Quantum adiabaticity in many-body systems

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Overview

1. Quantum many-body adiabaticity and orthogonality catastrophe
   - Quantum adiabaticity
   - Orthogonality catastrophe
   - Relation between adiabaticity and orthogonality catastrophe
   - Gap condition and many-body quantum adiabaticity
   - Grover adiabatic search
2. Genuine many-body adiabaticity *vs* thermodynamic adiabaticity
   - Thermodynamic adiabaticity
   - An impurity particle in a 1D quantum fluid
3. Summary and outlook

Quantum Adiabatic Theorem

Quantum Adiabatic Theorem (QAT) – colloquially:

A system evolving under a time-dependent Hamiltonian can be kept arbitrarily close to the Hamiltonian's instantaneous ground state provided that the parameters of the Hamiltonian vary *slowly enough*.

Key question: What is the exact meaning of *"slowly enough"*?

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- adiabatic quantum computers and quantum annealers
- topological quantum pumps
- quasi-Bloch oscillations of a mobile impurity in a 1D fluid
- Quantum Field Theory
- *etc*

Notations and definitions

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- Simplest case: linear driving, $\lambda(t) = \Gamma t$, Γ being the driving rate. In general, $\Gamma = \partial\lambda/\partial t$.
- Use λ instead of t as the evolution parameter. Schrodinger equation:

$$i\,\Gamma \frac{\partial}{\partial\lambda}\Psi_\lambda = \hat{H}_\lambda\,\Psi_\lambda.$$

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- One calls the evolution adiabatic as long as Ψ_λ remains close to Φ_λ, or in other words if the fidelity

$$\mathcal{F}(\lambda) = |\langle \Phi_\lambda | \Psi_\lambda \rangle|^2$$

  is close to one.

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Q1. For a given driving rate Γ, how long the adiabaticity can be maintained with a given accuracy ε?

Q2. How small should the driving rate Γ be for a given target λ and a given accuracy ε?

Orthogonality catastrophe

Orthogonality catastrophe – colloquially:

Given a many-body Hamiltonian $\hat{H}_\lambda$, two ground states corresponding to slightly different λ's can become nearly orthogonal with growing size of the system, N [Anderson, 1967].

Orthogonality catastrophe

Orthogonality catastrophe – rigorously. *Orthogonality overlap* in the leading order in λ:

$$\mathcal{C}(\lambda) \equiv |\langle \Phi_\lambda | \Phi_0 \rangle|^2 = e^{-C_N \lambda^2}.$$

The orthogonality catastrophe takes place whenever $C_N \to \infty$ in the thermodynamic limit (TL), $N \to \infty$. The behavior of C_N is determined by the type of driving and the gap.

Orthogonality catastrophe: scaling

$$\mathcal{C}(\lambda) \equiv |\langle \Phi_\lambda | \Phi_0 \rangle|^2 = e^{-C_N \lambda^2}.$$

	Local driving	Bulk driving
gapless systems	$C_N \sim \log N$	$C_N \sim N$
gapped systems	$\lim_{N\to\infty} C_N$ is finite	$C_N \sim N$

Key idea

In a many-body system subject to orthogonality catastrophe

$$\mathcal{F}(\lambda) \simeq \mathcal{C}(\lambda)$$

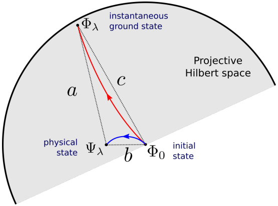
up to times sufficiently long for the adiabaticity to completely break down.

Central result

$$|\mathcal{F}(\lambda) - \mathcal{C}(\lambda)| \le \mathcal{R}_\lambda$$

$$\mathcal{R}_\lambda \equiv \Gamma^{-1} \int_0^\lambda \sqrt{\langle \hat{H}^2_{\lambda'} \rangle_0 - \langle \hat{H}_{\lambda'} \rangle_0^2}\, d\lambda',$$

where $\langle \cdots \rangle_0 \equiv \langle \Psi_0 | \cdots | \Psi_0 \rangle$.



For $\hat{H}_\lambda = \hat{H}_0 + \lambda \hat{V}$, one gets a simplified $\mathcal{R}_\lambda$:

$$R_\lambda = \lambda^2 \, \delta V_N / (2\Gamma) \quad \text{with} \quad \delta V_N \equiv \sqrt{\langle \hat{V}^2 \rangle_0 - \langle \hat{V} \rangle_0^2}.$$

OL, O. Gamayun, V. Cheianov
PRL 119, 200401 (2017)

Adiabaticity breakdown time (Question 1)

Define the adiabaticity breakdown time t_* and parameter $\lambda_* \equiv \lambda(t_*)$:

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Relation between adiabaticity and orthogonality catastrophe implies

$$\lambda_* = 1/\sqrt{C_N}$$

up to small corrections, as long as $\mathcal{R}(C_N^{-1/2}) \ll 1$. The latter is guaranteed for sufficiently large system since

$$\frac{\delta V_N}{C_N} \to 0 \quad \text{for} \quad N \to \infty.$$

Necessary condition for many-body adiabaticity (Q 2)

If the orthogonality catastrophe is present, the adiabaticity can be maintained for finite systems only as long as $\mathcal{R}(\lambda_*)$ is large enough to make inequality

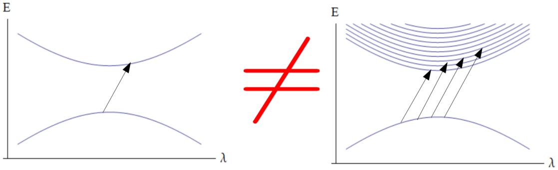
$$|\mathcal{F}(\lambda) - \mathcal{C}(\lambda)| \leq \mathcal{R}_\lambda$$

trivial.

This entails a **necessary adiabatic condition:**

$$\Gamma_N < \frac{\delta V_N}{2C_N} \frac{1}{1 - e^{-1} - \varepsilon}.$$

Adiabaticity and a gap



In a two-level system adiabaticity is governed by the gap (Landau-Zener). The adiabatic condition:

$$\Gamma \ll \Delta E_{\min}.$$

A Landau-Zener-type guess is typically **completely wrong** for bulk driven many-body systems! The adiabatic condition:

$$\Gamma \ll f_N \, \Delta E_{\min} \quad \text{with} \quad \lim_{N\to\infty} f_N = 0.$$

Take-home message

Maintaining adiabaticity in a many-body system is challenging.

More challenging than one may assume based on naive considerations employing energy gap.

Application to Grover adiabatic search

Applying our necessary adiabatic condition to the Grover adiabatic search algorithm gives a lower bound for the run time $\sim \sqrt{N}$ (N – number of database elements)

This scaling reproduces the scaling of the explicitly known optimal run time.

OL, Journal of Russian Laser Research 2018

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But is it really *necessary*?

Thermodynamic adiabaticity

Thermodynamic adiabaticity (see e.g. Polkovnikov, Gritsev, Nature Phys. 2008):

$$\text{energy gaps between levels} \ll \Gamma \ll \text{all intensive energy scales}$$

Thermodynamic *vs* genuine adiabaticity

For gapped spin systems *thermodynamic adiabaticity* is enough for expectation values of local operators to stay close to their ground state values (theorem by Bachmann, De Roeck, Fraas PRL 2017), even if the *genuine adiabaticity* is already broken.

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Can it be that the genuine adiabaticity is completely irrelevant in the many-body setting?

This is not always the case!

Thermodynamic *vs* genuine adiabaticity

In fact, whether the thermodynamic adiabaticity is enough for an "adiabatic" phenomenon to occur, or a genuine many-body adiabaticity is necessary, is a tricky question.

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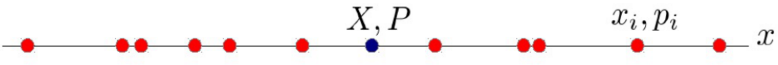
We consider dynamics of an impurity in a 1D quantum fluid as an examples.

Mobile impurity in a 1D fluid

A mobile impurity particle in a translation-invariant 1D gas of N fermions



$$\hat{H}_t = \frac{P^2}{2m} + \sum_{n=1}^{N} \frac{p_n^2}{2m} + g_t \sum_{i=1}^{N} \delta(x_i - x)$$

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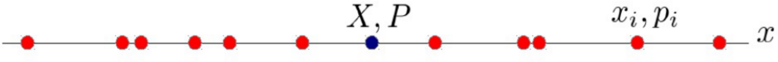

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$$|\mathrm{in}\rangle = |e^{i\, m\, v_0\, X}\rangle \otimes |\mathrm{Fermi\ sea}\rangle$$

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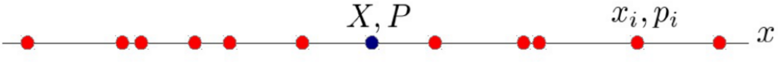

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Coupling is slowly switched on up to a value g:

$$g_t = \Gamma t\,(k_\mathrm{F}/m), \qquad t \in [0, \tau], \qquad g_\tau \equiv g$$

Mobile impurity in a 1D fluid

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$$g_t = \Gamma t, \qquad t \in [0, \tau] \tag{1}$$

Genuine many-body adiabaticity:

$$\Gamma \ll \Delta E \lesssim E_{\rm F}/N$$

Thermodynamic adiabaticity:

$$E_{\rm F}/N \ll \Gamma \ll E_{\rm F}$$

Mobile impurity in a 1D fluid

$$\begin{aligned} \hat{H}_t =& \frac{P^2}{2m} + \sum_{n=1}^{N} \frac{p_n^2}{2m} + g_t \sum_{i=1}^{N} \delta(x_i - x) \\ g_t =& \Gamma t, \qquad t \in [0, \tau] \end{aligned} \tag{1}$$

Genuine many-body adiabaticity:

$$\Gamma \ll \Delta E \lesssim E_{\mathrm{F}}/N$$

Thermodynamic adiabaticity:

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Will the local physical observables at $t \gg \tau$ differ?

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Local observable: v_∞ (velocity of the impurity at $t = \infty$).

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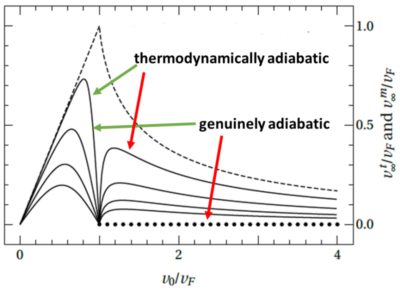

$v_0 < v_F$:

$$v_\infty^{\text{thermodynamic}} = v_\infty^{\text{genuine}}$$

$v_0 > v_F$:

$$v_\infty^{\text{genuine}} = 0$$

$$v_\infty^{\text{thermodynamic}} > 0$$

Gamayun, OL *et al* PRL 2018

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- Scaling of t_* with the number of particles, N, is determined by the nature of driving (global or local) and by presense/absense of gapless excitations.
- Contrary to the common belief, whenever driving is of bulk type, even a finite gap is not able to protect adiabaticity in the thermodynamic limit!
- Necessary adiabatic condition for finite systems derived – the most stringent to date, to the best of our knowledge!

Summary and outlook II

- For some "adiabatic" phenomena genuine many-body adiabaticity is not actually required. Instead, thermodynamic adiabaticity is sufficient.

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- For some "adiabatic" phenomena genuine many-body adiabaticity is not actually required. Instead, thermodynamic adiabaticity is sufficient.
- For others, genuine many-body adiabaticity is strictly necessary.
- The discrimination between the two scenarios is subtle and is currently done on the case-by-case basis. General theoretical understanding is lacking!

Published in:

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Thank you for your attention!