Adaptive direct numerical simulation with spatially-anisotropic wavelet-based refinement

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1 Methodology

In the wavelet-based adaptive multi-resolution approach to the numerical simulation of turbulent flows, the separation between resolved energetic structures and unresolved flow motions is achieved through the application of a wavelet thresholding filter. For very small threshold values, the effect of residual motions upon the resolved flow dynamics can be completely neglected, which leads to the adaptive Wavelet-based Direct Numerical Simulation (W-DNS) approach. The method allows for the direct solution of the organized flow motions, which consist of both large-scale and small-scale coherent structures with non-negligible energy, e.g. [5, 7].

Due to the ability to identify and efficiently represent energetic dynamically important turbulent eddies, the method has been proven reliable and effective for the simulation of unsteady external flows [6, 8]. However, when dealing with flow around obstacles, one of the main challenges of the traditional W-DNS approach is the requirement of high spatial grid resolution in both the near-wall and the wake regions. Furthermore, when the presence of the obstacle is mimicked by means of

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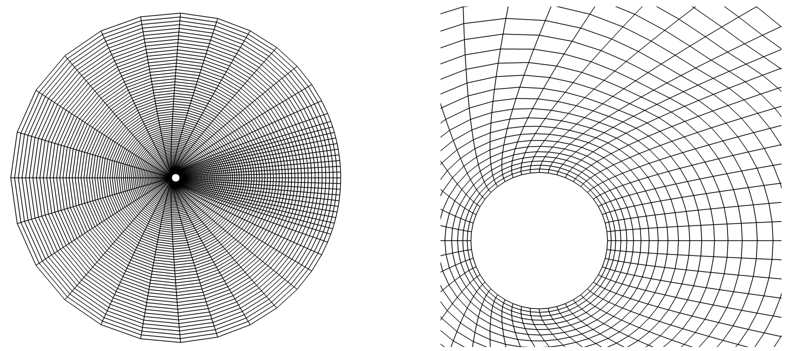


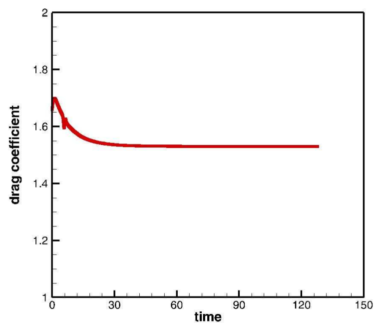
Fig. 1 Example of spatially non-adaptive anisotropic two-dimensional mesh: (left) global and (right) close-up views of the near wake zone.

the volume-penalization technique, e.g. [4, 11], for the accurate estimation of the wall stresses, and thus the aerodynamic loads, the thin boundary layer inside of the porous region representing the obstacle also needs to be accurately resolved. The isotropic mesh refinement, which is characteristic of classical wavelet-based methods, results in the simultaneous grid refinement in all directions, irrespective of the actual requirement, even in situations where just one particular direction is involved. This represents a strong constraint of realizability and limits the application of W-DNS to moderate Reynolds number flows. In this study, a novel approach that overcomes this limitation is exploited.

The new W-DNS methodology is developed by making use of the adaptive wavelet transform on curvilinear grids recently introduced in [3]. The traditional wavelet methods suffer from the "curse of anisotropy," due to the isotropic wavelet refinement procedure and the inability to deal with mesh elements with spatially varying aspect ratio and orientation. The new approach utilizes a spatially-anisotropic wavelet-based refinement, which takes advantage of coordinate mapping between the physical space, where the curvilinear numerical mesh is defined, and the computational space, where the adaptive rectilinear wavelet collocation grid is used. The new approach permits to construct dynamically adaptive body-fitted meshes, thus avoiding the use of the volume penalization technique.

2 Numerical experiments

In this work, the flow around a circular cylinder is considered as a prototype for wall-bounded external flows. The curvilinear approach makes it possible to construct stretched body-fitted O-meshes, differently from [2], where the same flow was simulated by exploiting uniform rectilinear meshes in conjunction with a volume penalization approach. Moreover, the introduction of a suitable mapping between computational and physical spaces allows for a particular arrangement of the grid points that permits a more efficient representation of both the wall and the wake



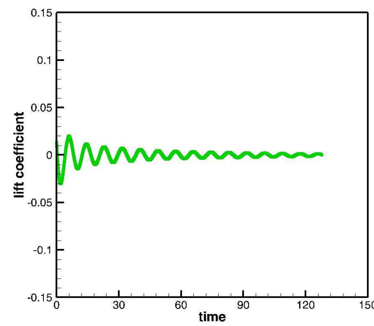


Fig. 2 Two-dimensional cylinder flow at $Re_D = 40$: time histories of (left) the drag and (right) the lift force coefficients.

regions. In the current work, a more favorable mesh anisotropy is imposed using the wake envelope mapping proposed in [1]. For example, a non-adaptive spatially anisotropic two-dimensional mesh is illustrated in Figure 1, along with the close-up view of the grid in the near wake region.

The newly proposed W-DNS method is demonstrated for both the laminar steady separated two-dimensional flow at a low Reynolds number, which is $Re_D = 40$, and the three-dimensional turbulent flow at a sub-critical Reynolds number, which is $Re_D = 1000$, where the Reynolds number is based on the cylinder diameter D. For the low Reynolds number simulation, five levels of resolution are used to simulate the vortex shedding flow, which corresponds to employing five nested wavelet collocation grids in the computational space ($J = 5$). Based on previous experience, the wavelet thresholding level is prescribed at the value of $\varepsilon = 5 \times 10^{-4}$.

Looking at the aerodynamic loads on the cylinder, the time histories of the drag and the lift force coefficients are reported in Figure 2. After the transient period, during which the regular shedding flow develops starting from initial conditions, the drag coefficient achieves the constant value of $C_D = 1.52$, which is very close to the reference value of 1.51 provided in [10]. As to the lift coefficient, predictably, it tends towards zero, with oscillations of decreasing amplitude. The present method allows for the exact enforcement of the no-slip condition at the body surface, whereas, with the volume penalization approach, the same condition could be only approximated. In that case, the inexact nature of the wall boundary condition manifested itself in higher resolution requirement to compensate for the velocity slip error at the body surface [2]. Due to the adaptivity of the method, the number of retained wavelets, and thus the computational cost, nearly follow the flow evolution. After the initial increase caused by the evolution of the wake region, the number of grid points remains practically constant for fully developed flow.

The key characteristic of the proposed W-DNS method stands in the possibility to effectively control the accuracy of the numerical solution. On the one hand, the spatial resolution can be increased by adding further levels of resolution. On the

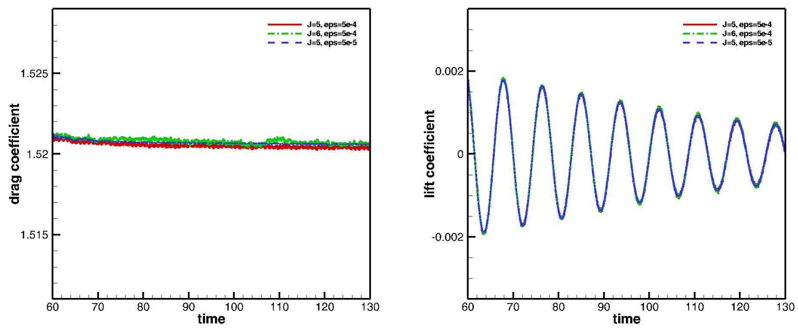


Fig. 3 Two-dimensional cylinder flow at $Re_D = 40$: time histories of (left) the drag and (right) the lift coefficients for three different resolutions that are ($J = 5$; $\varepsilon = 5 \times 10^{-4}$) (solid line), ($J = 6$; $\varepsilon = 5 \times 10^{-4}$) (dash-dotted line) and ($J = 5$; $\varepsilon = 5 \times 10^{-5}$) (dashed line).

other hand, for a given number of wavelet collocation grids, the thresholding level can be properly reduced. In this work, two additional simulations are carried out, starting form the previous baseline solution at the non-dimensional time $tU/D = 60$, where U stands for the freestream velocity, by either using a further level of resolution ($J = 6$) or choosing a lower wavelet threshold that is $\varepsilon = 5 \times 10^{-5}$. The time histories of the drag and the lift force coefficients for three different simulations with different resolutions are reported in Figure 3. While the use of an extra level of resolution, without changing ε, results in a more noisy solution, the use of a lower threshold undoubtedly results in a more accurate solution. This demonstrates that the direct numerical solution is actually achieved for a sufficiently low level of thresholding.

The present method has been developed for the accurate and efficient simulation of wall-bounded turbulent flows. Some preliminary experiments for the unsteady three-dimensional W-DNS solution of the turbulent flow past a circular cylinder are conducted for the sub-critical flow regime, where the boundary layer exhibits laminar separation and the transition to turbulence occurs in the shear layers developing on the cylinder side, e.g. [9]. The calculation is performed at $Re_D = 1000$, by using seven nested rectilinear wavelet collocation grids in the computational space. The associated anisotropic O-meshes in the physical space are constructed following the same approach of the previous two-dimensional solution in the cross-section planes, while no mapping is used in the third spanwise homogeneous direction, where uniform grid spacing is used. The adaptive method provides a non-uniform spatial resolution, which is actually varying in time following the dynamic evolution of the turbulent flow structures in the three spatial dimensions. This is illustrated in Figure 4, where the contours of the vorticity magnitude and the numerical mesh, colored by the level of resolution, in the mid-plane, are reported at a given time instant. The anisotropic refinement results in a more efficient representation of the flow field at the wall region, which, in turn, translates into the decrease of the number of active wavelet collocation points and, ultimately, into the reduction of the computational

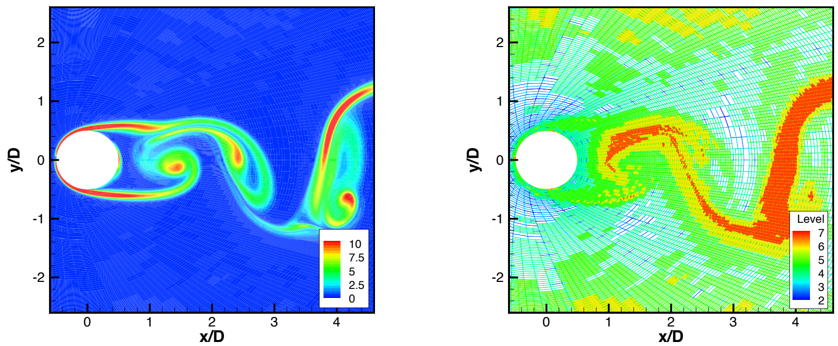


Fig. 4 Three-dimensional cylinder flow at $Re_D = 1000$: (left) instantaneous vorticity contours and (right) adaptive mesh in the mid plane, colored by the level of resolution.

cost. In fact, the use of anisotropically stretched mesh elements close to the surface reduces the number of wavelet levels that are actually needed to resolve the local flow structures. In particular, the maximum level of resolution ($J = 7$) is only involved in very limited zones, compared to excessively high resolution requirement in the near-wall region for the volume penalization approach [3]. Finally, in order to demonstrate how the complex three-dimensional vortex structures in the wake behind the cylinder are well represented by the W-DNS solution, the instantaneous iso-surfaces of the second invariant of the velocity gradient tensor, $Q = 0.4U^2/D^2$, are shown in Figure 5.

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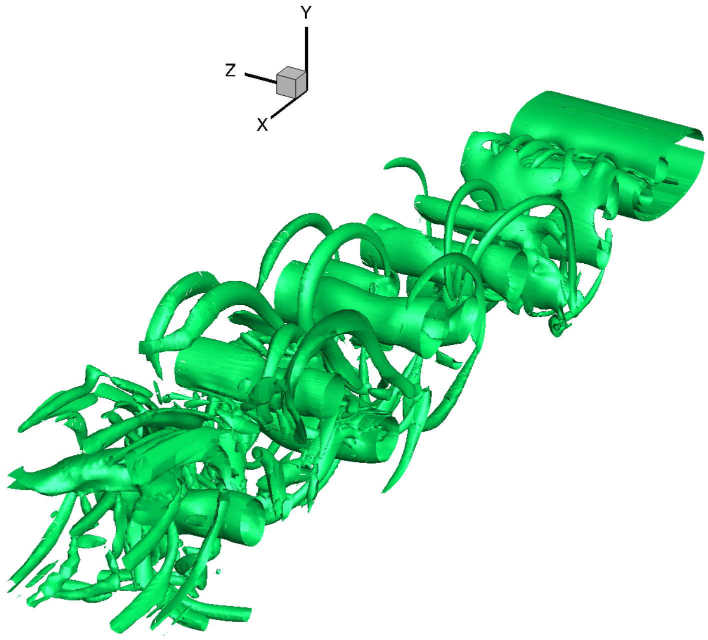

Fig. 5 Three-dimensional cylinder flow at $Re_D = 1000$: main vortical structures in the near wake of the cylinder identified by the iso-surfaces of Q.

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