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A Column-and-Constraint Generation Algorithm to Find Nash Equilibrium in Pool-Based Electricity Markets☆

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ABSTRACT

Equilibrium analysis is crucial in electricity market designs, with Nash equilibrium recognized as the most powerful one. Its most prominent hindrance, however, is an efficient methodology to compute an equilibrium point in large-scale systems. In this work, a Column-and-Constraint Generation (CCG) algorithm is proposed to tackle this challenge. More precisely, the master problem finds a candidate for Nash equilibrium and the oracle identifies whether this candidate point is indeed an equilibrium. A set of numerical experiments was conducted, comparing its computational performance with the solution of an Equilibrium Problem with Equilibrium Constraint (EPEC). We identify that the proposed algorithm overcomes the benchmark in the magnitude of 20 times on average and more than 30 times in the most demanding instances. Furthermore, the scalability of the EPEC formulation is challenged even for medium-scale instances, whilst the proposed algorithm was able to handle all tested instances in a reasonable computational time.

1. Introduction

Most power systems worldwide have evolved in the last decades towards the sedimentation of competitiveness in many of their spheres (e.g., generation, transmission, and distribution), with supply competition being recognized as the most mature among them [1,2]. Its main structure comprises a day-ahead pool-based marketplace, where Generation Companies (GENCOs) submit pairwise linked blocks of price and quantity offers to a market operator that identifies both the market-clearing electricity price and a day-ahead scheduled production for each GENCO. Each competitor is then financially compensated by the respective electricity price for each unit of production scheduled [3].

In this competitive context, several challenges of different nature materialize both from the viewpoint of a particular GENCO as well as from the perspective of the whole market and regulatory stability. On the one hand, the income from day-ahead electricity markets usually represents a significant share of the total cash flow source of most GENCOs. As a consequence, market participants should carefully adjust their supply offer in order to extract from the market sufficient amounts of income aiming at achieving secure levels of financial stability. On the other hand, market regulators need to continuously screen for market power evidence in order to avoid significant unilateral influence on the market outcome, ensuring thus high efficiency and social welfarism [4]. In this context, equilibrium models, particularly *Nash Equilibrium* ones, emerge as a powerful tool to support both GENCOs and regulators. More precisely, such models attempt to mimic the dynamics of the market, aiming at reflecting the rational behavior of all market participants. Therefore, regulators may use this modeling framework to monitor market activities, filtering singular strategic behavior; and GENCOs can refine their offers following the Nash equilibrium solution [5–7].

Based on Non-Cooperative Game Theory, Nash equilibrium is characterized by a set of feasible strategies (e.g., offers in the pool-based market) such that neither competitor can improve its outcome from the market by modifying its offer if the remaining competitors "play" the equilibrium [8,9]. As a consequence, a Nash equilibrium

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Nomenclature

For expository and didactic purposes, in this section, the main sets, functions, and constants/variables used in this paper are highlighted and appropriately described.

Sets

$\mathcal{N}$ Set of players competing in the pool-based electricity market;
$\mathcal{M}(\cdot)$ Set of optimal dispatch and respective uniform electricity price;
Q_j Set of feasible quantity offers of player $j \in \mathcal{N}$;

Functions

$R_j(\cdot)$ Net revenue function of player $j \in \mathcal{N}$ in the pool-based electricity market.

Constants

c_j Marginal production cost of player $j \in \mathcal{N}$;
d Inelastic demand;
$\bar{q}_j$ Power capacity of player $j \in \mathcal{N}$;

Decision Variables

g_j Dispatch of player in the pool-based electricity market;
q_j Quantity offered in the pool-based electricity market by the player $j \in \mathcal{N}$
π Uniform electricity price

point can be interpreted as a market status from which no competitor has (unilateral) incentives to deviate from. Although such market equilibrium philosophy is extremely powerful, one of its main drawbacks is its computational burden. Several approaches to deal this problem have been studied in technical literature, but an efficient method to compute equilibria in real-case applications is still needed [10,11].

The mathematical structure of typical electricity market equilibrium problems belongs to the class of *Multi-Leader-Common-Follower* games [12]. In this setting, the *multi-leaders* represent the strategic offering model of each market participant and the *common-follower* describes the market-clearing process [13]. By exploring this modeling structure, we convert the bi-level model into a large-scale single-level equivalent optimization problem by making use of the Karush-Kuhn-Tucker (KKT) optimally conditions of the common-follower (market clearing) problem. The resulting large-scale model can be interpreted as an *Equilibrium Problem with Equilibrium Constraints* (EPEC) [14], suitable for the column-and-constraint-generation (CCG) algorithm [15].

Therefore, in this work, we design a two-stage (master-oracle) iterative CCG algorithm to efficiently compute Nash equilibria in pool-based electricity markets. From an algorithmic viewpoint, such primal-based decomposition technique has recently attracted considerably attention in technical literature due to its distinct computational performance and capability of handling more general optimization structures. In power systems applications, for instance, [16] adapt the CCG technology to solve the proposed robust bidding in pool-based electricity markets model. Additionally, [17] apply this decomposition technique to handle the proposed energy and reserve scheduling model and [18,19] similarly make use of this approach to tackle transmission expansion planning models.

In our proposed CCG algorithm, at a given iteration, the master problem finds a feasible offer for each GENCO (market participant), candidate for an equilibrium point, and, then, an oracle verifies if this (candidate) set of offers is indeed a Nash equilibrium. If it is not an equilibrium, a primal cut is introduced into the master problem removing the previous solution from its feasible set. The procedure iterates until a Nash equilibrium point is recovered. We highlight that the proposed master-oracle-based solution approach aims at iteratively recover a collection of feasible strategies for each GENCO that encompass the umbrella-set of strategies sufficient to recover the Nash equilibrium point. A salient feature of our methodology is the flexibility to specify an objective function to rank and select the best equilibrium point among the many that might exist.

To validate the efficiency of our method, a set of numerical experiments are performed and the computational time is compared with the solution of a large-scale EPEC using an off-the-shelf Mixed Integer Linear Programming (MILP) solver. The proposed solution approach overcomes the benchmark in the magnitude of 20 times on average and more than 30 times in the most demanding instances. The scalability of the full EPEC formulation is challenged even for medium-scale instances (i.e., by systems with more than 30 players), whilst the proposed approach was able to handle all instances in a reasonable computational time.

1.1. Objectives and contributions regarding existing literature

The main objective of this work is to devise an efficient methodology to identify a Nash equilibrium in pool-based electricity markets. For this purpose, a two-stage iterative algorithm is designed based on the CCG techniques [15]. From the perspective of computing Nash equilibria in pool-based electricity markets, several methods can be found in technical literature. For instance, [6,7,20] propose to solve the large-scale EPEC using specialized algorithms and relaxations. Additionally, by making use of the so-called Nikaido-Isoda function [21], the authors in [22,23] derived methodologies to identify Nash-Cournot equilibria in hydrothermal electricity markets. Finally, fixed-point-based algorithms, such as the one described in [24], are also popular in this particular application. Nevertheless, we argue that these methods may be challenged to scale to real systems. In fact, on the one hand, solve an EPEC involves handling a large-scale non-convex optimization problem and, on the other hand, both the use of Nikaido-Isoda functions and the fixed-point-based algorithms are recognized to suffer from convergence difficulties. It is worth mentioning that the methodology proposed in this work tackles both issues by handling iteratively small-scale non-convex optimization problems and also has convergence guarantees.

2. Pool-based electricity markets

The financial pillar of most power systems around the globe is supported by an auction-based market that feeds the system's physical operation. In this paper, we consider a market comprised of a sealed-bid uniform-price auction for which power producers submit a stack of pairwise linked price and quantity offers. An inelastic demand is assumed and the market clears at maximum social welfare. This price/quantity offer stack represents the declared production cost that producers are willing to sell their energy in the market, up to the respective quantity offered. The market is cleared by solving a least-cost supply/

demand matching problem, indicating the amount of energy each generator should produce and the market uniform price. In this work, for expository purposes, we consider that the strategic decisions are all concentrated on the amount of energy offered into the market, assuming that all players offer their marginal costs as price offers[2].

Formally, let n be the number of GENCOs competing in the market. We assume that the set of feasible offers of each player $j \in \mathcal{N} \triangleq \{1, \cdots, n\}$ is a bounded subset of integers, i.e.,

$$Q_j = \{q_j \in \mathbb{Z} \mid 0 \le q_j \le \bar{q}_j\}, \tag{1}$$

where q_j stands for the amount of energy offered into the pool-based electricity market and $\bar{q}_j$ denotes the power capacity of player $j \in \mathcal{N}$. The market clearing design considered in this work encompasses a single-period (one-step ahead) single-node economic dispatch [3] as presented next.

$$\min_{g_i} \sum_{i \in \mathcal{N}} c_i g_i \tag{2}$$

subject to:

$$\textstyle\sum_{i \in \mathcal{N}} g_i = d; \quad : \pi \tag{3}$$

$$0 \le g_i \le q_i, \quad :(\underline{\beta}_i, \bar{\beta}_i) \quad \forall\ i \in \mathcal{N}. \tag{4}$$

Problem (2)–(4) is a linear and continuous mathematical programming problem, which seeks for the most economical dispatch $\boldsymbol{g} \triangleq (g_1, \cdots, g_n)$ to meet an inelastic demand d with $\boldsymbol{c} \triangleq \{c_1, \cdots, c_n\}$ indicating the marginal production cost of each player. The set of constraints in (4) assures that, for each player $j \in \mathcal{N}$, the quantity cleared in the market is non-negative and upper-bounded by the respective maximum offered amount q_j. It worth to highlight that, although for each player $j \in \mathcal{N}$ we assume an integer feasible offer set Q_j, the market-clearing dispatch $\boldsymbol{g}$ is a continuous variable, as usual in most electricity markets [6,7,16]. To ease presentation, we identify the Lagrange multipliers $(\pi, \bar{\boldsymbol{\beta}}, \underline{\boldsymbol{\beta}})$ of each constraint after colons for future reference. We highlight that, following the uniform-price-auction theory [25], multiplier π recovers the price for electricity. Next, we formally outline the competitors strategic behavior in the pool-based electricity market (2)–(4).

2.1. Strategic behavior in pool-based electricity market

Typically, the main objective of economic agents is to achieve high profit levels in the market they are competing in by strategically adjusting their game plan. In the particular context of this work, the GENCOs' profit basically resumes to the amount of power that is cleared in the market by the price for electricity discounted by the respective marginal production costs. Formally, for a given set of offers $\boldsymbol{q} \triangleq \{q_1, \cdots, q_n\}$, let $\mathcal{M}(\boldsymbol{q})$ to denote the set of optimal dispatch of each player in the market and the respective uniform electricity price, i.e.,

$$\mathcal{M}(\mathbf{q}) \triangleq \{(\mathbf{g}, \pi) \in \mathbb{R}^n_+ \times \mathbb{R} \mid (\mathbf{g}, \pi) \text{ solves (2)-(4)}\}. \tag{5}$$

Then, the net revenue of a given player $j \in \mathcal{N}$ in the pool-based electricity market can be written as the following *Mathematical Program with Equilibrium Constraints* (MPEC) [13]:

$$R_j\left(q_j, \boldsymbol{q}_{-j}\right) = \max_{\mathbf{g}, \pi} \left\{(\pi - c_j) g_j \mid (\mathbf{g}, \pi) \in \mathcal{M}(\boldsymbol{q})\right\}, \tag{6}$$

where, following the game-theoretical standard notation, $\boldsymbol{q}_{-j}$ stands for the quantity offers from all GENCOs but company $j \in \mathcal{N}$. Although intuitive, the payoff function (6) cannot be efficiently computed due to: (i) the non-representable format of the optimal solution set (5); and (ii) the bilinear objective function: $(\pi - c_j) g_j$.

To tackle the first issue, recall that the clearing problem (2)–(4) is linear and continuous. Thus, the set of optimal solution points (5) can be exactly represented[3] by the KKT system of the clearing problem:

$$\mathcal{M}(\boldsymbol{q}) = \{(\boldsymbol{g}, \pi) \in \mathbb{R}^n_+ \times \mathbb{R} \mid \exists\ (\bar{\boldsymbol{\beta}}, \underline{\boldsymbol{\beta}}) \in \mathbb{R}^n_+ \times \mathbb{R}^n_+;$$

$$c_i - \pi + \bar{\beta}_i - \underline{\beta}_i = 0, \quad \forall\ i \in \mathcal{N}; \tag{7}$$

$$\sum_{i \in \mathcal{N}} g_i = d; \tag{8}$$

$$0 \le g_i \le q_i, \quad \forall\ i \in \mathcal{N}; \tag{9}$$

$$\bar{\beta}_i (q_i - g_i) = 0, \quad \forall\ i \in \mathcal{N}; \tag{10}$$

$$\underline{\beta}_i g_i = 0, \quad \forall\ i \in \mathcal{N}; \}. \tag{11}$$

Secondly, in order to tackle the bilinear objective function $(\pi - c_j) g_j$, we follow a similar procedure described in [28] and [16]. For each player $j \in \mathcal{N}$, we firstly multiply Eq. (7) by g_j, leading to:

$$(\pi - c_j) g_j = \bar{\beta}_j g_j - \underline{\beta}_j g_j, \quad \forall\ j \in \mathcal{N}. \tag{12}$$

Furthermore, by adapting Eqs. (10) and (11) into (12), we have that the bilinear objective function in (6) can be replaced, for a given set of quantity offers $\boldsymbol{q}$, by the following linear equation:

$$(\pi - c_j) g_j = \bar{\beta}_j q_j, \quad \forall\ j \in \mathcal{N}. \tag{13}$$

Finally, an undesired byproduct of the representation of the optimality set through a KKT system is the need to efficiently handle primal-dual complementarity conditions. Although many techniques have been discussed in technical literature to tackle this source of non-linearity, in this work, we make use of standard Fortuny-Amat approach [29]. In this context, the net revenue function (6) of a given player $j \in \mathcal{N}$ can be re-written as the following MILP problem

$$R_j(q_j, \boldsymbol{q}_{-j}) = \max_{\substack{\mathbf{g}, \pi, \bar{\boldsymbol{\beta}}, \underline{\boldsymbol{\beta}}, \\ \bar{\boldsymbol{\eta}}, \underline{\boldsymbol{\eta}}}} \bar{\beta}_j q_j \tag{14}$$

subject to:

$$c_i - \pi + \bar{\beta}_i - \underline{\beta}_i = 0, \quad \forall\ i \in \mathcal{N}; \tag{15}$$

$$\sum_{i \in \mathcal{N}} g_i = d; \tag{16}$$

$$0 \le \bar{\beta}_i \le M\ \bar{\eta}_i, \quad \forall\ i \in \mathcal{N}; \tag{17}$$

$$0 \le q_i - g_i \le M(1 - \bar{\eta}_i), \quad \forall\ i \in \mathcal{N}; \tag{18}$$

$$0 \le \underline{\beta}_i \le M\ \underline{\eta}_i, \quad \forall\ i \in \mathcal{N}; \tag{19}$$

$$0 \le g_i \le M(1 - \underline{\eta}_i), \quad \forall\ i \in \mathcal{N}; \tag{20}$$

$$\bar{\eta}_i, \underline{\eta}_i \in \{0, 1\}, \quad \forall\ i \in \mathcal{N}. \tag{21}$$

Problem (14)–(21) is a computationally tractable formulation of (6), suitable for direct implementation on commercial solvers. Eqs. (17)–(21) represent the Fortuny-Amat exact representation of the non-linear set of constraints (9)–(11), where M is a sufficient large number and $(\bar{\boldsymbol{\eta}}, \underline{\boldsymbol{\eta}})$ are binary vectors determining the complementary statuses of primal constraints and dual variables.

2.2. Nash equilibrium in pool-based electricity markets

In the context of this work, a Nash equilibrium can be roughly interpreted as a set of offers such that the net revenue of each generation

[2] This competitive representation is usually referred to as a *Cournot Competition* and the respective equilibrium called *Nash-Cournot Equilibrium* [20].

[3] We refer to [26] and [27] for the main properties and a wide discussion regarding this representation.

company $j \in \mathcal{N}$ at the equilibrium point can not be improved if only company j deviates from this point by choosing a different quantity offer. Formally, a sufficient condition for a set of offers $\boldsymbol{q}^{(e)} \triangleq (q_1^{(e)}, \cdots, q_n^{(e)})$ to be characterized as a Nash equilibrium point is presented in (22).

$$R_j\left(q_j^{(e)}, \boldsymbol{q}_{-j}^{(e)}\right) \geq R_j\left(q_j, \boldsymbol{q}_{-j}^{(e)}\right), \quad \forall\ q_j \in \mathcal{Q}_j,\ j \in \mathcal{N}. \tag{22}$$

Note that, to find an equilibrium point $\boldsymbol{q}^{(e)}$, it is necessary to solve a large-scale system of inequalities with a combinatorial number of equations. This fact steams from the enumeration of all feasible offers of each generation company $j \in \mathcal{N}$ in the right-hand-side of (22). In technical literature, a typical approach to computationally handle this combinatorial system of inequalities is to embed it into an optimization framework and make use of specialized algorithms to identify an equilibrium point [6,7,13]. For instance, we can co-optimize the total net revenue of all GENCOs under the equilibrium point, i.e., solve the following optimization problem:

$$\max_{\boldsymbol{q}^{(e)}} \sum_{j \in \mathcal{N}} R_j\left(q_j^{(e)}, \boldsymbol{q}_{-j}^{(e)}\right) \tag{23}$$

subject to:

$$R_j\left(q_j^{(e)}, \boldsymbol{q}_{-j}^{(e)}\right) \geq R_j\left(q_j, \boldsymbol{q}_{-j}^{(e)}\right), \qquad \forall\ q_j \in \mathcal{Q}_j,\ j \in \mathcal{N}; \tag{24}$$

$$q_j^{(e)} \in \mathcal{Q}_j, \qquad \forall\ j \in \mathcal{N}. \tag{25}$$

Note that different objectives can be chosen, e.g., minimize the spot price for electricity at equilibrium [6]. Regardless of this choice, we highlight that solving the optimization problem is still challenging, even for medium-sized systems. Therefore, to tackle this issue, in the next section, we present a master-oracle decomposition algorithm based on the column-and-constraint generation framework [15]. The core of this approach is to iteratively identify, for each generation company $j \in \mathcal{N}$, a subset $\hat{\mathcal{Q}}_j \subset \mathcal{Q}_j$ of offers such that the solution of (23)–(25) with $\hat{\mathcal{Q}}_j$ in (24) ensures that (22) holds.

3. Solution methodology

Generally speaking, the main purpose of decomposition-based algorithms is to iteratively recover important features of the original (large-scale) problem, such that solving a (potentially smaller) problem considering only these features is sufficient to achieve optimality of the original problem. In technical literature, one of the most popular structures of this class of algorithms is based on constructing a so-called master/oracle iterative process. More precisely, on the one hand, the *master problem* is designed to identify a candidate feasible solution, for instance, by solving a relaxed formulation of the original problem; and, one the other hand, the *oracle* is usually built to check if this candidate solution belongs indeed to the optimality set of the original problem. If not, then a feature of the original problem (usually referred to as *cuts* in technical literature) is recovered and appended into the master problem. The procedure thus iterates until convergence [30].

The solution methodology proposed in this work follows this problem-decomposition iterative rationale. More specifically, at a given iteration k, the master problem is designed to identify a candidate feasible point $\boldsymbol{q}^{(k)}$. In the sequel, the oracle identifies the best response of each GENCO $j \in \mathcal{N}$ to its rival's offer $\boldsymbol{q}_{-j}^{(k)}$ and checks if the candidate point $\boldsymbol{q}^{(k)}$ is, in fact, a Nash equilibrium. If not, a *primal cut* is included into the master problem and a new iteration resumes. In Fig. 1, an overview of the proposed CCG algorithm is presented and, in the next two subsections, we carefully describe both the master and oracle problems. For didactic purposes, we begin with the oracle problem.

3.1. Oracle problem

The main goal of the oracle problem in the proposed algorithm is to verify if a candidate point $\boldsymbol{q}^{(k)}$ is indeed a Nash equilibrium. To achieve this purpose, for each player in the market, we need to identify their best response against the rivals' strategy of playing the current candidate point. In other words, for each generation company $j \in \mathcal{N}$, we need to solve the following optimization problem

$$q_j^* \in \arg\max_{q_j \in \mathcal{Q}_j} \left\{ R_j\left(q_j, \boldsymbol{q}_{-j}^{(k)}\right) \right\}. \tag{26}$$

If, for each GENCO $j \in \mathcal{N}$, the respective net revenue on the best response q_j^* is not strictly higher than the value at the candidate point, i.e., $R_j\left(q_j^*, \boldsymbol{q}_{-j}^{(k)}\right) \leq R_j\left(q_j^{(k)}, \boldsymbol{q}_{-j}^{(k)}\right)$, than no economic agent has incentive change its offer, configuring thus a Nash equilibrium. In our particular context, solving (26) for a given generation company $j \in \mathcal{N}$ is

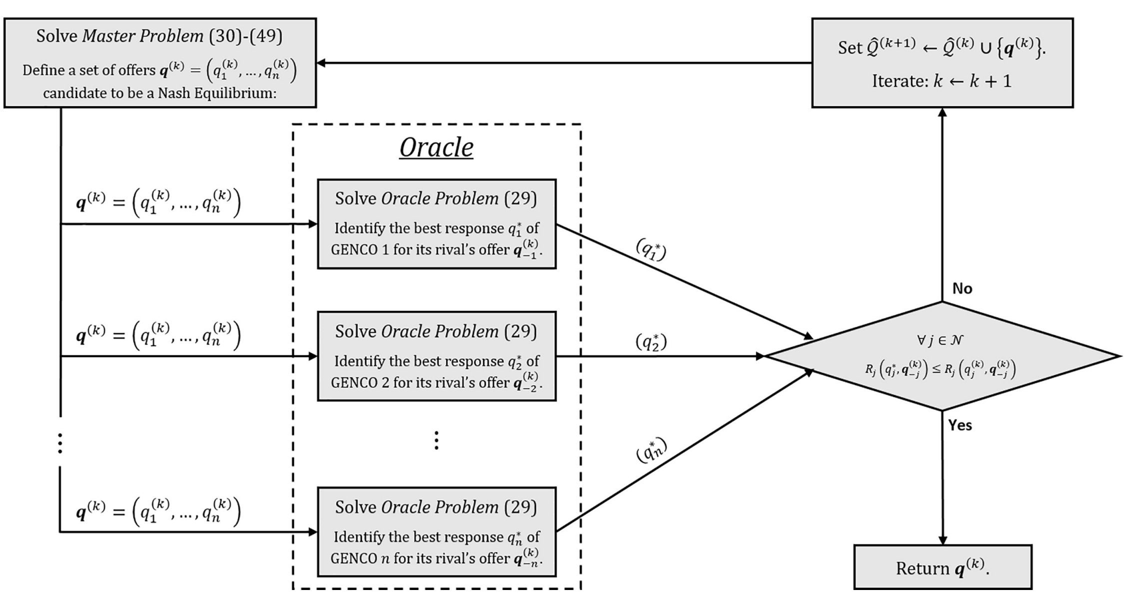

Fig. 1. Overview of the proposed CCG algorithm.

challenging due to the bilinear objective function $\overline{\beta}_j q_j$. To address this new source of non-linearity, firstly recall that the feasible set of offers of a given GENCO $j \in \mathcal{N}$ is assumed to be a bounded subset of integers, which can be conveniently expressed by the following binary expansion:

$$q_j = \sum_{l=1}^{\lfloor \log_2(\overline{q}_j) \rfloor + 1} 2^{(l-1)} \gamma_{j,l} \in [0, \overline{q}_j], \tag{27}$$

with γ_j a binary vector. Therefore, the bilinear product $\overline{\beta}_j q_j$ can be recast by a set of linear equations using exact linearization schemes via disjunctive inequalities[4] [31]. More precisely, for each $j \in \mathcal{N}$, we can identify the bilinear product as follows $\overline{\beta}_j q_j \leftrightarrow \tau_j$, with τ_j feasible within the following set:

$$\mathcal{T}(\overline{\beta}_j, \boldsymbol{\gamma}_j) = \Big\{ \tau_j \in \mathbb{R} \mid \exists\ \boldsymbol{\phi}_j \in \mathbb{R}_+^{\lfloor \log_2(\overline{q}_j) \rfloor + 1}; \\ 0 \le \phi_{j,l} \le M \gamma_{j,l}, \quad \forall\ l \in \{1, \cdots, \lfloor \log_2(\overline{q}_j) \rfloor + 1\}; \\ 0 \le \overline{\beta}_j - \phi_{j,l} \le M(1 - \gamma_{j,l}), \quad \forall\ l \in \{1, \cdots, \lfloor \log_2(\overline{q}_j) \rfloor + 1\}; \\ \tau_j = \sum_{l=1}^{\lfloor \log_2(\overline{q}_j) \rfloor + 1} 2^{(l-1)} \phi_{j,l}; \Big\}. \tag{28}$$

In this formulation, $\boldsymbol{\phi}_j$ exactly recovers the bilinear product between the binary vector γ_j and the continuous variable $\overline{\beta}_j$. Therefore, the non-convex optimization problem (26) resumes to the following MILP problem:

$$q_j^* \in \underset{\substack{q_j, \boldsymbol{\gamma}_j, \boldsymbol{\phi}_j, \tau_j, \\ \mathbf{g}, \pi, \overline{\boldsymbol{\beta}}, \underline{\boldsymbol{\beta}}, \overline{\boldsymbol{\eta}}, \underline{\boldsymbol{\eta}}}}{argmax} \left\{ \tau_j \ \middle|\ \begin{array}{c} \text{Constraints (15)-(21) and (27);} \\ \tau_j \in \mathcal{T}(\overline{\beta}_j, \boldsymbol{\gamma}_j); \\ \boldsymbol{\gamma}_j \in \{0, 1\}^{\left\lfloor \log_2\left(\overline{q}_j\right) \right\rfloor + 1} \end{array} \right\}. \tag{29}$$

Next, we perform a similar procedure done in this section for the master problem presented.

3.2. Master problem

At a given iteration k, let $\hat{Q}_j^{(k)} \subset Q_j$ be a subset of quantity offers of a given generation company $j \in \mathcal{N}$. The master problem is thus designed to identify a novel feasible set of offers $\boldsymbol{q}^{(k)}$, candidate for being an equilibrium point. This task is done by solving the optimization problem (23)–(25) with $\hat{Q}_j^{(k)}$ in (24). Following the reformulation procedures made in the previous sections, problem (23)–(25) can be suitably formulated as the following mixed-integer linear programming problem:

$$\max_{\Xi^{(k)}, \Xi^{\left(q_j\right)}} \ \textstyle\sum_{j \in \mathcal{N}} \tau_j^{(k)} \tag{30}$$

subject to:

$$\tau_j^{(k)} \ge \overline{\beta}_j^{(q_j)} q_j, \qquad \forall\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}; \tag{31}$$

$$c_j - \pi^{(k)} + \overline{\beta}_j^{(k)} - \underline{\beta}_j^{(k)} = 0, \qquad \forall\ j \in \mathcal{N}; \tag{32}$$

$$c_j - \pi^{(q_j)} + \overline{\beta}_j^{(q_j)} - \underline{\beta}_j^{(q_j)} = 0, \qquad \forall\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}; \tag{33}$$

$$\sum_{j \in \mathcal{N}} g_j^{(k)} = d; \tag{34}$$

$$\sum_{i \in \mathcal{N}} g_i^{(q_j)} = d, \qquad \forall\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}; \tag{35}$$

$$0 \le \overline{\beta}_j^{(k)} \le M\ \overline{\eta}_j^{(k)}, \qquad \forall\ j \in \mathcal{N}; \tag{36}$$

$$0 \le \overline{\beta}_i^{(q_j)} \le M\ \overline{\eta}_i^{(q_j)}, \qquad \forall\ i \in \mathcal{N},\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}; \tag{37}$$

$$0 \le q_j^{(k)} - g_j^{(k)} \le M\left(1 - \overline{\eta}_j^{(k)}\right), \qquad \forall\ j \in \mathcal{N}; \tag{38}$$

$$0 \le q_j - g_j^{\left(q_j\right)} \le M\left(1 - \overline{\eta}_j^{\left(q_j\right)}\right), \qquad \forall\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}; \tag{39}$$

$$0 \le q_i^{(k)} - g_i^{\left(q_j\right)} \le M\left(1 - \overline{\eta}_i^{\left(q_j\right)}\right), \qquad \forall\ i \in \mathcal{N} \setminus \{j\},\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}; \tag{40}$$

$$0 \le \underline{\beta}_j^{(k)} \le M\ \underline{\eta}_j^{(k)}, \qquad \forall\ j \in \mathcal{N}; \tag{41}$$

$$0 \le \underline{\beta}_i^{(q_j)} \le M\ \underline{\eta}_i^{(q_j)}, \forall\ i \in \mathcal{N},\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}; \tag{42}$$

$$0 \le g_j^{(k)} \le M\left(1 - \underline{\eta}_j^{(k)}\right), \qquad \forall\ j \in \mathcal{N}; \tag{43}$$

$$0 \le g_i^{\left(q_j\right)} \le M\left(1 - \underline{\eta}_i^{\left(q_j\right)}\right), \qquad \forall\ i \in \mathcal{N},\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}; \tag{44}$$

$$q_j^{(k)} = \sum_{l=1}^{\left\lfloor \log_2\left(\overline{q}_j\right) \right\rfloor + 1} 2^{(l-1)} \gamma_{j,l}^{(k)} \in [0, \overline{q}_j], \qquad \forall\ j \in \mathcal{N}; \tag{45}$$

$$\tau_j^{(k)} \in \mathcal{T}\left(\overline{\beta}_j^{(k)}, \boldsymbol{\gamma}_j^{(k)}\right), \qquad \forall\ j \in \mathcal{N}; \tag{46}$$

$$\overline{\eta}_j^{(k)}, \underline{\eta}_j^{(k)}, \in \{0, 1\}, \qquad \forall\ j \in \mathcal{N}; \tag{47}$$

$$\overline{\eta}_i^{(q_j)}, \underline{\eta}_i^{(q_j)}, \in \{0, 1\}, \qquad \forall\ i \in \mathcal{N},\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}; \tag{48}$$

$$\boldsymbol{\gamma}_j^{(k)} \in \{0, 1\}^{\lfloor \log_2(\overline{q}_j) \rfloor + 1}, \qquad \forall\ j \in \mathcal{N}. \tag{49}$$

For the sake of brevity, we identify the decision variables in (30)–(49) with the vectors $\Xi^{(k)} = (\boldsymbol{q}^{(k)}, \boldsymbol{\gamma}^{(k)}, \boldsymbol{\phi}^{(k)}, \boldsymbol{\tau}^{(k)}, \mathbf{g}^{(k)}, \pi^{(k)}, \overline{\boldsymbol{\beta}}^{(k)}, \underline{\boldsymbol{\beta}}^{(k)}, \overline{\boldsymbol{\eta}}^{(k)}, \underline{\boldsymbol{\eta}}^{(k)})$ and $\Xi^{(q_j)} = \left(\mathbf{g}^{(q_j)}, \pi^{(q_j)}, \overline{\boldsymbol{\beta}}^{(q_j)}, \underline{\boldsymbol{\beta}}^{(q_j)}, \overline{\boldsymbol{\eta}}^{(q_j)}, \underline{\boldsymbol{\eta}}^{(q_j)}\right)$, $\forall\ q_j \in \hat{Q}_j^{(k)},\ j \in \mathcal{N}$. The MILP problem (30)–(49) can be interpreted as a particular instance of an EPEC, suitable for off-the-shelf commercial MILP solvers.

It worth to highlight that, by setting $\hat{Q}_j^{(k)} = Q_j,\ \forall\ j \in \mathcal{N}$, problem (30)–(49) resumes to the large-scale complete EPEC formulation that current technical literature solves [6,7,13]. Needless to say, this full enumerated problem is extremely challenging to solve for real-sized power systems due to the exponential number of constraints within a MILP problem, which advocate in favor of the CCG decomposition algorithm proposed in this work and thoroughly described in the next subsection.

[4] Note that this approach can be adapted to any other discretization of the set of offers with a constant step size.

Initialization:

Set $k \leftarrow 1$.

Choose an initial subset $\hat{Q}_j^{(1)} \subset Q_j, \ \forall \ j \in \mathcal{N}$.

Iteration $k \geq 1$:

Step 1 – *Master Problem*: Identify a feasible offer $\boldsymbol{q}^{(k)}$ by solving the MILP problem (30)–(49).

Step 2 – *Oracle Problem*: $\forall \ j \in \mathcal{N}$, identify the best response q_j^* for its rival's offer $\boldsymbol{q}_{-j}^{(k)}$ by solving the MILP problem (29).

if $\exists \ j \in \mathcal{N} \mid R_j(q_j^{(k)}, \boldsymbol{q}_{-j}^{(k)}) < R_j(q_j^*, \boldsymbol{q}_{-j}^{(k)})$

Update $\hat{Q}_j^{(k+1)} \leftarrow \hat{Q}_j^{(k)} \cup \{q_j^*\}, \ \forall \ j \in \mathcal{N}$.

Set $k \leftarrow k + 1$.

else

Set $\boldsymbol{q}^{(e)} \leftarrow \boldsymbol{q}^{(k)}$.

Return $\boldsymbol{q}^{(e)}$.

end if

Algorithm 1. – Column-and-Constraint Generation Algorithm.

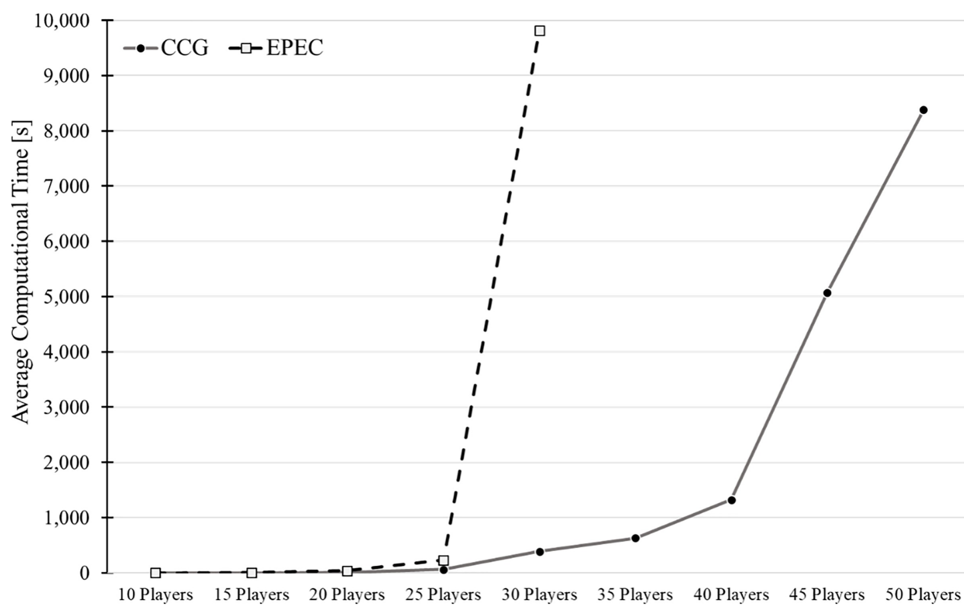

Fig. 2. Average computational time (seconds) to solve the sampled instances for market sizes of $n \in \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$.

3.3. Column-and-constraint generation algorithm

The proposed two-stage algorithm to identify a Nash equilibrium in pool-based electricity markets is based on the iterative solution of the master (30)–(49) and oracle (29) problems. This iterative process is carried out until a candidate for an equilibrium point, identified by the master problem, is verified as a Nash equilibrium by the oracle problem. The proposed solution algorithm, as illustrated in Fig. 1, is summarized in Algorithm 1.

It is worth highlighting two interesting features of our proposed solution methodology, which can be explored in future works. Firstly, **Step 2** of Algorithm 1 is suitable for parallel computing, since each optimization problem can be solved independently for each player $j \in \mathcal{N}$. Secondly, the computational efficiency of Algorithm 1 can also be significantly improved by an adequate choice of the initial subset $\hat{Q}_j^{(1)} \subset Q_j, \ \forall \ j \in \mathcal{N}$, e.g., by fast heuristic approaches. We highlight, however, that both parallel computing and heuristic initialization procedures are not in the scope of this work.

In the next section, a set of numerical experiments are conducted aiming at illustrating the capability of the proposed CCG algorithm, comparing its computational performance with the solution of an EPEC via MILP solvers.

4. Computational experiments

To illustrate the solution capability of the proposed methodology, in this section, we perform a computational comparison between the CCG method proposed in this work and the direct solution of the fully-enumerated EPEC formulation via an off-the-shelf commercial MILP solver. In this numerical experiment, we analyze the computational effectiveness of each method as the number of players (GENCOs) in the market (n) increases. The instances, described by the parameters $(d, \boldsymbol{c}, \bar{\boldsymbol{q}})$, were designed to scale with the number of players in a

Table 1
Average computational time (in seconds) of the worst 4 instances for each market size in both solution approaches analyzed.

	Number of Players (GENCOs)								
	10	15	20	25	30	35	40	45	50
CCG	1.83	2.14	39.96	242.27	1483.08	1916.11	4022.13	18863.70	30480.64
EPEC	1.34	4.92	151.03	927.69	48167.07	-	-	-	-

meaningful manner as described next. For a given n, the demand for electricity is scaled by the number of players in the magnitude of 20, i.e., $d = 20n$ MW. Then, each power plant capacity was sampled such that the system total capacity is equal to $1.2d$. Additionally, in order to assure dispatch feasibility in all instances regardless of the strategic behavior of the GENCOs, we assume a deficit generator with marginal cost of 1000 $/MW and total capacity equals to the system demand. Furthermore, each player marginal cost follows an Uniform distribution between 0 and 100 $/MW (i.e, $c_j \sim \mathcal{U}(0, 100), \ \forall \ j \in \mathcal{N}$).

Following this structure, a total of 20 instances were generated for the following market sizes $n \in \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$ and a running time limit of 172,800 seconds (48hrs) per instance were allowed. All numerical results were obtained on a Dell Precision® T7600 Xeon® E5-2687W 3.10 GHz with 128 GB of RAM machine, with Gurobi® Solver 8.0 under JuMP®.

Fig. 2 presents the average computing time to solve sampled instances for each market size considered in this study. We highlight that the performance of both solution approaches (CCG and EPEC) are indistinguishable up to $n = 20$ players. Nevertheless, the computational scalability of the EPEC formulation, (30)–(49), is rapidly challenged due to the exponential increase when more than 20 players are considered. For instance, the full EPEC approach is 20 times slower, on average, than the proposed CCG algorithm, for $n = 30$. Additionally, for more than 30 players in the market, almost all instances sampled could not be solved by the full EPEC formulation. On the other hand, the proposed CCG algorithm was able to solve all instances of every market size analyzed in a reasonable computational time.

In order to evaluate the potential computational difficulty that each method may face to solve adverse instances, in Table 1, we present the average computational time of the worst four instances for each market size n analyzed. Note that the discrepancy between both methods from this worst-case viewpoint is even higher. For instance, the solution time of the full EPEC approach is 4 times slower than the proposed CCG algorithm for $n = 20$ and $n = 25$, and reaches roughly 30 times for $n = 30$.

Finally, we present a comparative analysis for the strategic and perfect competition equilibria[5] for the particular instance with $n = 35$ players. Firstly, we highlight that, as the master problem aims at identifying an equilibrium with the highest net revenue for each GENCO, the Nash equilibrium found thus explores the deficit cost as the marginal generator in every instance sampled. As a consequence, the spot prices at equilibrium are equal to the deficit marginal cost and, since the numerical experiments were designed to have an excess of capacity, some GENCOs withhold capacity in the Nash equilibrium. In fact, we observed that the average offer represents roughly 90% of the GENCOs capacity. Additionally, the lowest offers on each instance are on average 10% of the generators capacities and, when filtering the players that do not offer full capacity, the average offer is approximately 40% of the GENCO's capacity. On the other hand, considering the perfect competition, the spot price is, on average, 85% lower than the most expensive generator.

[5] We refer to [32] for a wide discussion on perfect competition equilibria.

5. Conclusion

Competition among power generators in pool-based electricity markets represents a key element on the restructuring design occurred in most power systems around the globe. In this context, equilibrium analysis, in particular the Nash equilibrium, is of utmost importance for both GENCOs and regulators. However, an important hindrance to adequately perform such equilibrium analysis is an efficient methodology to identify the Nash equilibrium in large-scale power systems. To tackle this issue, in this work, we adapt the Column-and-Constraint Generation (CCG) techniques to design a decomposition-based iterative algorithm to efficiently compute Nash equilibrium in large-scale pool-based electricity markets. More specifically, the proposed solution approach is based on a master-oracle decomposition, in which, the master problem identifies a set of offers candidate for a Nash equilibrium point and the oracle checks if this candidate point is indeed an equilibrium by computing the best response of each GENCO, given their rivals "play" the equilibrium.

Numerical experiments comparing the computational performance of the proposed CCG algorithm with the benchmark approach, namely, the solution of a large-scale EPEC via MILP solvers, corroborates the effectiveness of the proposed method. We identify that the proposed solution approach overcomes the benchmark (based on the full EPEC formulation) in the magnitude of 20 times on average and more than 30 times for the most demanding instances. Additionally, the scalability of the benchmark is challenged as it was not able to solve most of the medium-scale instances (i.e., systems with more than 30 players), whilst the proposed CCG algorithm could handle all instances considered in the experiment in a reasonable computational time.

It is important to highlight that the solution methodology proposed in this work is an exact method and, after convergence, it precisely solves the Nash equilibrium problem defined in (23)–(25). Nevertheless, ongoing research includes parallel computing and the combination of the CCG algorithm here proposed with heuristic-based solution techniques aiming at enhancing the computational capability of the proposed method.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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