Computational Complexity of Adaptive LES with Variable Fidelity Model Refinement

Alireza Nejadmalayeri, Oleg V. Vasilyev, and Alexei Vezolainen

1 Introduction

Adaptive methods with both mesh and polynomial order refinements have been used extensively in computational fluid dynamics to achieve optimal accuracy with the minimal computational cost. However *hp*-refinement by itself is not sufficient for numerical simulation of turbulent flows of engineering interest. For instance, even for the extreme *hp*-refinement such as spectral DNS, the requirement to resolve Kolmogorov length-scale results in a daunting computational cost. LES is a much less expensive approach, but for high Reynolds number turbulent flows only large scales of the flow are captured and most of the dissipation is provided by the SGS model. The marginally resolved LES with small ratio of SGS and the total dissipations resolves more of the flow physics, but scales approximately the same as DNS in the limit of high Reynolds numbers, thus, making it impractical.

The quest for an appropriate criteria to identify the hierarchical change of scale for multi-scale simulations brought us to define the turbulence resolution in a broader perspective rather than the structure-size distinction as in classical LES, or the extreme case of resoling Kolmogorov length-scale as in DNS, or decomposing deterministic-coherent and stochastic-incoherent modes as in CVS, or even capturing more/less energetic structures as in SCALES. This new definition is based on the measure that is required in practical applications: "how much the flow-physics is modeled/resolved?" In essence, maintaining the percentage of modeled and resolved physically important quantity (e.g. turbulent kinetic energy, dissipation, or enstrophy) at a constant level implies that the methodology should exhibit synergistic transition between various levels of fidelity both in space and time as well as take advantage of spatial and temporal flow intermittency. This dynamically adaptive transition between different regimes necessitates the model adaptation.

Alireza Nejadmalayeri · Oleg V. Vasilyev · Alexei Vezolainen
Department of Mechanical Engineering, University of Colorado, Boulder, CO 80309, USA, e-mail: `{Alireza.Nejadmalayeri,Oleg.Vasilyev,Alexei.Vezolainen}@Colorado.edu`

Therefore, the missing component for turbulence simulation is not either *h*- or *p*-refinement but **coupling the model with the numerics**. That is to say, the selection and adjustments of the model fidelity, computational mesh, and/or the order of the numerical method need to be dynamically adaptive in order to take into account the intermittency of the turbulent flow filed. This new concept of *model*-refinement, which is named *m*-refinement [4], is utilized to perform Stochastic Coherent Adaptive Large Eddy Simulation (SCALES) of linearly-forced homogeneous turbulence at various fixed levels of turbulence resolution.

2 Computational Framework

The SCALES equations that govern evolution of coherent energetic structures are obtained by filtering the Navier-Stokes equations using wavelet-thresholding filter [2]. In this study, homogeneous turbulence with linear forcing [3] applied in the physical space over the whole range of wavenumbers [1] is investigated. The objective is to control the turbulence resolution, defined as the local fraction of SGS dissipation, $\mathscr{F}$: $\frac{\Pi}{\varepsilon_{\text{res}}+\Pi}$, where $\varepsilon_{\text{res}} = 2\nu \overline{S}_{ij}^{>\varepsilon} \overline{S}_{ij}^{>\varepsilon}$ is the resolved viscous dissipation and $\Pi = -\tau_{ij}^{*} \overline{S}_{ij}^{>\varepsilon}$ is the local SGS dissipation. This ratio of the SGS dissipation to the total dissipation, can be viewed as turbulence resolution since it indicates how much the flow is modeled/resolved. Therefore, by controlling $\mathscr{F}$, one can explicitly control the percentage of the flow physics that is desired to be resolved. To maintain the turbulence resolution at a constant level, the spatially variable thresholding methodology [4, 5] is used. This approach automatically provides the required numerical resolution and the model-fidelity in a space/time adaptive fashion based on a two-way coupling of numeric and physics. This method dynamically tracks the regions of interest in spatial and time space and not only adapts the grid but adjusts the model as well (*hm*-refinement).

In the classical non-adaptive explicitly filtered LES, the filter-width is priori user-defined based on which the resolution is determined; therefore, both the CFD engine (through the resolution) and the filtering mechanism (via the filter-width) depend on priori defined filter-width, which is not fine-tuned based on the results (Figure 1a). The original SCALES has improved this by its dynamically adaptive wavelet-filtering mechanisms via constantly adapting both the numerical grid and the filter-width based on the instantaneous flow field (Figure 1b). However, the wavelet thresholding filter (WTF) uses a priori user-defined threshold-level and as a result of filtering the velocity-filed with this constant threshold, the WTF is indeed imposing a feedback based on a constant level of resolved kinetic energy. This limitation has been recently removed [4] by means of constructing a fully adaptive wavelet thresholding filter [5]. The new *m*-refined SCALES requires a priori user-defined level of resolution/fidelity based on which the threshold is dynamically adapted in order to maintain the fidelity constant as user has requested. In original SCALES, the filtering mechanism is a function of the results (kinetic energy) and a constant threshold, while in the newly developed *m*-refined SCALES, threshold

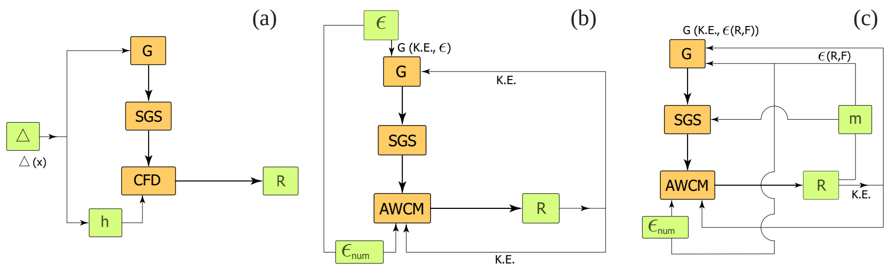

Fig. 1 Dependency diagram for (a) classical explicitly filtered LES, (b) original SCALES, and (c) the variable-fidelity SCALES. Notation: G – filter, R – results, m – model refinement, Δ – user provided LES filter width, ε – wavelet threshold for model adaptation, ε_{num} – wavelet threshold controlling the accuracy of the solution, F – an arbitrary dynamically important physical quantity to be controlled, e.g., $\mathscr{F}$.

itself is a function of the results (any physical quantity and not limited to kinetic energy) and the user-defined fidelity. All in all, *m*-SCALES integrates all components of the computational methodology including numerics, models, and physics altogether to construct a fully dynamically adaptive computational framework (Figure 1c).

3 Reynolds Number Scaling

To construct the Reynolds number scaling statistics, a series of simulations where the Reynolds number is progressively increased are performed. SCALES of linearly forced homogeneous turbulence [1] with linear forcing constant coefficient $Q = 20/3$ are performed in the computational domain of $[0, 2\pi]^3$ on a dynamically adaptive dyadic grid with effective nonadaptive resolutions of 256^3, 512^3, 1024^3, and 2048^3. These correspond to Taylor micro-scale Reynolds number of $Re_\lambda \cong 70, 120, 190, 320$ based on viscosities of $\nu = 0.09, 0.035, 0.015, 0.006$. These choices of viscosities are based on maintaining the ratio of Kolmogorov length-scale to the smallest grid-spacing constant, i.e., $\frac{\eta}{\Delta_{\min}} = 2$, to ensure the resolution required for a well-resolved DNS.

In order to study the influence of the fidelity of simulation on the Reynolds number scaling of SCALES, a series of simulations of different turbulence resolution is conducted. The different fidelity is achieved by using spatially variable thresholding approach [5] with different goal values of $\mathscr{F}$, namely $\mathscr{G} = 0.2, 0.25, 0.32, 0.4, 0.5$. It is observed that in the logarithmic scale the slope of Re_λ scaling of SCALES spatial modes at least up to 1024^3 remains approximately the same regardless of the level of turbulence resolution, Figure 2. In other words, the scaling exponent of constant-fidelity *m*-SCALES is nearly insensitive to the level of fidelity.

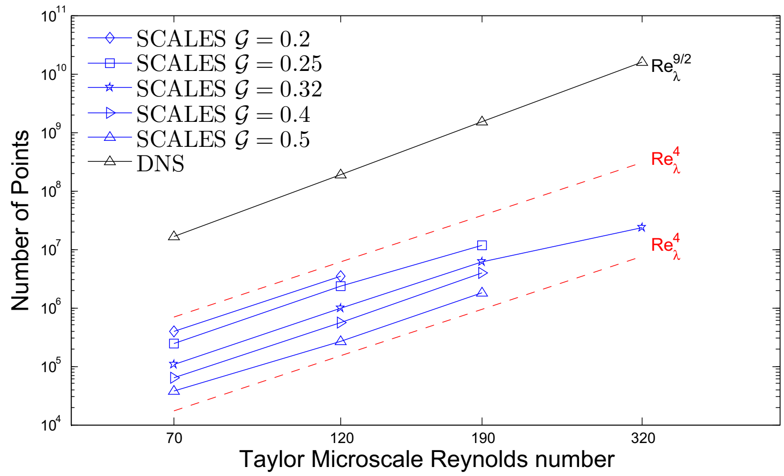

Fig. 2 Reynolds scaling of constant-dissipation SCALES at various goal values.

The scaling statistics presented by this work proves that the developed model can resolves more flow-physics phenomena yet with profoundly smaller number of spatial modes compared with marginally resolved LES. It is demonstrated that depending on what flow physics is desired to be captured, the same model and the same numerical method result in different Reynolds scaling. Therefore, the broad message of this computational complexity work is not to advertise the wavelet-based methods but to promote the physics-based turbulence modeling as a marriage of model and numerics. This *m*-refinement concept can be easily implemented into the existing adaptive Large Eddy Simulation methodologies in order to construct continuously variable fidelity LES. The possible implementation can be illustrated as Figure 3. Such an LES would include an additional feedback mechanism from the results (any physical quantity) in order to incorporate a filter-width/model adaptation preferably coupled with adaptation of the numerical resolution as well. Hence, both filtering-mechanism/model (via the filter-width) and CFD-engine/numerics (through the resolution) should be dynamically coupled based on any objective physics-based fidelity measure.

Acknowledgements This work was supported by the United States National Science Foundation (NSF) under grants No. CBET-0756046 and CBET-1236505. This support is gratefully acknowledged. Authors are also thankful for the computing time on the Janus supercomputer, which is supported by the National Science Foundation (award number CNS-0821794) and the University of Colorado Boulder. The Janus supercomputer is a joint effort of the University of Colorado Boulder, the University of Colorado Denver and the National Center for Atmospheric Research.

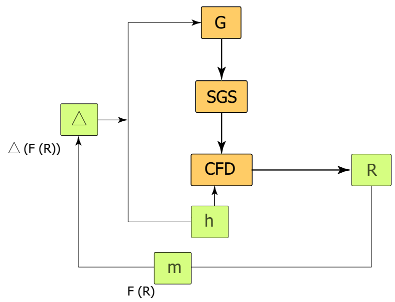

Fig. 3 Proposed dependency diagram for a possible variable-fidelity LES.

References

1. De Stefano, G., Vasilyev, O.V.: Stochastic coherent adaptive large eddy simulation of forced isotropic turbulence. *Journal of Fluid Mechanics* **646**, 453–470 (2010)
2. Goldstein, D.E., Vasilyev, O.V.: Stochastic coherent adaptive large eddy simulation method. *Physics of Fluids* **16(7)**, 2497–2513 (2004)
3. Lundgren, T.S.: Linearly forced isotropic turbulence. *Annual Research Briefs*, Center for Turbulence Research, NASA Ames/Stanford University, 461–473 (2003)
4. Nejadmalayeri, A.: Hierarchical multiscale adaptive variable fidelity wavelet-based turbulence modeling with Lagrangian spatially variable thresholding. *PhD Thesis*, University of Colorado Boulder, Boulder, CO, USA (2012)
5. Nejadmalayeri, A., Vasilyev, O.V., Vezolainen, A., De Stefano, G.: Spatially variable thresholding for stochastic coherent adaptive LES. In: Kuerten, H., Geurts, B., Armenio, V., Fröhlich, J. (eds.) Proceedings of Direct and Large-Eddy Simulation Workshop 8: Eindhoven University of Technology, the Netherlands, July 7-9, 2010, pp. 95–100. Springer (2011)