transient time is very short, less than half the eddy turnover time of the turbulence.

The transition from the initial resolution towards the desired one is well represented by the corresponding evolution of the energy spectra, shown in Figures 3 and 4, for the two different initial levels, respectively. For $\varepsilon_0 = 0.60$, since the turbulence resolution is initially insufficient, the energy associated to smaller scales increases as time passes, while the energy spectrum adjusts to the expected shape. On the other hand, for $\varepsilon_0 = 0.20$, where the turbulence is initially over-resolved, the energy associated to smaller scales reduces in time. Note that the energy spectra after two time-units practically correspond to the same solution. In practice, after the initial transient, the two solutions can be thought as two different realizations of the same SCALES solution with the prescribed resolution. By looking at Figure 5, where the different contributions to total dissipation are reported for $\varepsilon_0 = 0.20$, one can see that the constraint $\mathscr{R} \approx 0.40$ is maintained over long time integration, while the total dissipation is in good agreement with the reference spectral solution (SDNS).

As a conclusion, the use of the present time-dependent thresholding strategy makes it possible to achieve the objective separation of resolved energetic and unresolved flow structures in SCALES, for a given desired turbulence resolution. That is particularly promising for the adaptive simulation of complex unsteady turbulent

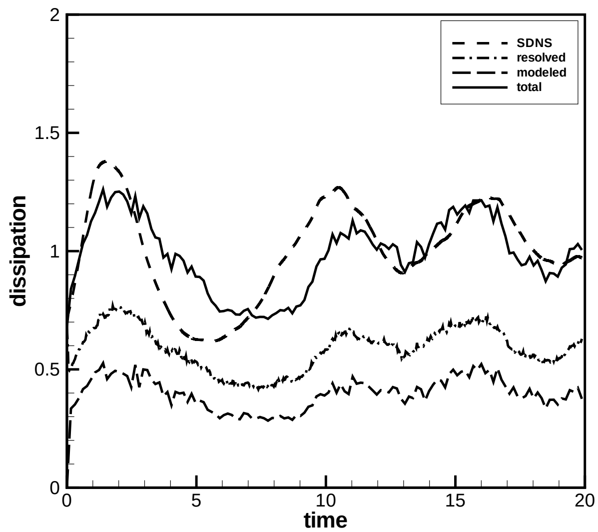

Fig. 5 Resolved, modeled and total dissipation evolution for $\varepsilon_0 = 0.20$, compared to spectral reference solution.

flows, where the energetic level of the dominant flow structures can significantly vary in time so that the desired level of turbulence resolution could not be conserved by using a constant wavelet thresholding level.

Acknowledgements This work was supported by the United States National Science Foundation (NSF) under grant No. CBET-0756046 and the United States Department of Energy (DOE) under grant No. DE-FG02-07ER64468.

References

1. Schneider K, Vasilyev O V (2010) Wavelet methods in computational fluid dynamics. Annu Rev Fluid Mechanics 42:473–503
2. De Stefano G, Vasilyev O V, Goldstein D E (2008) Localized dynamic kinetic-energy-based models for stochastic coherent adaptive large eddy simulation. Phys Fluids 20:045102.1–045102.14
3. De Stefano G, Vasilyev O V (2010) Stochastic coherent adaptive large eddy simulation of forced isotropic turbulence. J Fluid Mech 646:453–470
4. Pope S B (2004) Ten questions concerning the large-eddy simulation of turbulent flows. New J of Physics 6:1–24
5. Lundgren T S (2003) Linearly forced isotropic turbulence. Annual Research Briefs CTR 2003:461–473