Spatially Variable Thresholding for Stochastic Coherent Adaptive LES

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1 Introduction

The properties of wavelet transform, viz. the ability to identify and efficiently represent temporal/spatial coherent flow structures, self-adaptiveness, and de-noising, have made them attractive candidates for constructing multi-resolution variable fidelity schemes for simulations of turbulence [10]. Stochastic Coherent Adaptive Large Eddy Simulation (SCALES) [6] is the most recent wavelet-based methodology for numerical simulations of turbulent flows that resolves energy containing turbulent motions using wavelet multi-resolution decomposition and self-adaptivity. In this technique, the extraction of the most energetic structures is achieved using wavelet thresholding filter with a priori prescribed threshold level.

SCALES is a methodology, which inherits the advantages of both Coherent Vortex Simulations (CVS) [5] and Large Eddy Simulation (LES) while overcoming the shortcomings of both. Unlike coherent/incoherent and large/small structures decomposition in CVS and LES respectively, in SCALES the separation is between more and less energetic structures. Therefore, unlike CVS, the effect of background flow can not be ignored and needs to be modeled similarly to LES. As a result of using SGS models, the number of degrees-of-freedom is smaller than CVS and consequently a higher grid-compression can be achieved.

Ever since the emergence of the wavelet-based multi-resolution schemes for simulations of turbulence, there has been a major limitation for all wavelet-based techniques: the use of a priori defined global (both in space and time) thresholding-

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parameter. In this work the robustness of the SCALES approach is further improved by exploring the spatially and temporally variable thresholding strategy, which allows more efficient representation of intermittent flow structures.

2 Stochastic Coherent Adaptive Large Eddy Simulation

To address the shortcomings of LES and CVS, SCALES uses a wavelet thresholding filter to dynamically resolve and track the deterministic most energetic coherent structures while the effect of less energetic unresolved modes is modeled. The unresolved less energetic structures have been shown to be composed of a minority of deterministic coherent modes that dominate the total SGS dissipation and a majority of stochastic incoherent modes that, due to their decorrelation with the resolved modes, add little to the total SGS dissipation [6, 1]. In the current implementation, similar to the classical LES, only the effect of coherent part of the SGS modes is modeled using deterministic SGS models. The use of stochastic SGS models to capture the effect of the incoherent SGS modes will be the subject of future investigations. The most significant feature of SCALES is the coupling of modeled SGS dissipation and the computational mesh: more grid points (active wavelets) are used for SGS models with lower levels of SGS dissipation. In other words, the SCALES approach compensates for inadequate SGS dissipation by automatically increasing the local resolution and, hence, the level of resolved viscous dissipation. Another noticeable feature of the SCALES method is its ability to match the DNS energy spectra up to the dissipative wavenumber range using very few degrees of freedom.

2.1 Wavelet Thresholding Filter

In the wavelet-based approach to the numerical simulation of turbulence the separation between resolved energetic structures and unresolved residual flow is obtained through nonlinear multi-resolution wavelet threshold filtering (WTF). The filtering procedure is accomplished by applying the wavelet-transform to the unfiltered velocity field, discarding the wavelet coefficients below a given threshold (ε) and transforming back to the physical space. This results in decomposition of the turbulent velocity field into two different parts: a coherent more energetic velocity field and a residual less energetic coherent/incoherent one, i.e., $u_i = \overline{u}_i^{>\varepsilon} + u_i'$, where $\overline{u}_i^{>\varepsilon}$ stands for the wavelet-filtered velocity at level ε

$$\overline{u}_i^{>\varepsilon}(\mathbf{x}) = \sum_{\mathbf{l}\in\mathscr{L}^0} c_{\mathbf{l}}^0 \phi_{\mathbf{l}}^0(\mathbf{x}) + \sum_{j=0}^{+\infty} \sum_{\mu=1}^{2^n-1} \sum_{\substack{\mathbf{k}\,\in\,\mathscr{K}^{\mu,j} \\ |d_{\mathbf{k}}^{\mu,j}| > \varepsilon \|u_i\|_{\mathrm{WTF}}}} d_{\mathbf{k}}^{\mu,j} \psi_{\mathbf{k}}^{\mu,j}(\mathbf{x}), \tag{1}$$

where $\psi_{\mathbf{k}}^{\mu,j}$ are wavelets of family μ at j level of resolution, $d_{\mathbf{k}}^{j}$ are the coefficients of the wavelet decomposition, and $\phi_{\mathbf{l}}^{0}$ are scaling functions at zero level of resolution.

The key role in the wavelet-filter definition is clearly played by the non-dimensional relative thresholding level ε that explicitly defines the relative energy level of the eddies that are resolved and, consequently, controls the importance of the influence of the residual field on the dynamics of the resolved motions. In this work we explore the use of spatially and temporary varying thresholding level ε, which follows the evolution of the turbulent velocity field.

2.2 *Wavelet-Filtered Navier-Stokes Equations*

By filtering the Navier-Stokes equations, the following SCALES equations that govern the evolution of coherent energetic structures are obtained:

$$\partial_{x_i}\overline{u}_i^{>\varepsilon} = 0, \tag{2}$$

$$\partial_t \overline{u}_i^{>\varepsilon} + \overline{u}_j^{>\varepsilon}\partial_{x_j}\overline{u}_i^{>\varepsilon} = -\partial_{x_i}\overline{P}^{>\varepsilon} + \nu\partial^2_{x_j x_j}\overline{u}_i^{>\varepsilon} - \partial_{x_j}\tau^*_{ij} + Q\overline{u}_i^{>\varepsilon}, \tag{3}$$

where $\tau_{ij} = \overline{u_i u_j}^{>\varepsilon} - \overline{u}_i^{>\varepsilon}\overline{u}_j^{>\varepsilon}$ are the unresolved "SGS stresses" that need to be modeled, $Q\overline{u}_i^{>\varepsilon}$ is the linear forcing term [8], which is applied in the physical space over the whole range of wavenumbers, and the superscript $(\cdot)^{>\varepsilon}$ denotes wavelet filtered quantities. The SCALES equations are similar to the LES ones with the exception that the nonlinear multiscale band-pass wavelet filter, which depends on instantaneous flow realization, is used. The unresolved SGS stresses represent the effect of "unresolved less energetic deterministic coherent and stochastic incoherent eddies" on the "resolved more energetic coherent structures". In this study the localized kinetic-energy-based model [4] is exploited to close the filtered momentum equations. The SCALES methodology involving both the filtered momentum and the SGS energy equations is implemented by means of the adaptive wavelet collocation method [11].

3 Spatially Variable Thresholding

Previous studies, e.g. [7], demonstrated that in SCALES, the SGS dissipation is proportional to ε^2; therefore, one can enhance SCALES by exploiting spatially-varying ε based on local SGS dissipation $\Pi = -\tau^*_{ij}\overline{S}_{ij}^{>\varepsilon}$. This implies that rate of local-transfer of energy from energetic-resolved-eddies to unresolved-less-energetic structures can be controlled by varying the thresholding-factor. Therefore, the idea is to locally vary ε wherever Π deviates from a priori defined goal-value. A decrease of the thresholding level results in the local grid refinement with subsequent rise of the resolved viscous dissipation, while an increase of ε coarsens the mesh resulting in the growth of the local SGS dissipation. However, in order to vary ε in a physically consistent fashion, it should follow the local flow structures as they evolve in space and time. This necessitates the Lagrangian representation of ε, which is achieved using the Lagrangian Path-Line Diffusive Averaging approach [12]:

$$\partial_t \varepsilon + \overline{u}_j^{>\varepsilon} \partial_{x_j} \varepsilon = -\text{forcing}_{\text{term}} + \nu_\varepsilon \partial^2_{x_j x_j} \varepsilon. \tag{4}$$

For the sake of efficiency, instead of solving Eq. (4) for the evolution of ε, the linear-interpolation along characteristics, similar to the idea of Meneveau et al. [9], is performed

$$\frac{1}{\Delta t}\left[\varepsilon^{\text{new}}\left(\mathbf{x}, t+\Delta t\right) - \varepsilon^{\text{old}}\left(\mathbf{x} - \overline{\mathbf{u}}^{>\varepsilon}\Delta t, t\right)\right] = -\text{forcing}_{\text{term}}. \tag{5}$$

The use of linear interpolation results in sufficient numerical diffusion, thus, eliminating the need for explicit diffusion. The proposed spatially variable thresholding strategy ensures that the wavelet threshold is not *a priori* prescribed but determined on the fly by desired turbulence resolution. In this work two different mechanisms for the forcing term are studied:

FT1 The local fraction SGSD (FSGSD) is defined as $\frac{\Pi}{\varepsilon_{\text{res}}+\Pi}$, where $\varepsilon_{\text{res}} = 2\nu \overline{S}_{ij}^{>\varepsilon} \overline{S}_{ij}^{>\varepsilon}$ is the resolved viscous dissipation. The idea is to maintain FSGD at a "Goal" value which means retain Π at $\varepsilon_{\text{res}} \frac{\text{Goal}}{1-\text{Goal}}$. The first forcing type (FT1) is an attempt to implement this while normalizing the forcing term based on its time average:

$$\text{forcing}_{\text{term}} = \varepsilon^{\text{old}}\left(\mathbf{x} - \overline{\mathbf{u}}^{>\varepsilon}\Delta t, t\right) C_{\text{f}_\varepsilon} \frac{\Pi - \varepsilon_{\text{res}} \frac{\text{Goal}}{1-\text{Goal}}}{\text{TAF}}, \tag{6}$$

where TAF stands for the time average of the forcing, is the time average of $|\Pi - \varepsilon_{\text{res}} \frac{\text{Goal}}{1-\text{Goal}}|$. The forcing constant coefficient, C_{f_ε}, is intentionally set to 400 in order to make the time response of FT1 about three to four times faster than large eddy turnover time which is discussed in the next section.

FT2 In this approach, the variations of local-FSGSD is controlled directly based on the goal-value in conjunction with a relaxation time parameter (time-scale), τ_ε,

$$\text{forcing}_{\text{term}} = \varepsilon^{\text{old}}\left(\mathbf{x} - \overline{\mathbf{u}}^{>\varepsilon}\Delta t, t\right) \frac{1}{\tau_\varepsilon} \left(\frac{\Pi}{\varepsilon_{\text{res}} + \Pi} - \text{Goal}\right). \tag{7}$$

Following the time-varying threshold studies [2], a time-scale associated to the characteristic rate-of-strain is chosen: $\tau_\varepsilon^{-1} = \langle |\overline{S}_{ij}^{>\varepsilon}| \rangle$.

4 Results

The proposed methodology has been tested for linearly forced homogeneous turbulence [3] with linear forcing constant coefficient $Q = 6$ at $Re_\lambda \cong 72$ (Taylor micro-scale Reynolds number) on an adaptive grid with effective resolution 256^3. Figures 1(a,b) demonstrate the preliminary results of this implementation for three

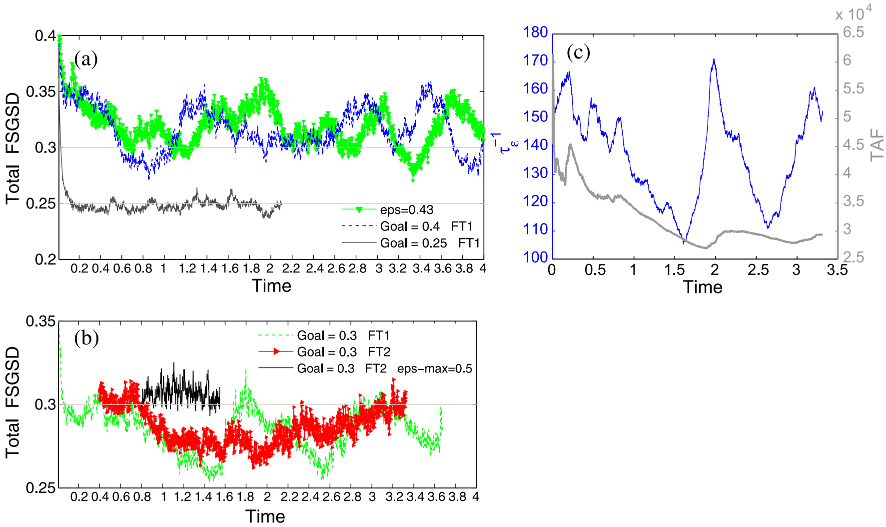

Fig. 1 Time-history of total fraction SGSD (a); Time-history of TAF and τ_ε^{-1} (b).

different goal-values $(0.25, 0.3, 0.4)$ for the local FSGSD with the upper and lower bound for epsilon set as 0.2 and 0.43 ($\varepsilon \in [0.2, 0.43]$). For reference, a constant-thresholding case of $\varepsilon = 0.43$ is included as well. The local and total FSGSD are defined respectively as $\frac{\Pi}{\varepsilon_{\text{res}}+\Pi}$ and $\frac{\langle\Pi\rangle}{\langle\varepsilon_{\text{res}}\rangle+\langle\Pi\rangle}$, where $\langle\Pi\rangle = \langle -\tau_{ij}^{*}\overline{S}_{ij}^{>\varepsilon}\rangle$ and $\langle\varepsilon_{\text{res}}\rangle = 2\nu\langle\overline{S}_{ij}^{>\varepsilon}\,\overline{S}_{ij}^{>\varepsilon}\rangle$ are respectively the volume-averaged SGS dissipation and the volume-averaged resolved viscous dissipation.

For the case of Goal=0.4, total-FSGSD never reaches the prescribed goal-value (0.4). The reason is that the total-FSGSD for the case of constant-thresholding with $\varepsilon = 0.43$ is smaller than 0.4 for most of the time. As a result, varying thresholding-factor with a "local-FSGSD goal-value" larger than the average FSGSD of constant-thresholding using the same ε and $\varepsilon_{\max}$ resulted in total-FSGSD, which was bellow the goal-value. Similarly to the previous case, the test case of the goal-value of 0.3 inherits a large-period oscillations due to capping ε at 0.43 level regardless of the forcing method. These oscillations are removed by increasing $\varepsilon_{\max}$ to 0.5. The success of this test with larger $\varepsilon_{\max}$ compared with the abovementioned two tests, where $\varepsilon_{\max}$ was 0.43, revealed that the upper bound of the interval for allowable threshold variations was not large enough to increase the SGS dissipation accordingly, which implies that with $\varepsilon \in [0.2, 0.43]$ flow was over resolved. Therefore, to achieve a FSGSD greater than the average of FSGSD corresponding to constant-thresholding at a certain $\varepsilon_{\text{constant-thresholding}}$, it is required to set the $\varepsilon_{\max} > \varepsilon_{\text{constant-thresholding}}$. This is further confirmed by considering the case with the goal set to 0.25, which illustrates how precisely the spatially variable thresholding methodology can maintain Π at a priori defined level. In addition, when $\varepsilon_{\max}$ is set up high enough, the SGS dissipation approaches the desired level within few eddy turnover times.

The time history of TAF and τ_ε^{-1} are shown in Fig. 1(c). The relaxation time parameter for FT2, τ_ε, is approximately one-tenth of the large eddy turnover time, $\tau_{\mathrm{eddy}} = \frac{u'^2}{\langle\varepsilon\rangle} = \frac{\frac{2}{3}K}{2KQ} = \frac{1}{3Q} = \frac{1}{18}$. While the relaxation time parameter for FT1, $C_{\mathrm{f}_\varepsilon}\mathrm{TAF}^{-1}$, is between one-third and one-fourth of τ_{eddy}. That is, FT2 has as much as 2 to 3 times faster response compared with FT1. This faster time response was able to partially recover the FSGSD. This improvement reveals the importance of very localized and fast mechanisms for the forcing term. The time-averaged term in FT1 destroys the localized Lagrangian nature of the algorithm; however, to smear out the effect of possible very localized FSGSD values, it is recommended to have some averaging mechanism. Hence, another approach, which is currently under investigation, is to track the forcing term itself within a Lagrangian frame so that the forcing term inherits the history of the flow evolution.

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