Quench and adiabatic dynamics of a mobile impurity in a one-dimensional quantum fluid

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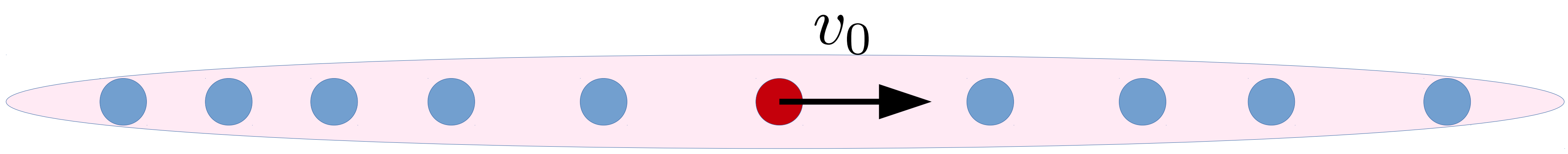
МИАН

Inject an impurity particle in a 1D quantum fluid at *T=0*
(1D fluid =1D gas for our purposes)



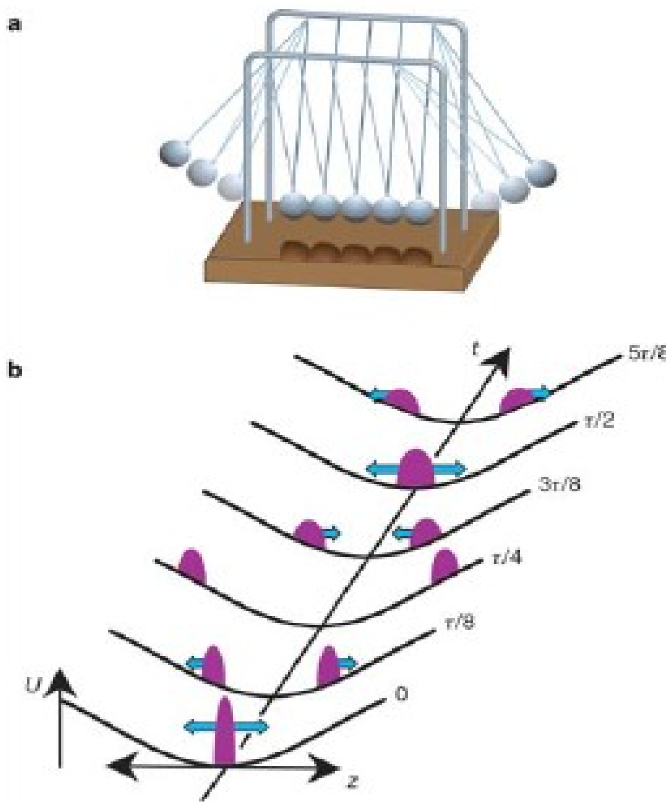
Question: will it ever stop ?

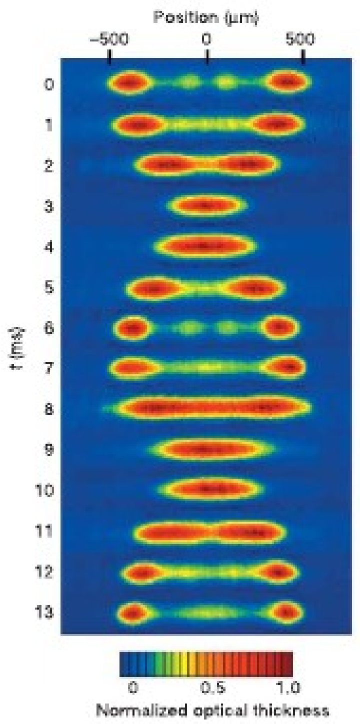
Fine print: the question makes sense for infinite system or periodic boundary conditions.

We stick to periodic boundary conditions.

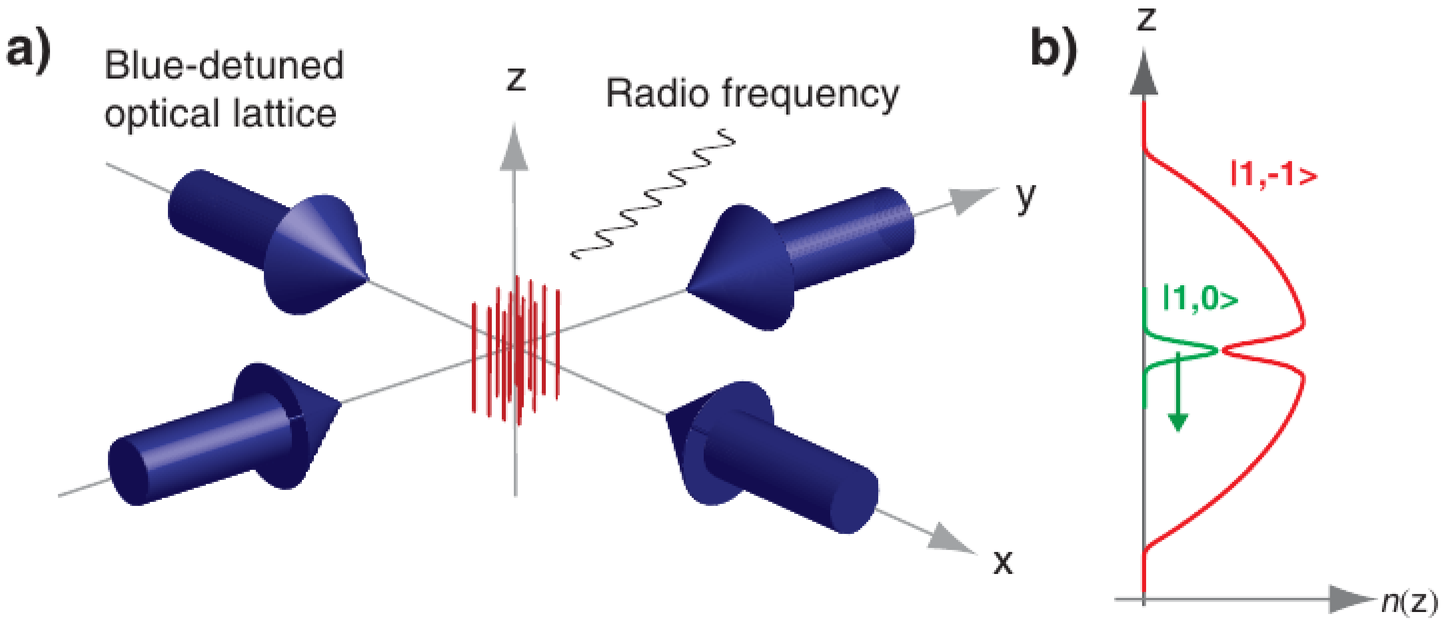
Experimental realizations of a1D quantum gas

A Quantum Newton's cradle (Nature 440, 900 (2006))





Experimental realization of a mobile impurity in a 1D quantum gas

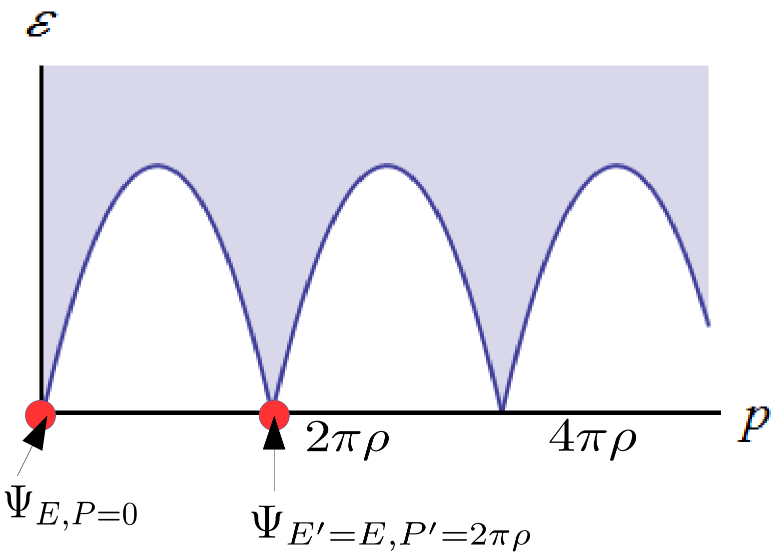


Phys. Rev. Lett. 103, 150601 (2009);
Phys. Rev. Lett. 109, 235301 (2012);
Nature Phys. 9, 235241 (2013).

Very recent: Science 356, 945–948 (2017)

Toroidal optical traps are also possible

Dispersion of a1D fluid



Momentum quantum: $\delta k = \frac{2\pi}{L}$

Particle density: $\rho = \frac{N}{L^D}$

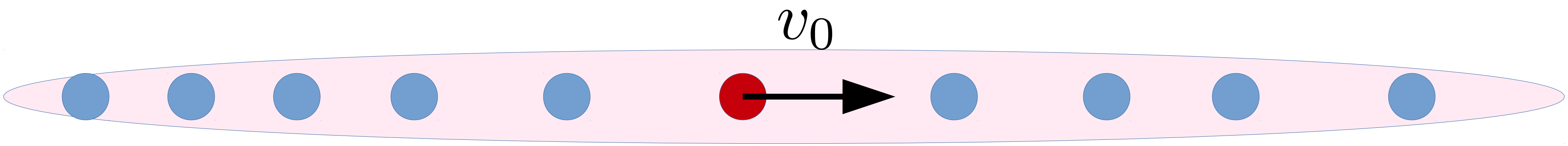
$$\Psi_{E,P=0} = \sum_{\{k_1,k_2\ldots,k_N\}} C_{\{k_1,k_2\ldots,k_N\}} a^\dagger_{k_1} a^\dagger_{k_2},\ldots, a^\dagger_{k_N} |\text{no particles}\rangle$$

$$\Psi_{E',P'} = \sum_{\{k_1,k_2\ldots,k_N\}} C_{\{k_1,k_2\ldots,k_N\}} a^\dagger_{k_1+\delta k} a^\dagger_{k_2+\delta k},\ldots, a^\dagger_{k_N+\delta k} |\text{no particles}\rangle$$

$$P' = P + N\delta k = P + 2\pi\rho L^{\underline{D-1}}$$

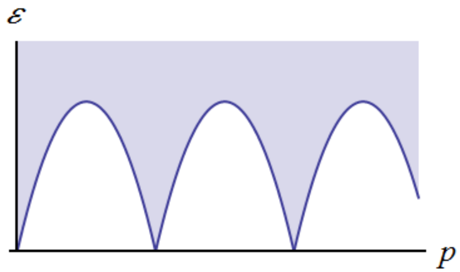
$$E' = E + \frac{(2\pi)^2}{2m}\rho L^{\underline{D-2}}$$

Question: will the impurity stop ?



Naive (and wrong) answer: eventually yes

No phase transitions in 1D



Landau critical velocity is zero in 1D

No superfluidity in 1D

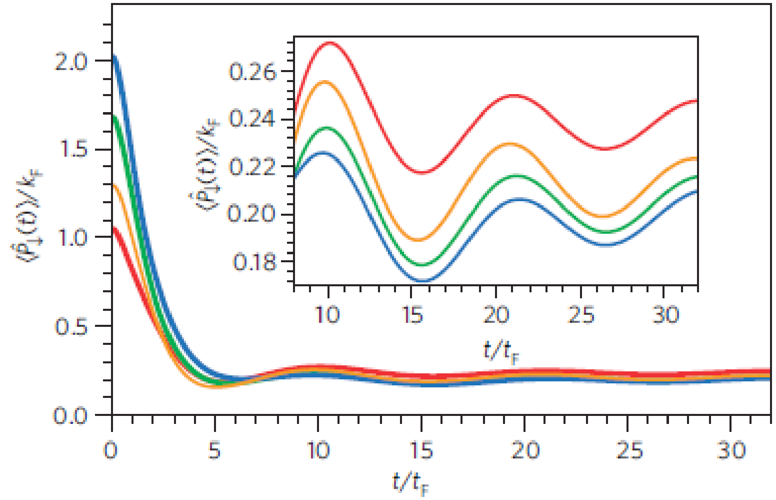


Figure 5 | Time evolution of $\langle\hat{P}_\downarrow(t)\rangle$ for $\gamma = 5$ and several values of Q. Initial momentum $Q = 1.05k_F$ (red), $1.35k_F$ (orange), $1.7k_F$ (green), $2k_F$ (blue). Inset: zoom in on the oscillations. The oscillations depend only weakly on Q. This implies that if the impurity was created in a wave packet state $\sum_k \alpha_k |\text{in}_k\rangle$ its momentum $\sum_k |\alpha_k|^2 \langle \text{in}_k|\hat{P}_\downarrow(t)|\text{in}_k\rangle$ would still oscillate with time for α_k not too broad in momentum space (see Supplementary Section S9). If the Raman beams have a finite width w in the set-up shown in Fig. 4, α_k is a Gaussian in momentum space centred around Q with width $1/w$.

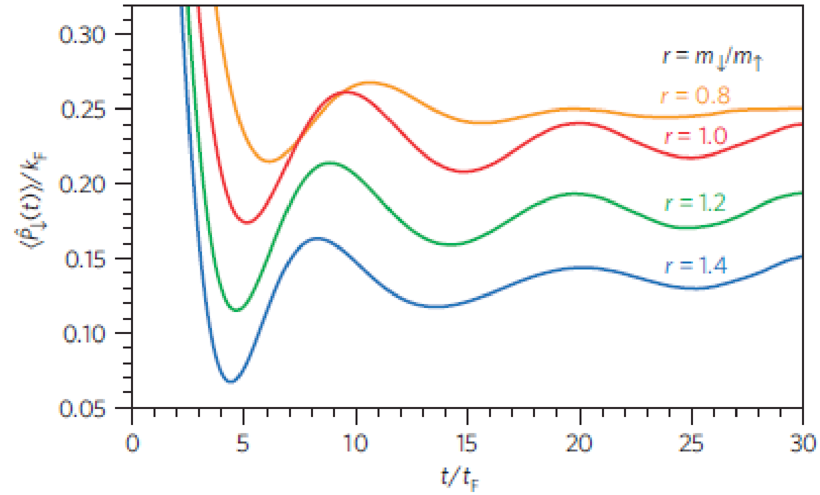
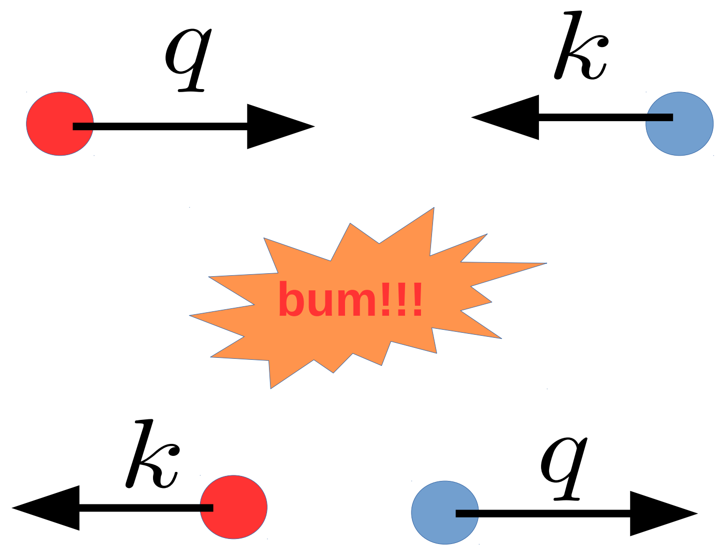


Figure 6 | Time evolution of $\langle\hat{P}_\downarrow(t)\rangle$ for several values of mass ratio $r = m_\downarrow/m_\uparrow$. Initial momentum $Q = 1.05k_F$, interaction strength $\gamma = 5$. These results are obtained by the variational approach discussed in the text. In the integrable case, $r = 1$, they agree quantitatively with those obtained by Bethe Ansatz (see Supplementary Section S8). One can see that the saturation of momentum loss and quantum flutter exist away from the integrable point. However, quantum flutter gets strongly damped for $r < 1$, whereas for $r > 1$ the damping depends on r only weakly.

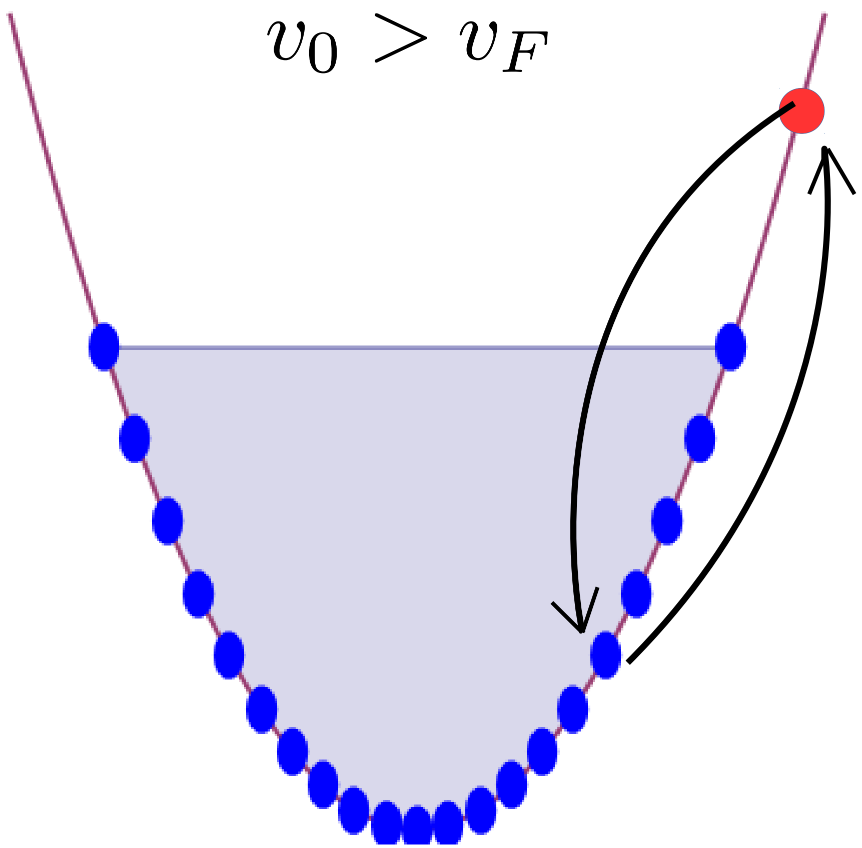
Illustration: impurity in a sea of noninteracting fermions of the same mass

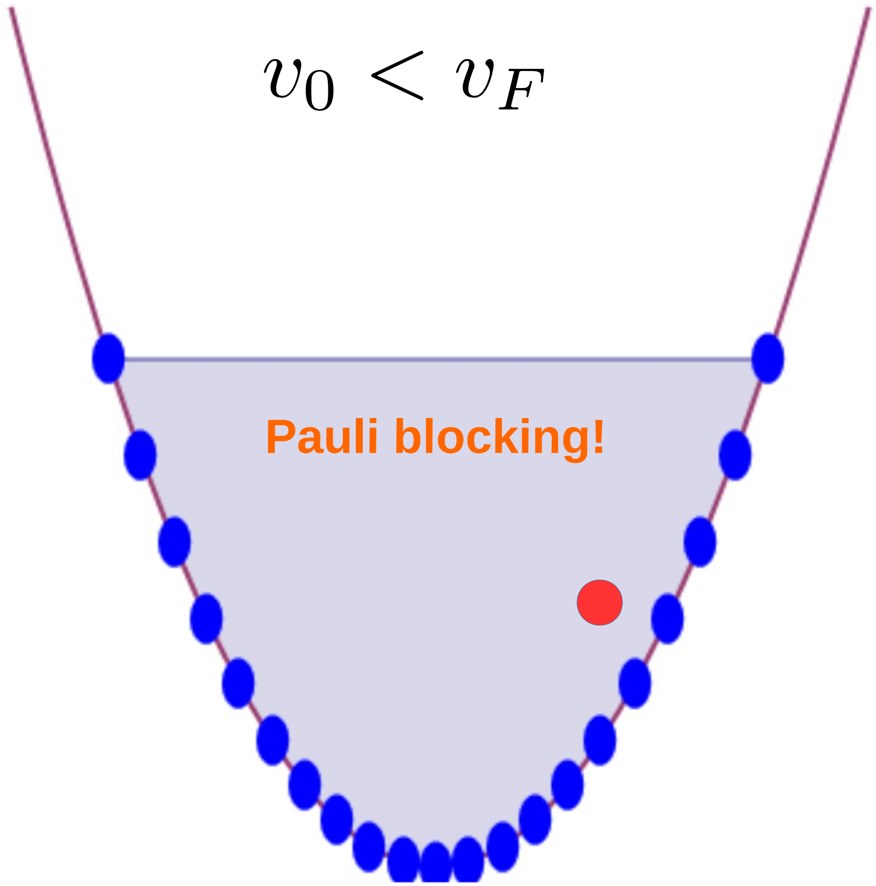
pairwise scattering in 1D, $m = m_f$



due to energy and momentum conservation the momenta of the impurity and fermion are merely exchanged

Illustration: impurity in a sea of noninteracting fermions of the same mass





$$v_\infty = v_0$$

What about:

- Three-body processes?
- Quantum interference?
- Interactions?
- Bosonic fluids?
- ...

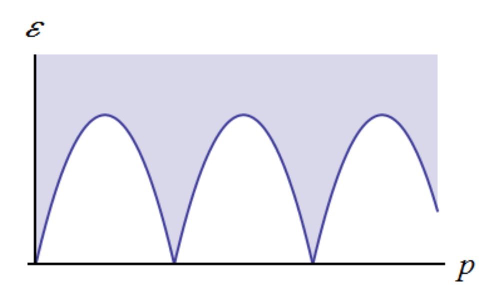
Theorem: the impurity still does not stop!

Scope and notations

- Hamiltonian of the impurity-fluid system: $\hat{H} = \hat{H}_{\mathrm{h}} + \hat{H}_{\mathrm{i}} + \hat{U}$.
  $|E\rangle$ - eigenstates of H.
- Host fluid consists of N particles in volume V with number density $\rho = N/V$. Hamiltonian of the host fluid $\hat{H}_{\mathrm{h}}$ is arbitrary.
- Dispersion of the fluid is the lower edge of spectrum of $\hat{H}_{\mathrm{h}}$:

$$\varepsilon(q) \equiv \inf_{\substack{\Phi: \\ \hat{\mathbf{P}}_{\mathrm{h}}\Phi = \mathbf{q}\Phi}} \langle \Phi | \hat{H}_{\mathrm{h}} | \Phi \rangle,$$

where $q \equiv |\mathbf{q}|$.



Initial condition

Initially the impurity is injected with some velocity $\mathbf{v}_0$ (with $v_0 \equiv |\mathbf{v}_0|$) into the host fluid at zero temperature:

$$|\mathrm{in}\rangle = |\mathrm{GS}\rangle \otimes |\mathbf{v}_0\rangle = |\mathrm{GS}, \mathbf{v}_0\rangle,$$

where $|\mathrm{GS}\rangle$ is the ground state of the fluid.

Generalized critical velocity

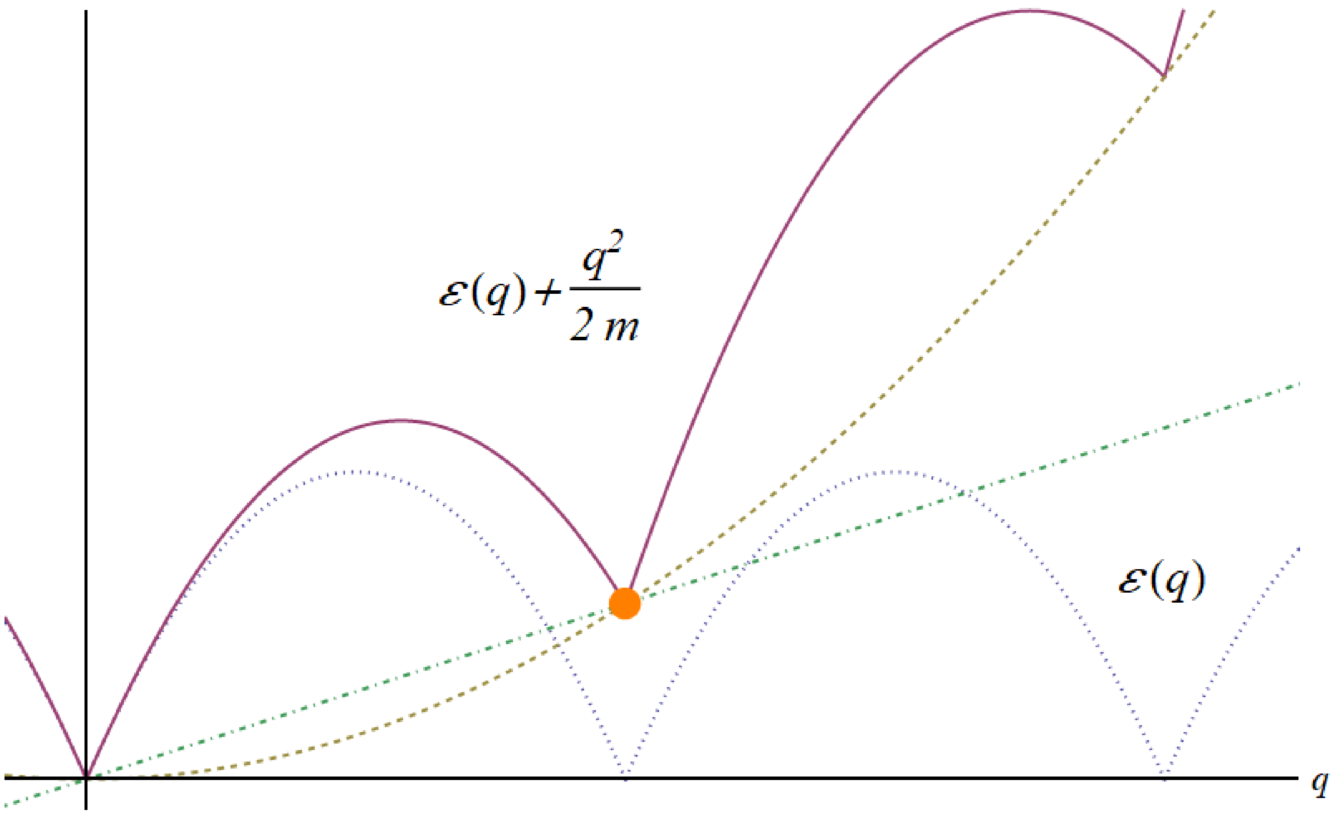
Generalized critical velocity depends on mass of the impurity [Rayfield, 1966]:

$$\boxed{v_c \equiv \inf_q \frac{\varepsilon(q) + \frac{q^2}{2m}}{q}.}$$

Physically, v_c is the minimal velocity which allows the impurity to create real excitations of the fluid (remind however that impurity-fluid interaction was ignored).

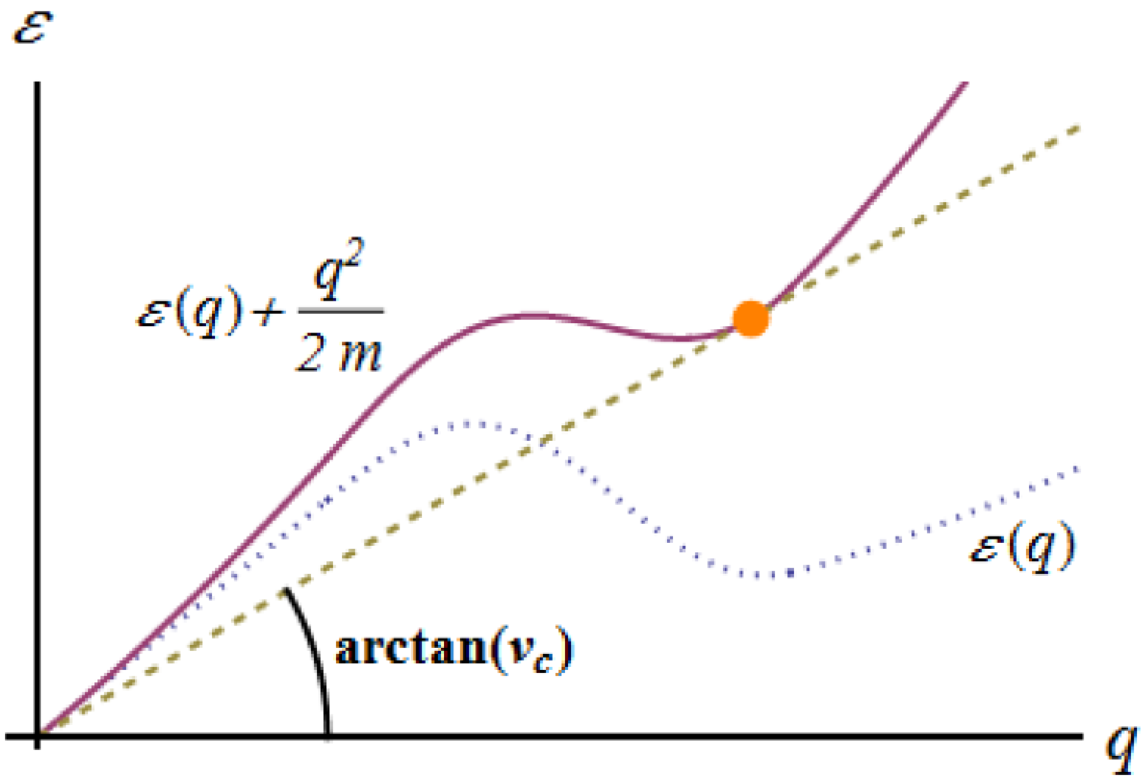
Geometrical meaning of the critical velocity

The line $v_c q$ is tangent to the curve $\varepsilon(q) + \frac{q^2}{2m}$:



Geometrical meaning of the critical velocity

The line $v_c q$ is tangent to the curve $\varepsilon(q) + \frac{q^2}{2m}$:



Rigorous bound on $|\mathbf{v}_0 - \mathbf{v}_\infty|$

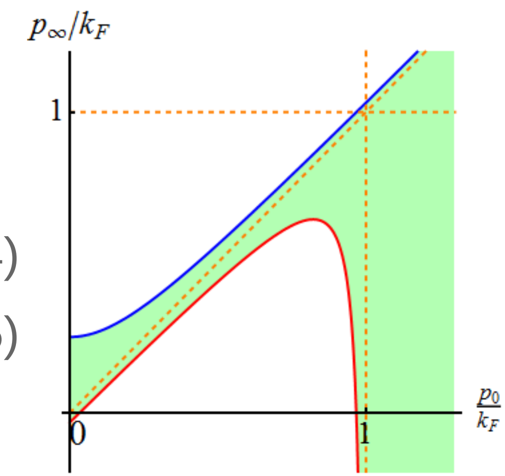
For an everywhere repulsive impurity-fluid interaction $U(x) \geq 0$ and for $v_0 \equiv |\mathbf{v}_0| < v_c$

$$\boxed{|\mathbf{v}_0 - \mathbf{v}_\infty| \leq \frac{\overline{U}}{m(v_c - v_0)},} \qquad (1)$$

where $\overline{U} \equiv \rho \int d\mathbf{r}\, U(|\mathbf{r}|)$.

OL, Phys. Rev. A 89, 033619 (2014)

OL, Phys. Rev. A 91, 040101 (2015)



How to reconcile this result with the absence of superfluidity in 1D?

Easy: superfluid flow through a static constriction is equivalent to the stationary motion of impurity of infinite mass. However,

$$v_c(m = \infty) = 0$$

and the bound collapses.

We can do more in certain cases:

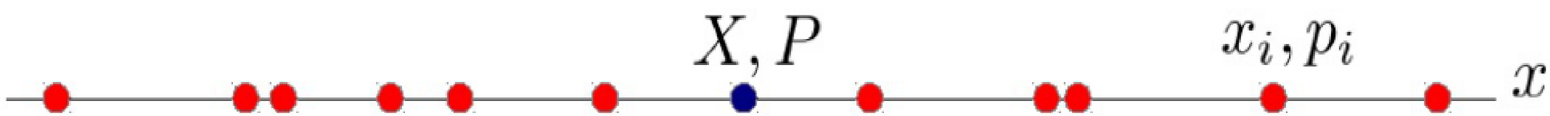
$$v_\infty = v_\infty(v_0)$$

explicitly

- Keldysh dynamical perturbation theory for small impurity-fluid coupling
- Bethe ansatz in the integrable point

Model description

External particle immersed into quantum gas



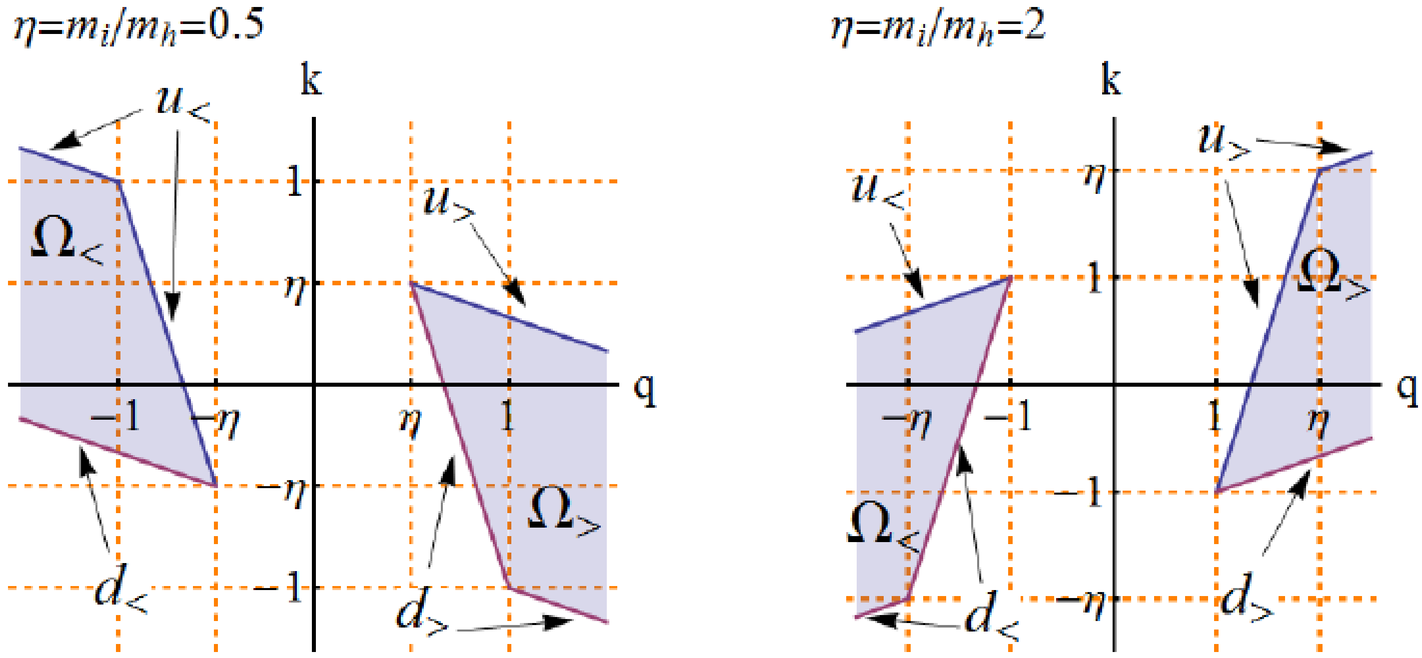
$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_h} + c \sum_{i<j} \delta(x_i - x_j) + \underbrace{\frac{P^2}{2m_i}}_{\text{impurity}} + g \sum_{i=1}^{N} \delta(x_i - X)$$

- Initial state $|\mathrm{in}\rangle = |\mathrm{vac}\rangle e^{ip_0 X}$
- $\langle P(t=0)\rangle = p_0, \quad \langle P(t)\rangle = ?$
- Thermodynamic limit $N \to \infty$, $L \to \infty$, $\rho = N/L = \mathrm{const}$
- Tonks-Girardeau (TG) limit $c \to \infty$: $|\mathrm{vac}\rangle = |\mathrm{FS}\rangle$
- The mass ratio $\eta = m_i/m_h$. System is integrable for $\eta = 1$.

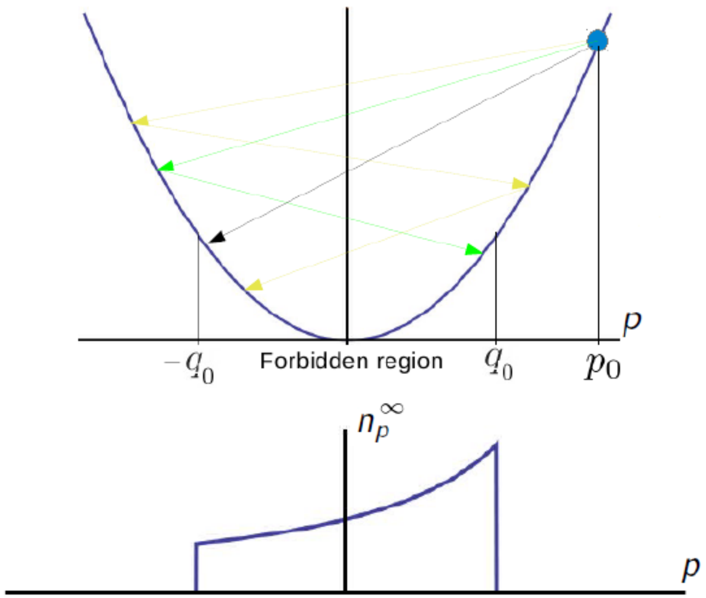
Quantum Boltzmann Equation (QBE)

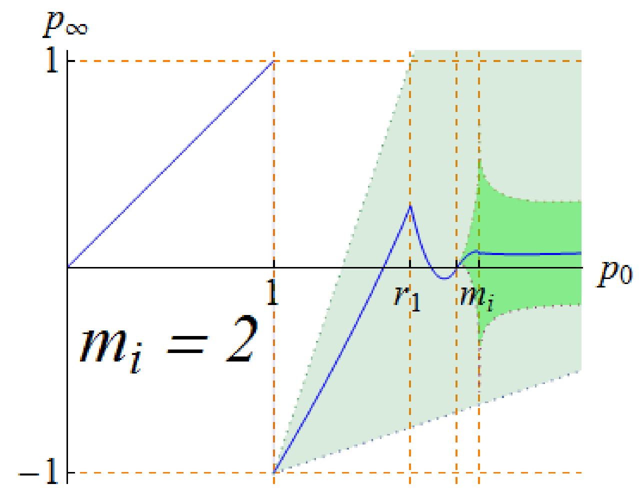
$$\frac{\partial}{\partial t} w_k(t) = -\Gamma_k w_k(t) + \sum_q \Gamma_{q \to k} w_q(t),$$

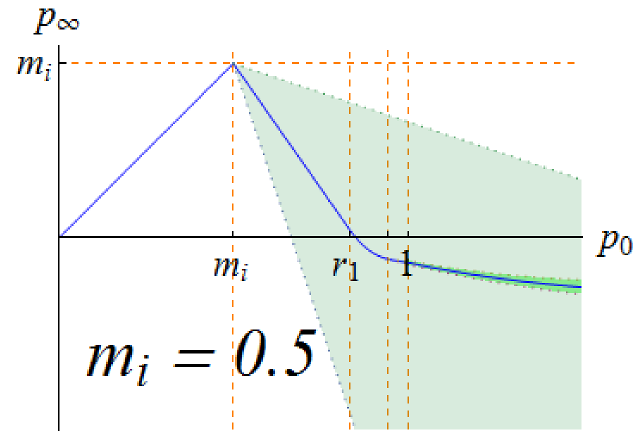
$$\Gamma_{q \to k} = \frac{\gamma^2}{\pi^2 m_h L} \frac{\theta_\Omega(q, k)}{|q - k|} \qquad \Gamma_k = \sum_q \Gamma_{k \to q}$$



QBE: iterative solution with controllable precision







O. Gamayun, O. L, V. Cheianov,
Phys. Rev. E 90, 032132 (2014)

Bethe ansatz solution

- Hamiltonian in the impurity rest frame

$$H_{p_0} = \frac{1}{2m_i}\left(p_0 - \sum_{i=1}^{N} p_i\right)^2 + \sum_{i=1}^{N}\frac{p_i^2}{2m_h} + g\sum_{i=1}^{N}\delta(x_i)$$

- Eigenfunctions in the impurity rest frame $H_{p_0}|f\rangle = E_f|f\rangle$
-

$$|f\rangle \sim \frac{1}{\sqrt{N!L^N}}\begin{vmatrix} e^{ik_1x_1} & \dots & e^{ik_{N+1}x_1} \\ \vdots & \ddots & \vdots \\ e^{ik_1x_N} & \dots & e^{ik_{N+1}x_N} \\ e^{ik_1L}-1 & \dots & e^{ik_{N+1}L}-1 \end{vmatrix}, \qquad E_f = \sum_{j=1}^{N+1}\frac{k_j^2}{2}$$

where $k_1, k_2, \dots k_{N+1}$ are deformed quasimomenta (Bethe roots), defined through the set of integers $n_1, n_2 \dots n_{N+1}$ as

$$k_j = \frac{2\pi}{L}\left(n_j - \frac{1}{\pi}R\left(\Lambda - \frac{4\pi}{gL}n_j\right)\right)$$

where the function $R(x) \in [0,\pi)$ is a solution of the transcendental equation

$$\frac{1}{\tan R(x)} = x + \frac{4R(x)}{gL}.$$

and parameter Λ is found from the condition

$$\sum_{j=1}^{N+1} k_j = p_0\,.$$

- Initial state in the impurity rest frame is just a filled Fermi-Sea $|\mathrm{in}\rangle = |\mathrm{FS}\rangle \equiv \det_{i,j\in[1,N]}(e^{2\pi(N+1-2j)x_i/2L})$

Bethe ansatz solution

$$v_i = \frac{1}{m} \sum_f \langle f | P_{\rm imp} | f \rangle | \langle f | {\rm in}_Q \rangle |^2$$

$$\langle f | P_{\rm imp} | f \rangle \equiv \mathcal{P}(\Lambda) = \frac{\partial E_Q}{\partial Q} = \frac{\Lambda}{\alpha} + \frac{1}{2\alpha} \frac{\ln \frac{1+(\alpha-\Lambda)^2}{1+(\alpha+\Lambda)^2}}{\arctan(\alpha - \Lambda)\arctan(\alpha+\Lambda)}, \qquad \alpha = 2v_F/g$$

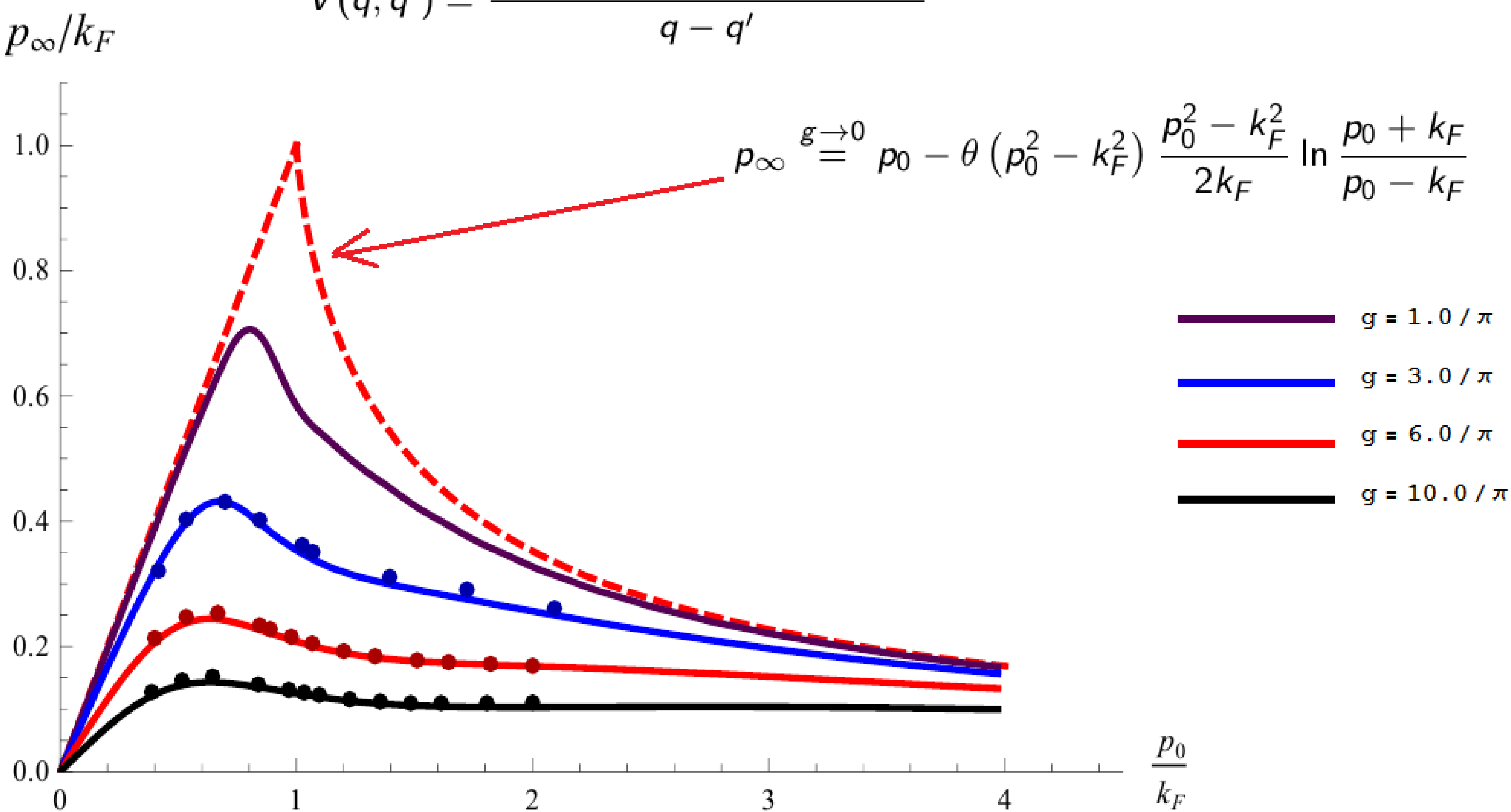
$$|\langle f | {\rm in}_Q \rangle| = Y \left(\frac{2}{L}\right)^N \left(\prod_{l=1}^{N+1} s_l\right) \begin{vmatrix} (k_1 - q_1)^{-1} & (k_2 - q_1)^{-1} & \ldots & (k_{N+1} - q_1)^{-1} \\ (k_1 - q_2)^{-1} & (k_2 - q_2)^{-1} & \ldots & (k_{N+1} - q_2)^{-1} \\ . & . & . & . \\ . & . & . & . \\ (k_1 - q_N)^{-1} & (k_2 - q_N)^{-1} & \ldots & (k_{N+1} - q_N)^{-1} \\ 1 & 1 & \ldots & 1 \end{vmatrix}$$

$$\frac{v_i}{v_F} = -i \int_{-\infty}^{\infty} \frac{d\Lambda}{\pi} \mathcal{P}(\Lambda) \int_0^{\infty} dx \, \sin(x v_0 / v_F) F(\Lambda, x)$$

Bethe Ansatz Results

$$p_\infty = \int P(\Lambda, g)\, d\Lambda \int \frac{dx}{2\pi} e^{-ip_0 x} \left[(h(x,g) - 1) \det{}_{[-k_F, k_F]}(1 + \hat{V}_1) + \det{}_{[-k_F, k_F]}(1 + \hat{V}_2) \right]$$

$$V(q, q') = \frac{e_+(q)e_-(q') - e_-(q)e_+(q')}{q - q'}$$



$$p_\infty \overset{g\to 0}{=} p_0 - \theta\left(p_0^2 - k_F^2\right) \frac{p_0^2 - k_F^2}{2k_F} \ln \frac{p_0 + k_F}{p_0 - k_F}$$

O. Gamayun, E. Burovski, V. Cheianov, O. L, M. Malcomson, M. Zvonarev, Phys. Rev. Lett. 120, 220605 (2018).

Turning on the impurity-fluid coupling *g* adiabatically

$$g(t) = \Gamma t$$

Integrability broken by time dependence!

Two types of adiabaticity in a many-body system:

$$\Gamma \ll E_F/N$$

genuine many-body-adiabatic regime
(many-body wave function stays close to the eigenfunction)

$$E_F/N \ll \Gamma \ll E_F$$

thermodynamically adiabatic regime
(local observables stay close to their ground state values)

Initial state is the product state:

$$|\mathrm{in}\rangle = |\mathrm{p}_0\rangle \otimes |\mathrm{Fermi\ sea}\rangle$$

Genuine many-body adiabatic regime

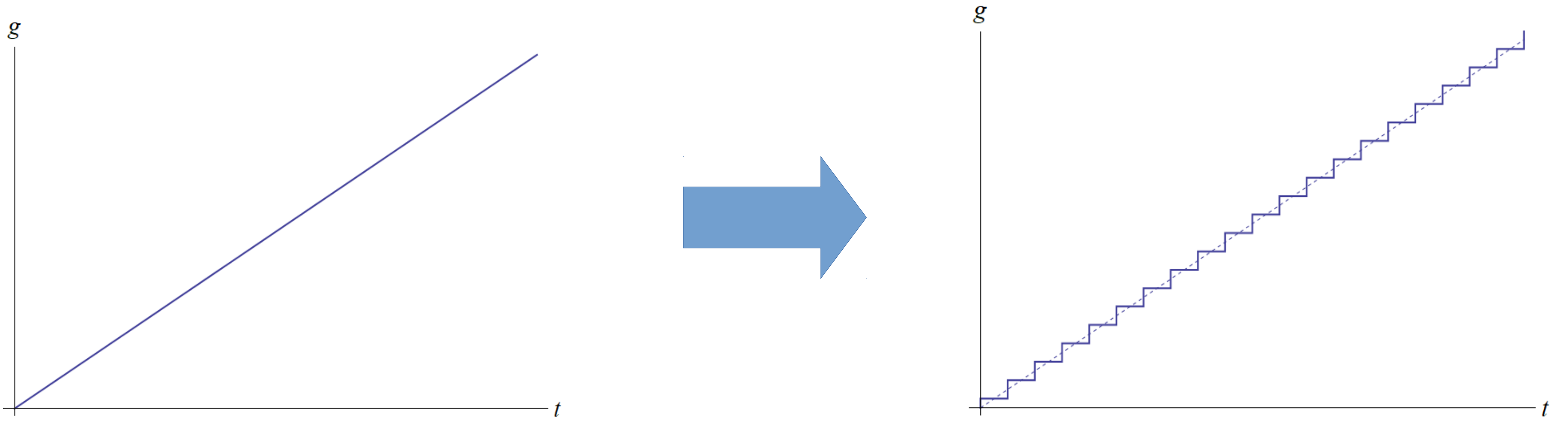
Adiabatic theorem at work,
the system stays in the many-body eigenstate.

Subtlety: degeneracy at $t=0$, the initial eigenstate should be chosen on continuity basis

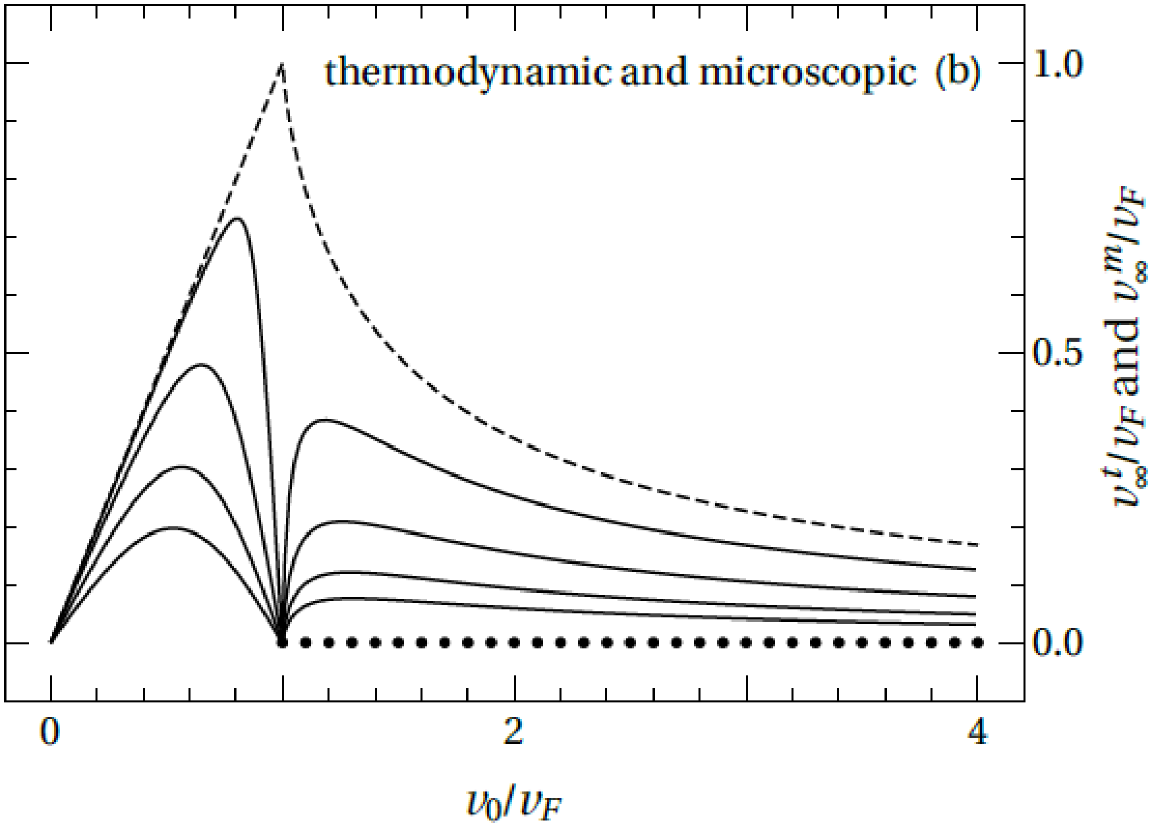
Thermodynamically adiabatic regime

Problem: integrability is broken, adiabatic theorem does not apply

Solution: *stepwise approximation* – slow growth of *g(t)* substituted by a stepwise sequence of small quenches



Thermodynamically adiabatic vs many-body adiabatic regimes



$v_0 < v_F$ **- no difference**

$v_0 > v_F$ **- dramatic difference**

O. Gamayun, E. Burovski, V. Cheianov, O. L, M. Malcomson, M. Zvonarev, Phys. Rev. Lett. 120, 220605 (2018).

Summary

- A mobile impurity of finite mass injected in a 1D quantum keeps moving forever.
- Apparent conflict with the Landau argument on the absence of superfluidity is resolved.
- General theorem bounds the steady state velocity from below.
- The steady state velocity is found explicitly in the weak coupling limit and in the integrable point.
- Integrable McGuire system – the simplest BA-solvable model. A lot of physics can be studied in detail!
- Thermodynamically adiabatic evolution can be studied in an “integrable” with the help of a stepwise substitution. Outcome can be drastically different from the many-body adiabatic case.

Literature

[1] C. J. M. Mathy, M. B. Zvonarev, and E. Demler, “Quantum flutter of supersonic particles in one-dimensional quantum liquids,” Nature Physics 8, 881–886 (2012).

[2] M. Knap, C. J. M. Mathy, M. Ganahl, M. B. Zvonarev, and E. Demler, “Quantum flutter: Signatures and robustness,” Phys. Rev. Lett. 112, 015302 (2014).

[3] O. Lychkovskiy, “Perpetual motion of a mobile impurity in a one-dimensional quantum gas“, Phys. Rev. A 89, 033619 (2014).

[4] O. Lychkovskiy, “Perpetual motion and driven dynamics of a mobile impurity in a quantum fluid“, Phys. Rev. A 91, 040101 (Rapid Communication) (2015).

[5] E. Burovski, V. Cheianov, O. Gamayun, and O. Lychkovskiy, “Momentum relaxation of a mobile impurity in a one-dimensional quantum gas”, Phys. Rev. A 89, 041601 (Rapid Communication) (2014).

[6] O. Gamayun, O. Lychkovskiy, V. Cheianov, “Kinetic theory for a mobile impurity in a degenerate Tonks-Girardeau gas”, Phys. Rev. E 90, 032132 (2014).

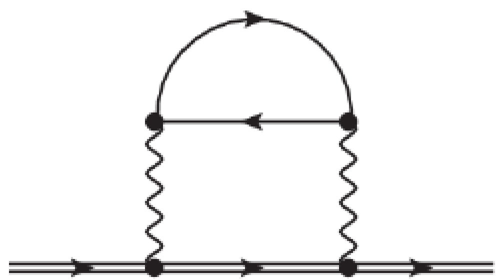
[7] O. Gamayun, E. Burovski, V. Cheianov, O. Lychkovskiy, M. Malcomson, M. Zvonarev, “Impact of the injection protocol on an impurity’s stationary state”, Phys. Rev. Lett. 120, 220605 (2018).

Quantum Boltzmann Equation (QBE)

Quantum kinetic equations $[\mathbb{G}]^{-1} = [\mathbb{G}^{(0)}]^{-1} - \hat{\Sigma}$

Pure Boltzmann

$\hat{\Sigma}^B =$

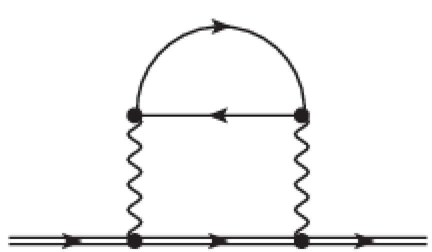


$$n_p^\infty = Z_{p_0} \frac{\theta(k_F - |p|)}{p_0 - p}, \quad p_0 > k_F \,.$$

$$p_\infty^B = p_0 - \theta\left(|p_0| - k_F\right) \frac{2k_F}{\ln \frac{p_0 + k_F}{p_0 - k_F}}$$

Multiple Scattering Events

$\hat{\Sigma}^L =$



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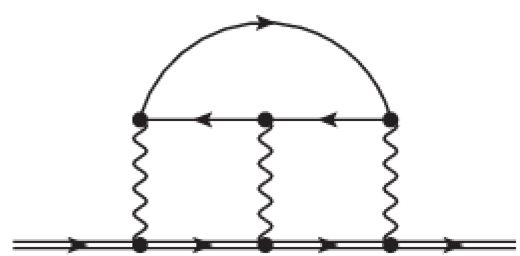

$+ \dots$

$$\gamma \to \frac{\gamma}{|\eta - 1|} \quad \overset{\eta \to 1 \; \gamma \to 0}{\Longrightarrow} \quad n_p^\infty = \tilde{Z}_{p_0} \frac{\theta(k_F - |p|)}{(p_0 - p)^2}, \quad p_0 > k_F \,.$$

$$p_\infty = p_0 - \theta\left(p_0^2 - k_F^2\right) \frac{p_0^2 - k_F^2}{2k_F} \ln \frac{p_0 + k_F}{p_0 - k_F}$$