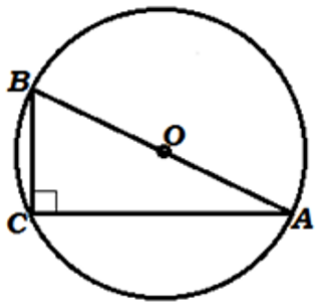
Problem 1

$\lim_{x \to \pi/2} \frac{1-sin(x)}{cos^2(x)} = ?$

Answer: $\frac{1}{2}$

Find radius of circle circumscribing rectangular triangle ABC, if its legs (cathetus's) equal to 24 and 10 cm.



Given:
ΔABC is a rectangular triangle.
AB is the hypotenuse
AC=24 cm.
BC=10 cm.
Circle O is circumscribing rectangular triangle.
Find: $r-?$

Answer: $r = 13$

Problem 2

$\int_0^{\sqrt{3}} \frac{xdx}{\sqrt{x^4+16}} = ?$

Answer: $\frac{\ln(2)}{2}$

Problem 3

Among 30 students in a group, 6 students received grade "5" for the exam, 10 students - "4", 9 students - "3", the rest - "2". Find the probability that 3 students, called to the blackboard, got grade "2" for the exam.

Answer: $P = 0.002$

Problem 4

What we know about the relation between second derivative $y = f''(x)$ with the original function $y = f(x)$?

Answer:

1. Function $y = f(x)$ is convex downward at intervals, where the second derivative is positive

$f''(x) > 0$.

2. Function $y = f(x)$ is convex upward at intervals, where the second derivative is negative

$f''(x) < 0$.

3. Function $y = f(x)$ has critical points of the second kind at the points, where the second derivative equals to zero or does't exist (only interior points of the domain of definition of a function. Points at the ends of the domain of definition are not considered).

4. Function $y = f(x)$ has inflection points at the points at which the second derivative changes its sign.

5. Taking into account that x_0 is a maximum point of $f(x)$ if $f'(x_0) = 0$ and $f''(x_0) < 0$, maximum points, if they exist, then, they are below the axis OX on the plot. Correspondingly, x^* is a minimum point of $f(x)$ if $f'(x_0) = 0$ and $f''(x_0) > 0$, thus minimum points, if they exist, then, they are above the axis OX on the plot.

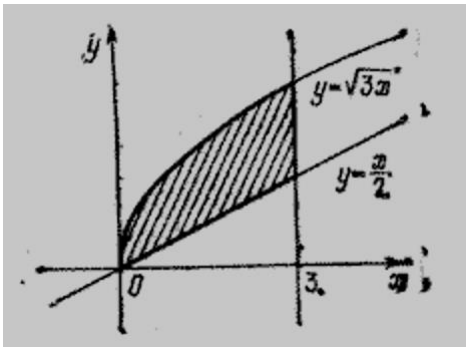
Problem 5

Construct equation of line, where the distance of every point (belonging to this line) from the point $A(1; 2)$ is two times greater than that from the point $B(4; 5)$.

Answer: $(x-5)^2 + (y-6)^2 = \left(2\sqrt{2}\right)^2$

Problem 6

Find the volume of the body formed by the rotation around the abscissa of a figure bounded by lines $y = \sqrt{3x}, y = \frac{x}{2}, x = 3$.



Answer: $V = \frac{45}{4}\pi$

Problem 7

Find the derivative in respect to x of a function

$$F(x) = \int_{\ln x}^{x^2} e^{t^2} dt$$

Answer: $\frac{2x^2 e^{x^4} - x^{\ln x}}{x}$

Problem 8

How many real solutions does this equation have?

$x^3 = 10 - x$

Answer: 1 solution (graphical representation can help to solve the problem).

Problem 9

Find the sum of an infinitely decreasing geometric progression: $12, 4, \frac{4}{3}, \ldots$

Answer: 18

Problem 10

Find the quotient z_1/z_2 from dividing two complex numbers

$z_1 = 2 + 5i$

$z_2 = 4 - 3i$

Answer: $\frac{z_1}{z_2} = -\frac{7}{25} + \frac{26}{25} i$

Problem 11

Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} -1 & -6 \\ 2 & 6 \end{pmatrix}$$

Answer: $\lambda_1 = 2, \lambda_2 = 3$

Problem 12

Find the sum of the angles of the convex

a) pentagon

b) hexagon

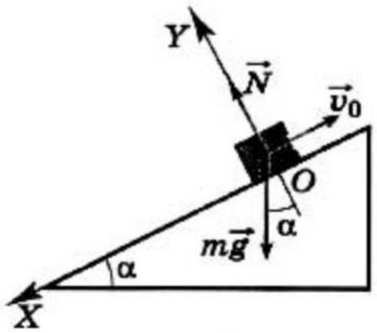
c) decagon

Answer: a) 540°; b) 720°; c) 1440˚.

PHYSICS

Problem 1

As a result of the push, a bar began to slide up over the inclined plane from point O with an initial velocity $v_0 = 4.4$ m/s. Determine the position of the bar relative to the point O in the time interval $t_1 - 2$s after the start of its movement, if the angle of inclination of the plane to the horizon is $\alpha = 30^o$. Do not take friction into account.



Answer: $X = 1$ m.

Problem 2

On a smooth horizontal surface there is a ball of mass *M*. This ball is bumped by another ball of mass *m*, moving with velocity $\vec{V}$. There is elastic central impact between them. Find the velocities $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ of the balls after the collision.

Answer:

$\boldsymbol{V_1 = \frac{m-M}{m+M}V} \quad \boldsymbol{V_2 = \frac{2m}{m+M}V}$

Problem 3

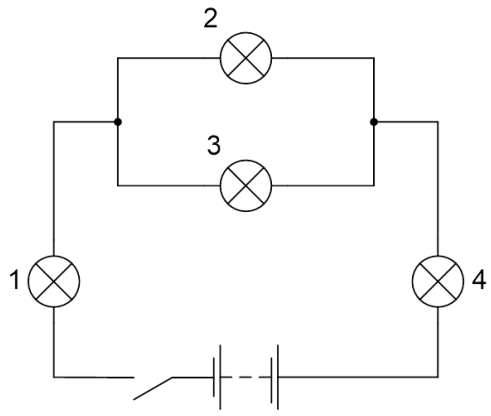
A mathematical pendulum of length 1 m oscillates with an amplitude of 1 cm. How long does it take to travel along the path of 1 cm, if the pendulum passes the equilibrium position at the initial moment?

Answer: Period of oscillations of pendulum is 2s. To travel the path that is equal to the amplitude, the pendulum spends 0.5 s.

Problem 4

Four identical lamps are connected to a constant voltage source (see figure below). Determine the current in each lamp if the voltage at the source is 30 V.

Given: $\boldsymbol{U = 30V}$, $R_1 = R_2 = R_3 = R_4 = 6\ Ohm$
Find: l_1, l_2, l_3, l_4



Answer: $l_1 = l_4 = 2A; l_2 = l_3 = 1A$

Problem 5

The plates of a flat capacitor are supplied with a voltage that varies linearly with time: U = k · t. Determine bias current density between the plates of capacitor. The distance between the plates is *d*, the dielectric constant of the substance between the plates is *ε*.

Given data: $k = 12\ V/s$, $\varepsilon = 2.7; d = 2 \cdot 10^{-6}$ m

Answer: Bias current density is 143 microampere

Problem 6

What are the directions of the electric and magnetic fields oscillations in a free electromagnetic wave?

Answer

Oscillations of electric and magnetic fields in a free electromagnetic wave occur in mutually perpendicular planes in directions perpendicular to the direction of wave propagation

Problem 7

What are the processes that occur in an isolated system when the entropy remains unchanged?

Answer

According to the second law of thermodynamics, if the system is isolated, then entropy does not change when reversible processes take place in it, and it increases in irreversible processes.

Problem 8, isochoric process

When the thick-walled metal cylinder with gas was taken out of the room to the street, the pressure in the cylinder fell by 10%. What is the temperature in the street, if the room is 20° C?

Answer: Temperature in street is $\boldsymbol{-9^oC}$

Problem 9

How will the average quadratic velocity of molecules change with increasing of temperature by 4 times?

Как изменится средняя квадратичная скорость движения молекул при увеличении температуры в 4 раза?

Answer: Increased 2 times

Problem 10

How magnetic m_l and orbital l quantum numbers are related according to the electronic structure of atoms?

Answer: Quantum number m_l takes integer values from $-l$ to $+l$ via zero, i.e. $2l + 1$ values.

Problem 11

Determine the period of rotation of an Earth satellite, moving in a circular orbit with a radius of $R = 8 \cdot 10^6$ m. The mass of Earth is $M_E = 6 \cdot 10^{24}$ kg.

Answer: 2 hours