

## Combined cooling, heat and power systems operation planning & design: Mixed Integer Non Linear Programming

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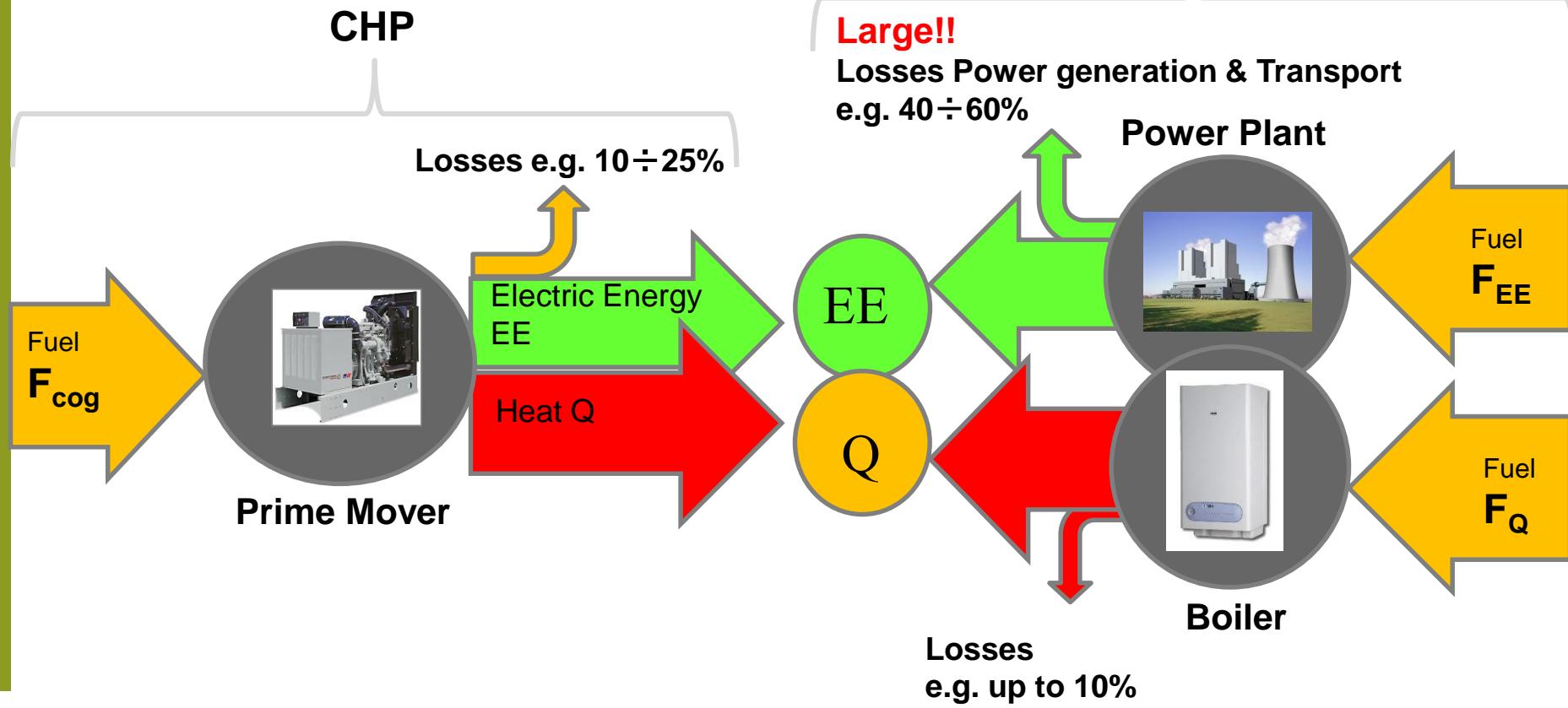


<http://www.gecos.polimi.it/>

# Combined Heat and Power (CHP)

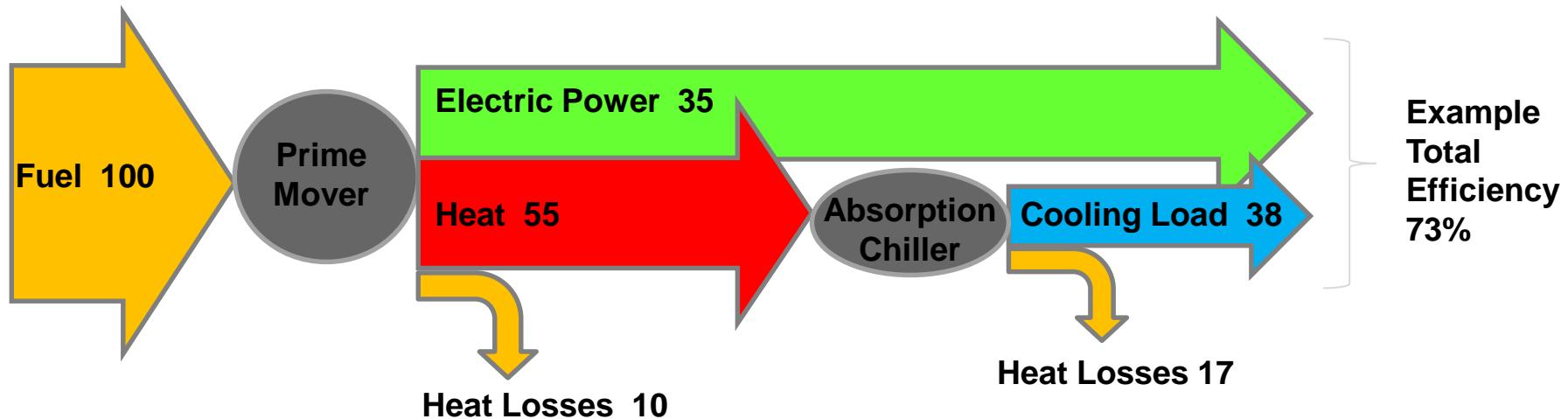
*Rational use of primary energy generating simultaneously heat, electric/mechanical power*

**Conventional**



# Combined Cooling, Heat and Power (CCHP)

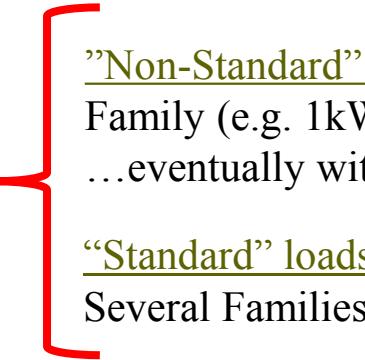
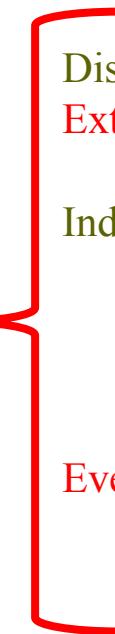
.....and refrigeration effect



- Effective way to reduce Primary Energy consumption & CO<sub>2</sub> emissions
- Advantageous for Electric Power Distribution system,
- No Construction of New Large Power Plants & High Voltage Network
- Reliability, Peakshaving, Power quality, liberalization of electricity market

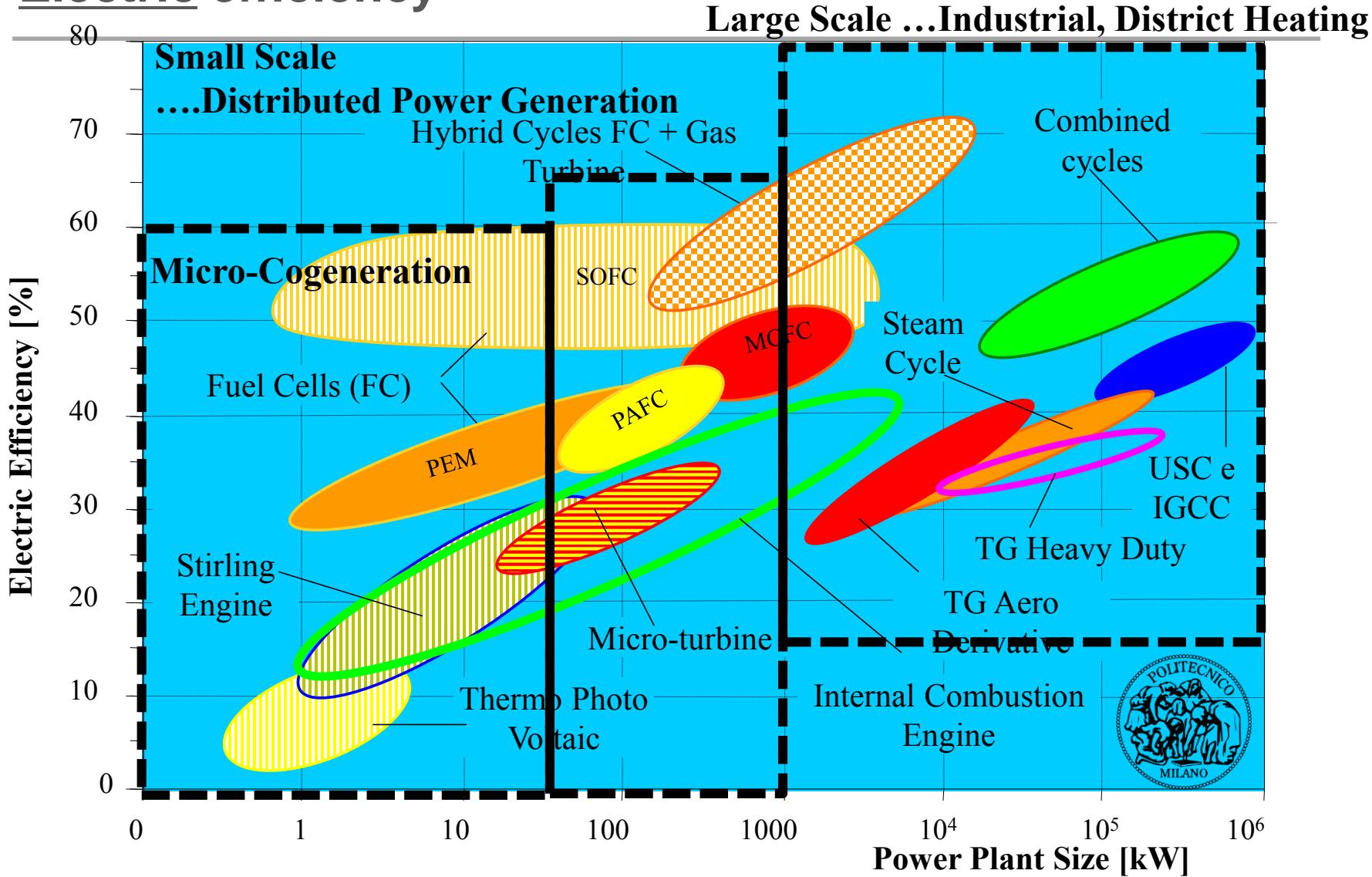
# Technologies: Size & Application Families

## Load/Customer

- **Micro, 50kW<sub>el</sub>** 
  - ”Non-Standard” loads  
Family (e.g. 1kWel), Small industries/Artisans  
...eventually with large variations ▶ Unforeseeable
  - “Standard” loads:  
Several Families, Tertiary sector, Small industries/Artisans, ▶ Foreseeable
- **Small Scale, < 1MW<sub>el</sub>** 
  - District Heating/Cooling  
Extra challenges due to both pressure & thermal load losses
  - Industrial Processes, Case-Specific!
    - Temperature the heat is required
    - N° of Temperature levels the heat is required
    - Thermal loads small dependency on the season
  - Eventually unforeseeable!
    - ▶ e.g. Oil & Gas,  
Project SNAM Recompression Power Stations
- **Large Scale**  
(Large size & small number e.g. <200 units represent > 20% of Italian Th.Power, about 75GW)

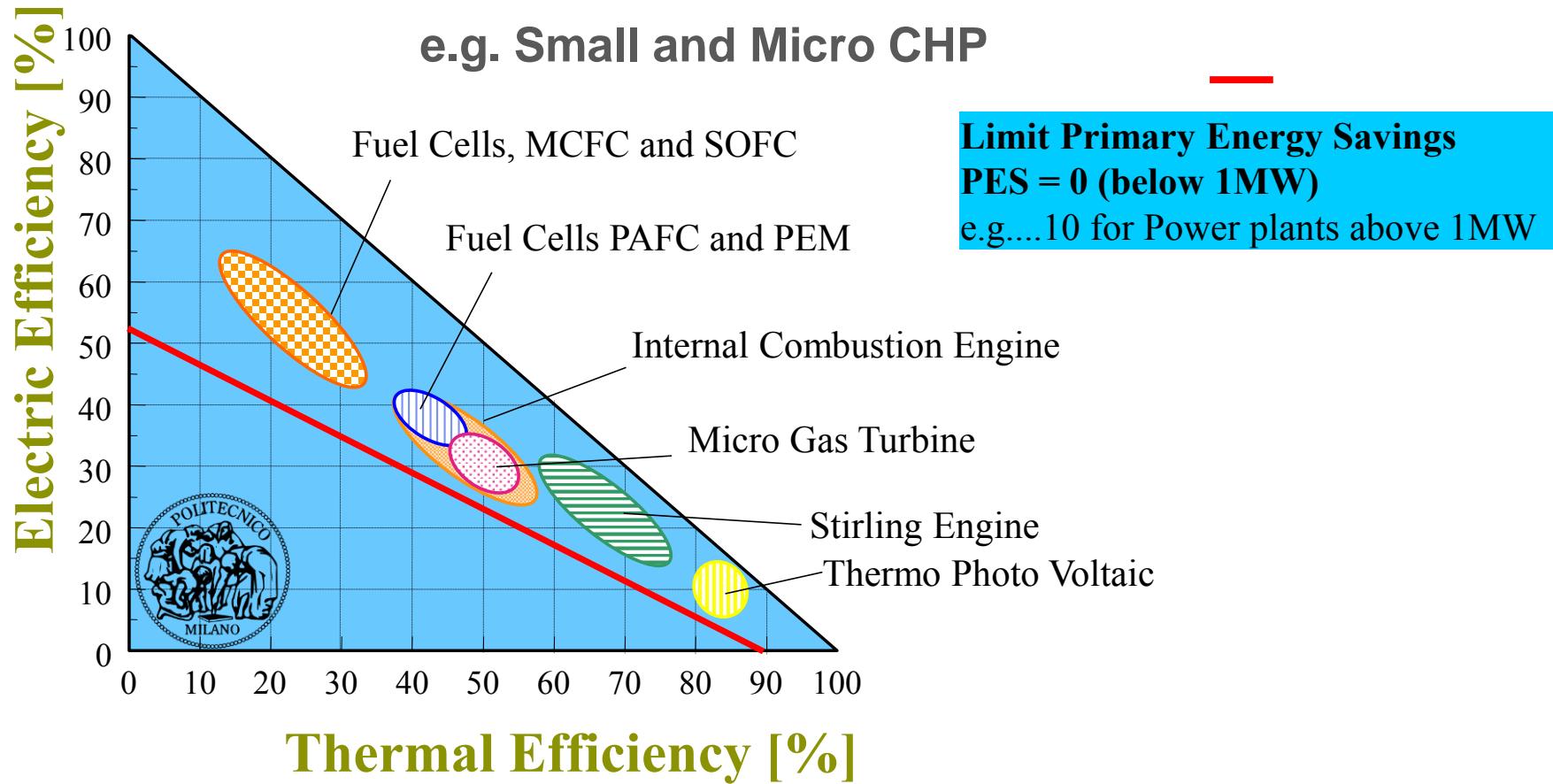
# Technologies: Size & Application Families

## Electric efficiency



# Technologies: Size & Application Families

## (Electric & Thermal efficiency)

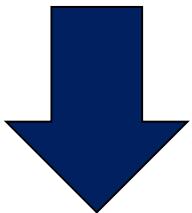


$$PES = \left( 1 - \frac{E_{fuel}}{\frac{E_{el}}{\eta_{el, ref} \cdot p} + \frac{Q_{rec}}{\eta_{th, ref}}} \right) \cdot 100$$

# CCHP Challenges

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- Larger Investment Costs
- Not Simultaneous Request of Electric Power & Heat  
**Storage...Thermal/Cooling Load Power!**
- Bureaucracy & deep knowlege of the Legislation, Gas & Power Tarifs  
**Vary on hourly basis & depend on monthly/yearly consumption!**
- Temperature Level .....  
    Different Level ▶ Different Unit & Thermal Efficiency  
    More than one Level/Downgrade

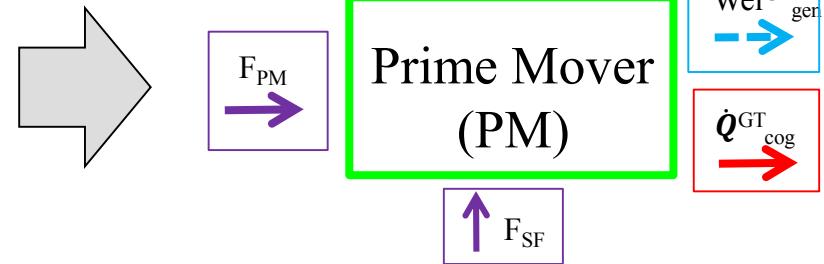
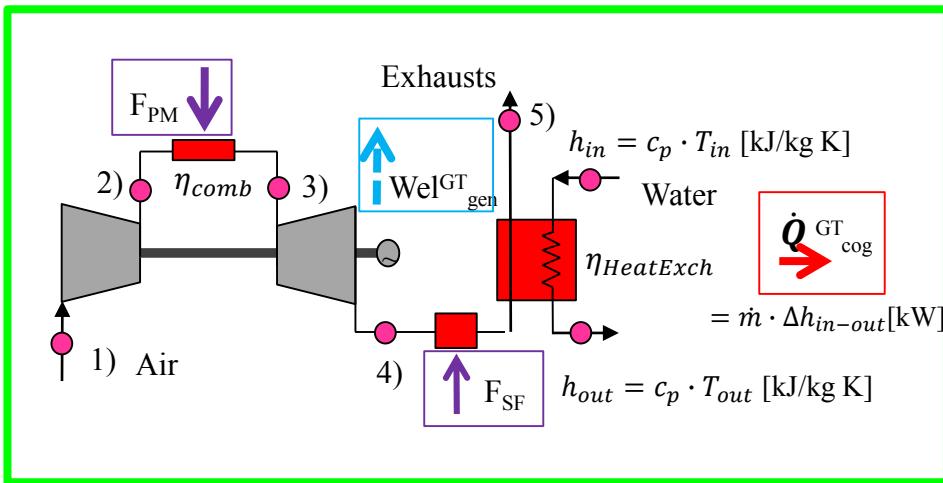


**Optimal Design & Scheduling**

# Co-generative Units Characterization,

→ Experimental Measurements

→ Thermodynamic cycle calculation (each point),

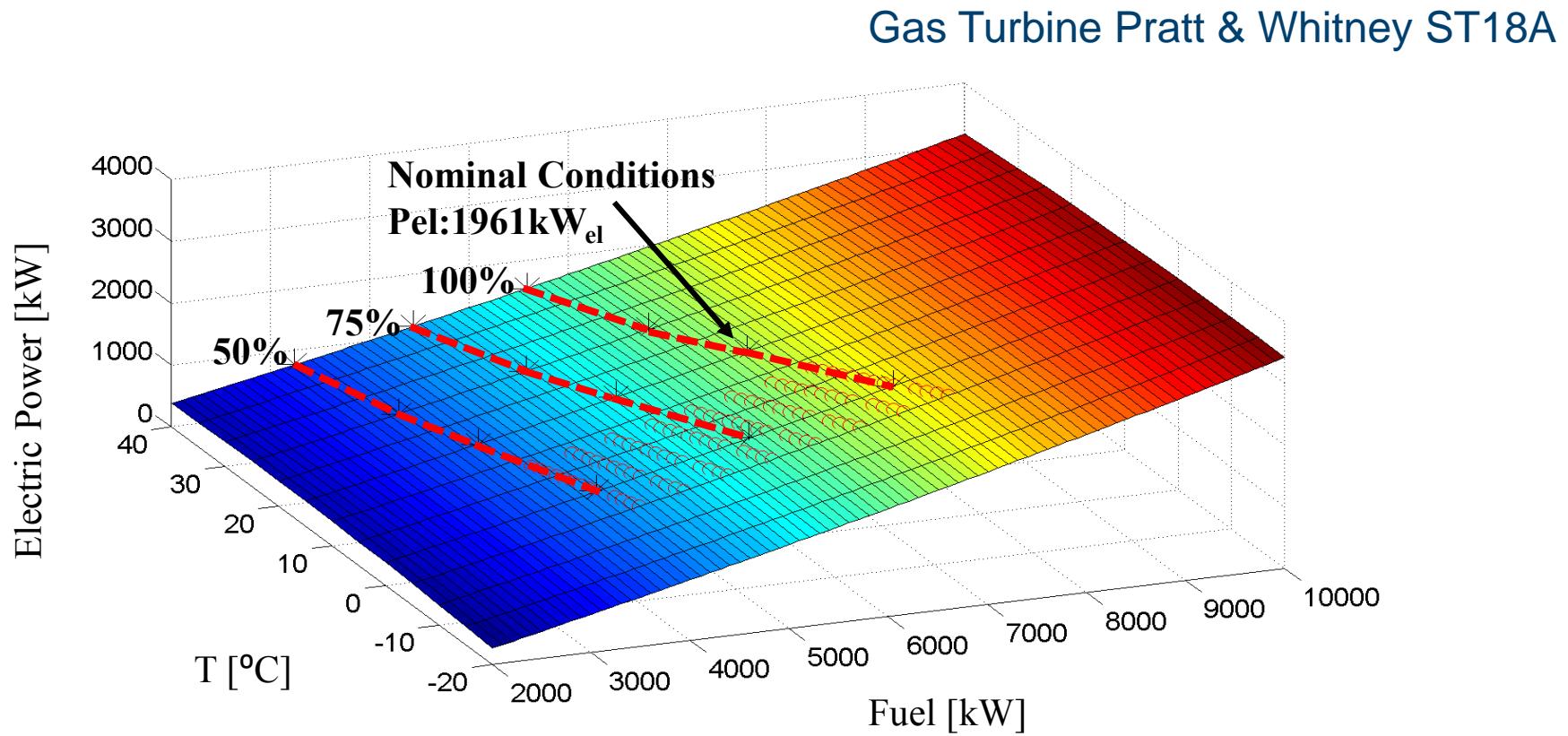


For several **Ambient Temperatures**

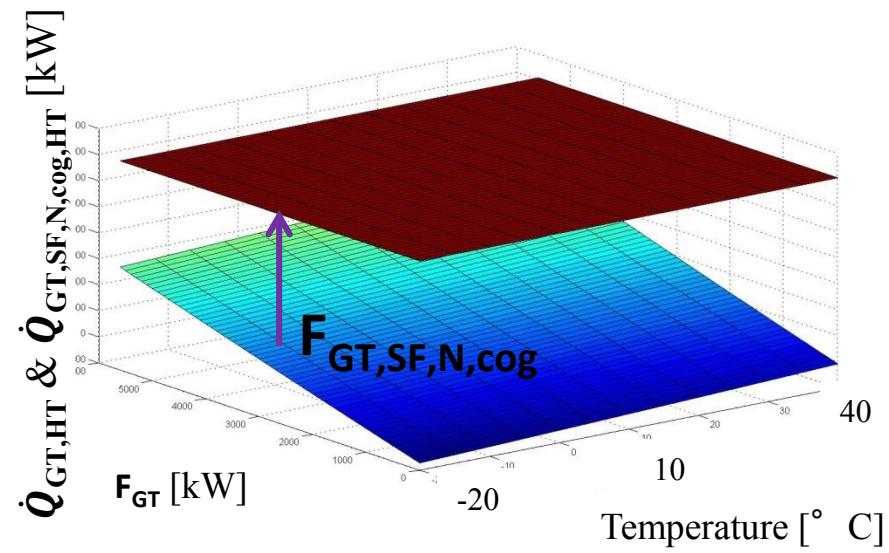
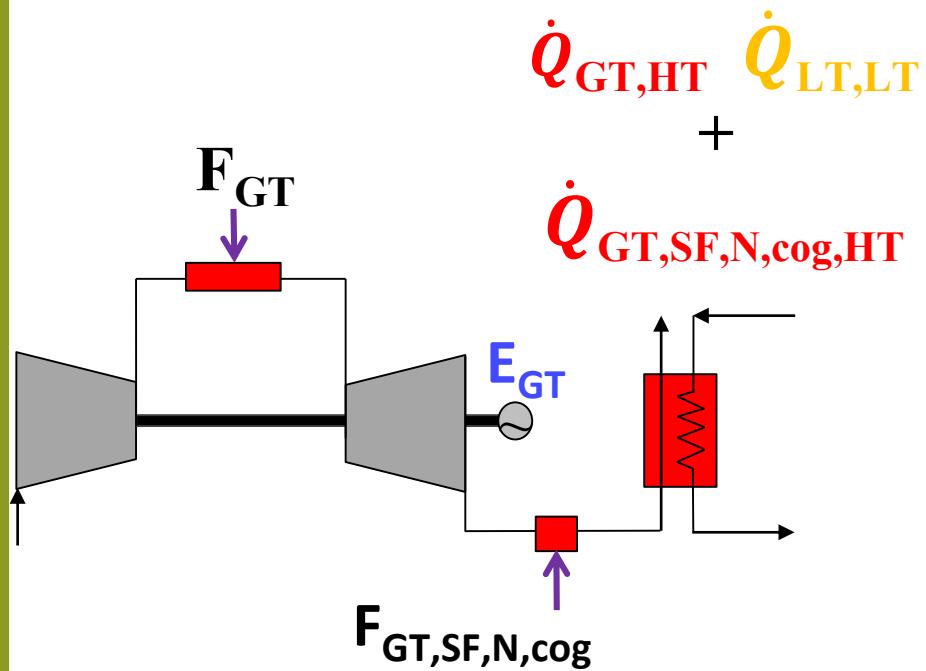
Associate characteristic curves to each unit, taking into consideration just the fluxes in & out.

For several **Fuel input**, from Minimum to Nominal load

# High Level of Detail for Units Characterization



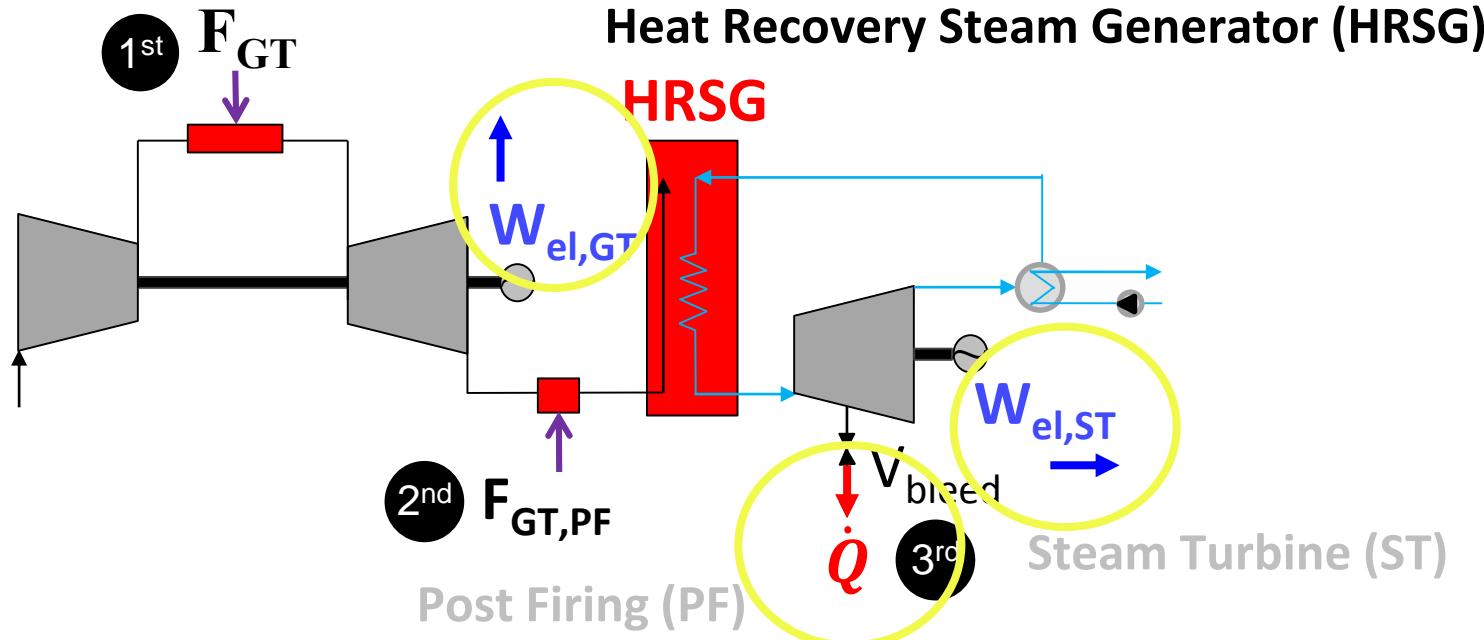
# “Two-degrees-of-freedom” units



# “Three-degrees-of-freedom” units

e.g. Natural Gas Combine Cycle with Post Firing

Gas Turbine (GT)



$$W_{el,GT} = f(F_{GT})$$

vs. “White Box” approach!

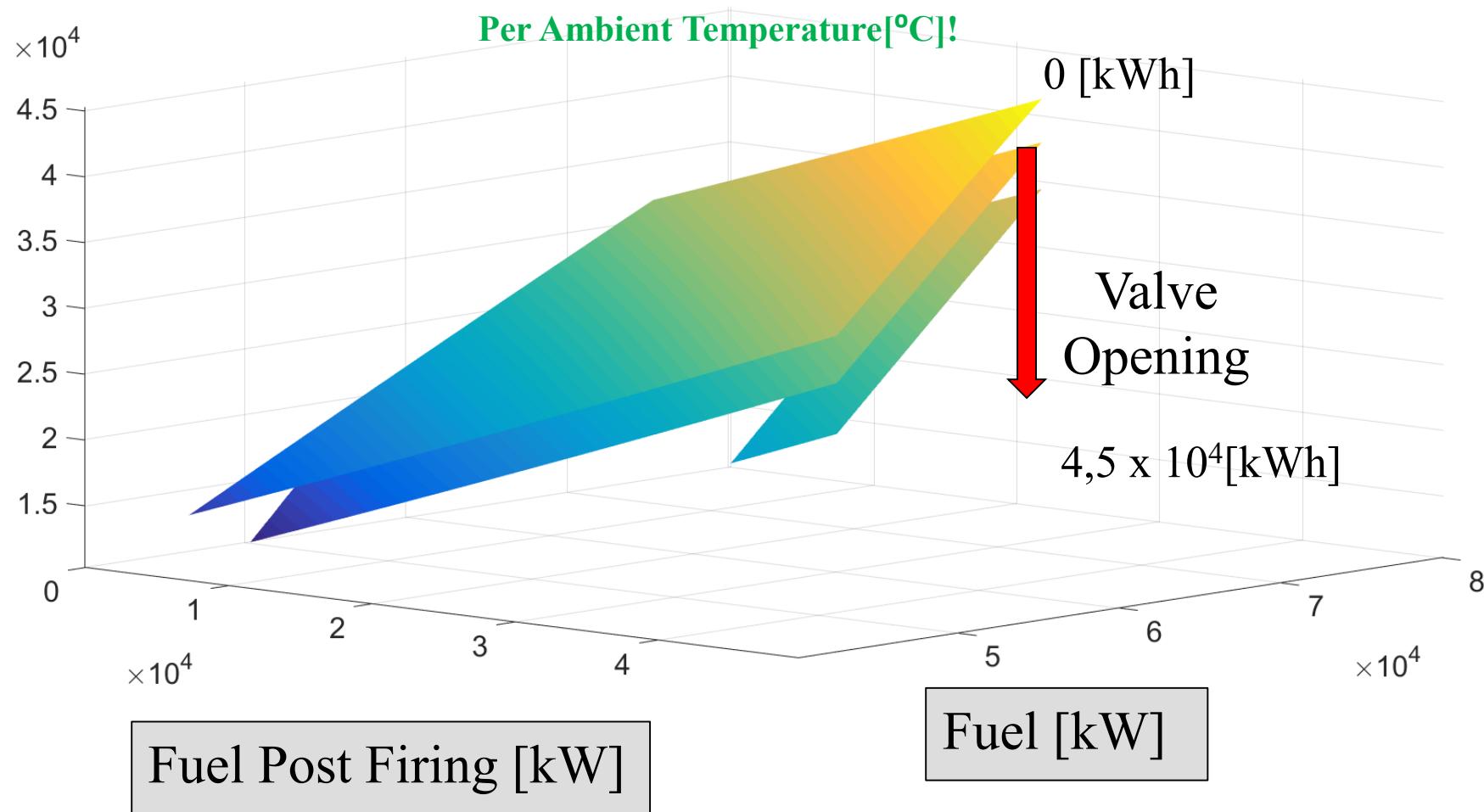
$$\dot{Q} = f(F_{GT}, F_{GT,PF}, V_{bleed})$$

$$W_{el,ST} = f(F_{GT}, F_{GT,PF}, V_{bleed})$$

# “Three-degrees-of-freedom” units

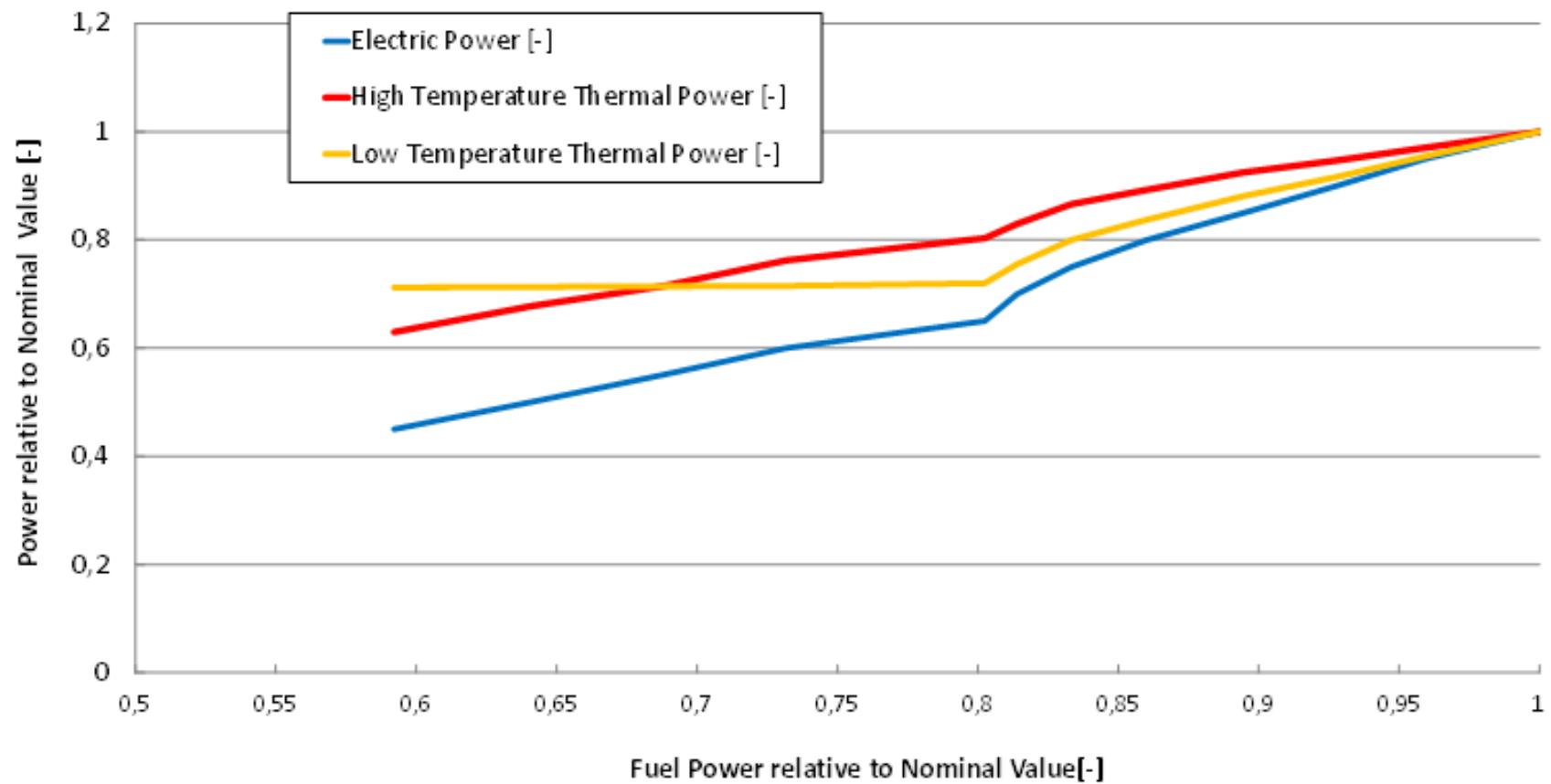
e.g. Natural Gas Combine Cycle with Post Firing

Electric Power [kW]



# Performance curves NON Smooth

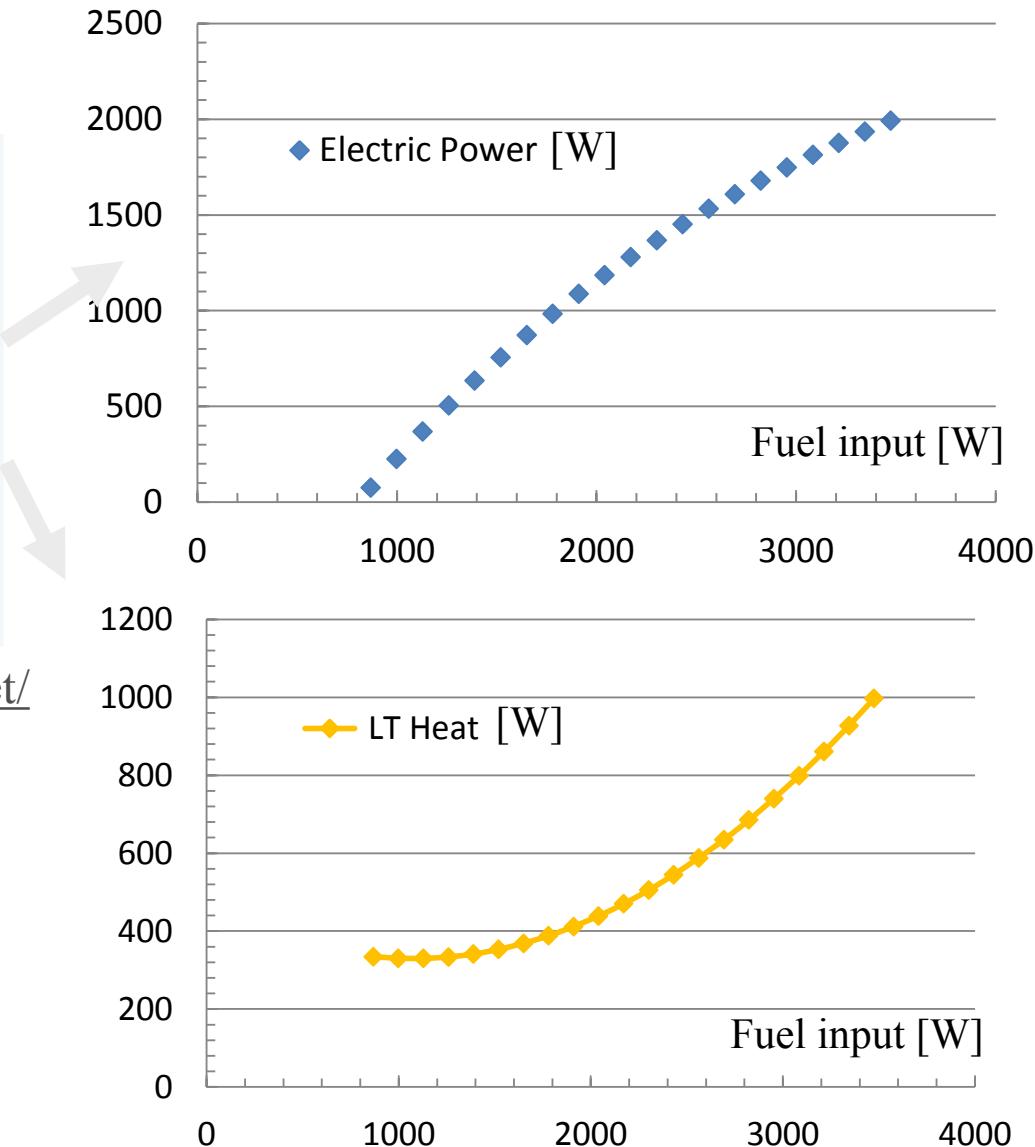
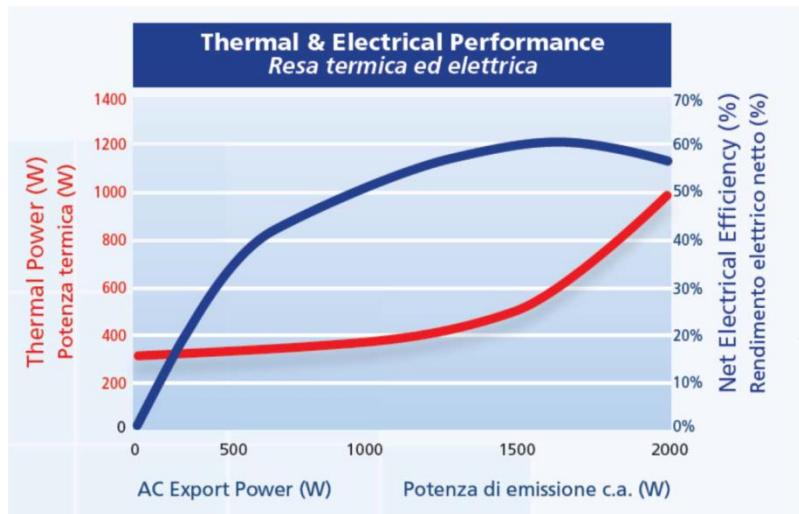
.....e.g. gas turbine (depends on the control strategy)



“*Natura non facit saltus*”....sometimes yes, but it is unusual

# Non-Concave Functions

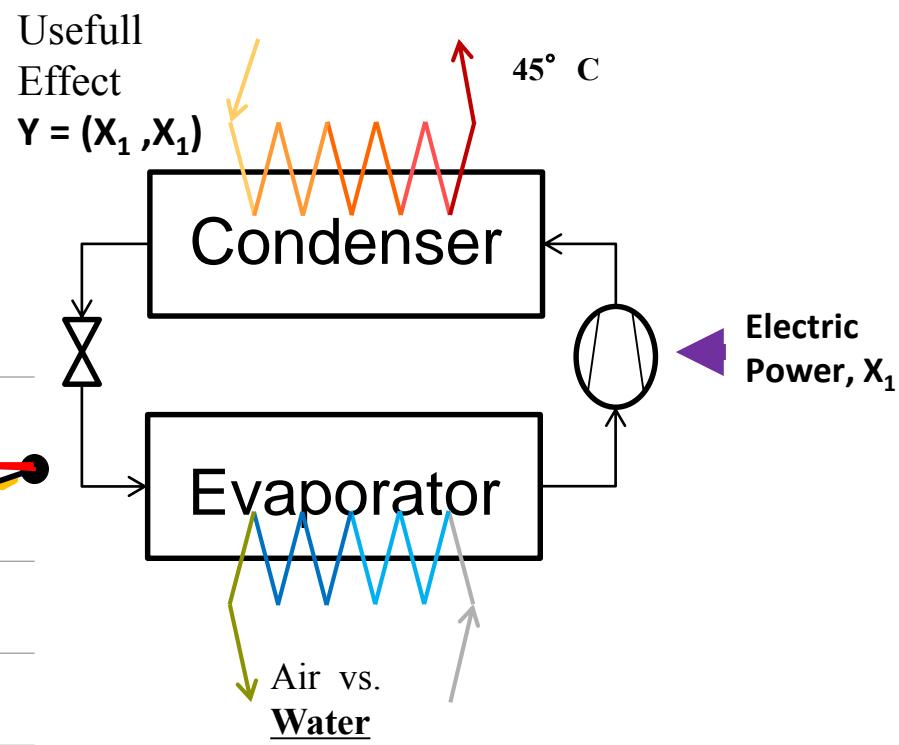
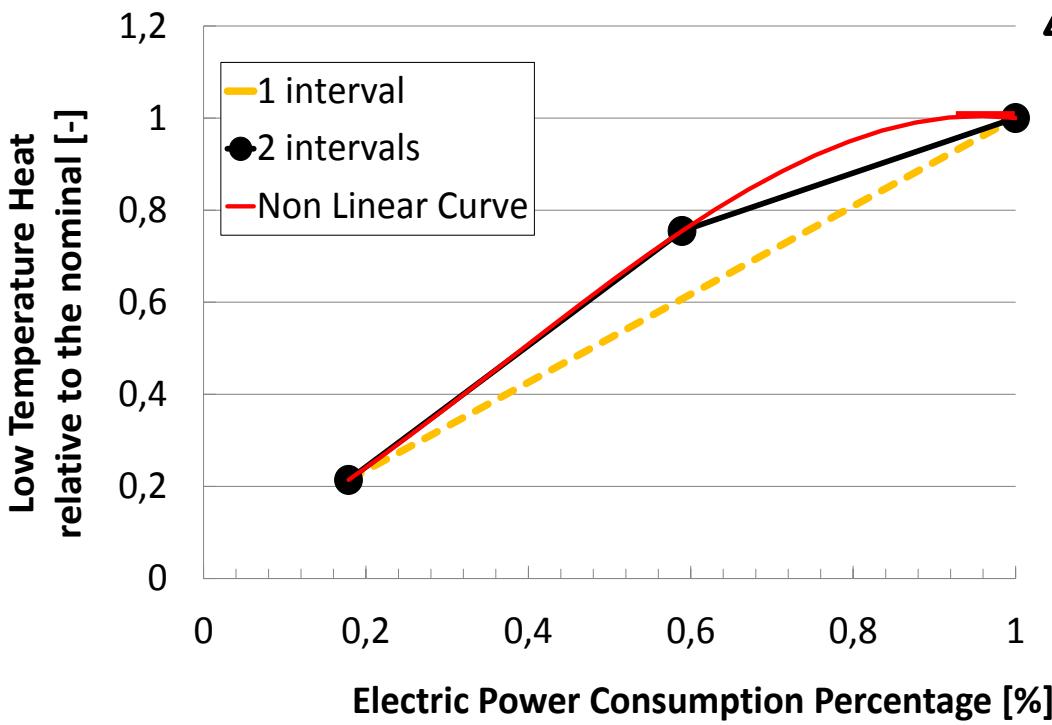
## e.g. Solid Oxyde Fuel Cell (SOFC)



Original Source, <https://www.bluegen.net/>

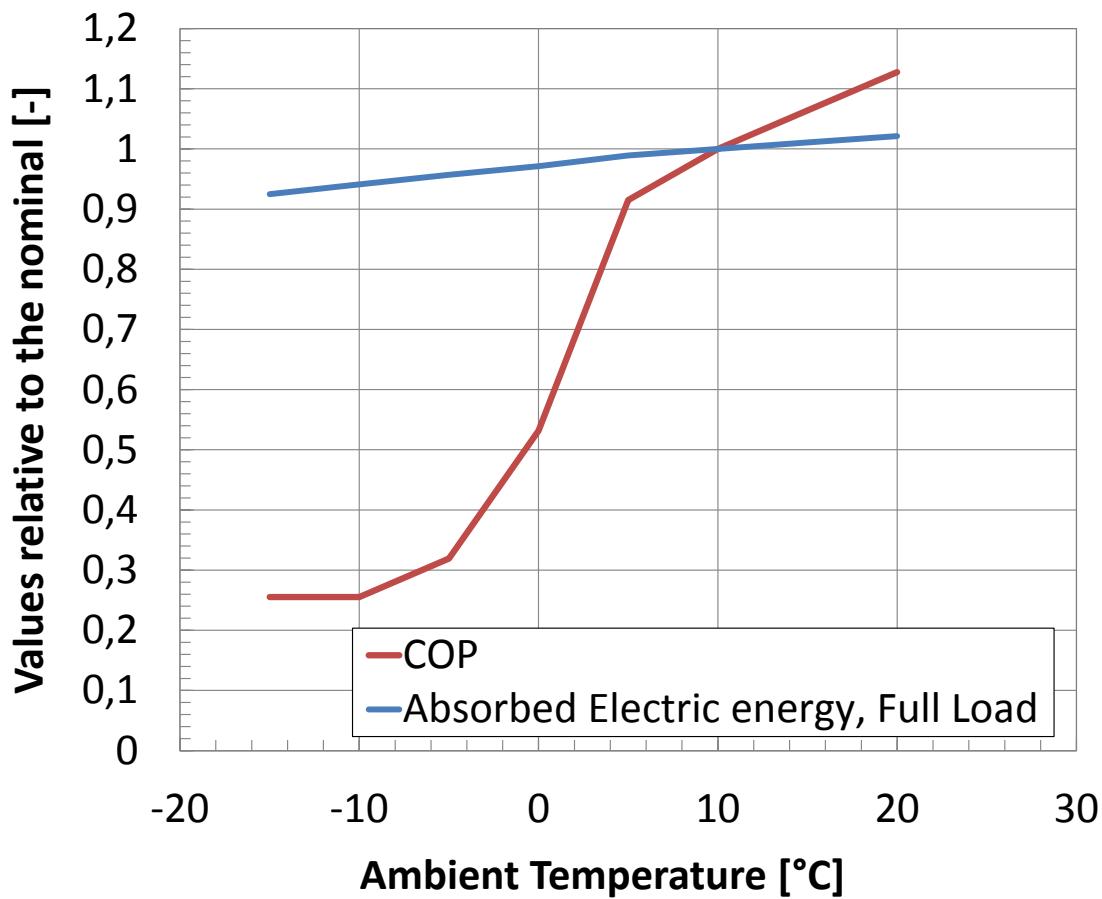
# Performance curves highly NON Linear

.....e.g. Heat Pump/Part Load



# Performance Strong Temperature Dependency

→ .....e.g. Heat Pump/Evaporator Temperature



## Scheduling Application

### Air Cooled Heat Pump

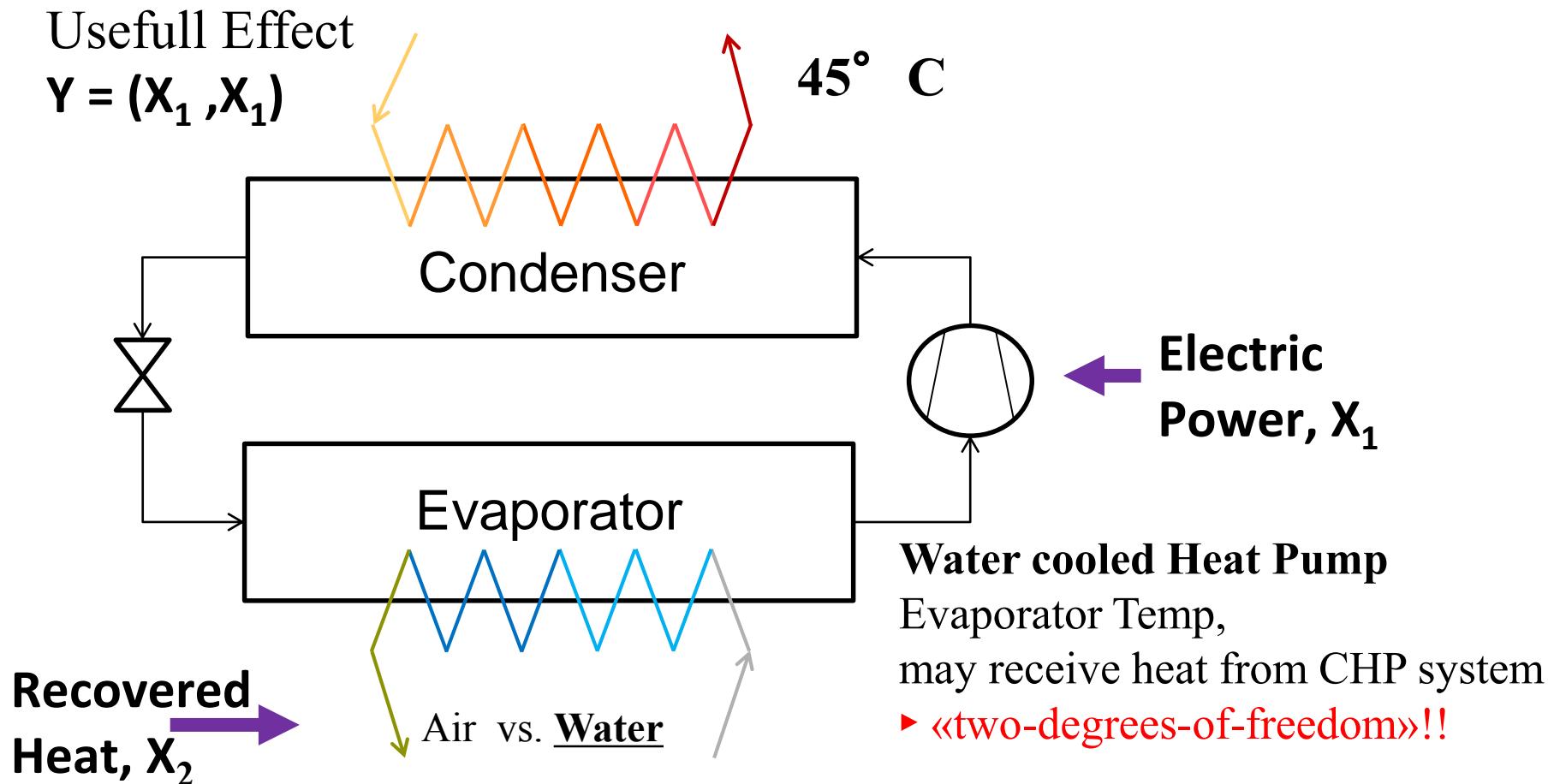
Evaporator Temperature =  $T_{amb}$ ,  
NOT a variable

$T_{amb}$  reduction▶

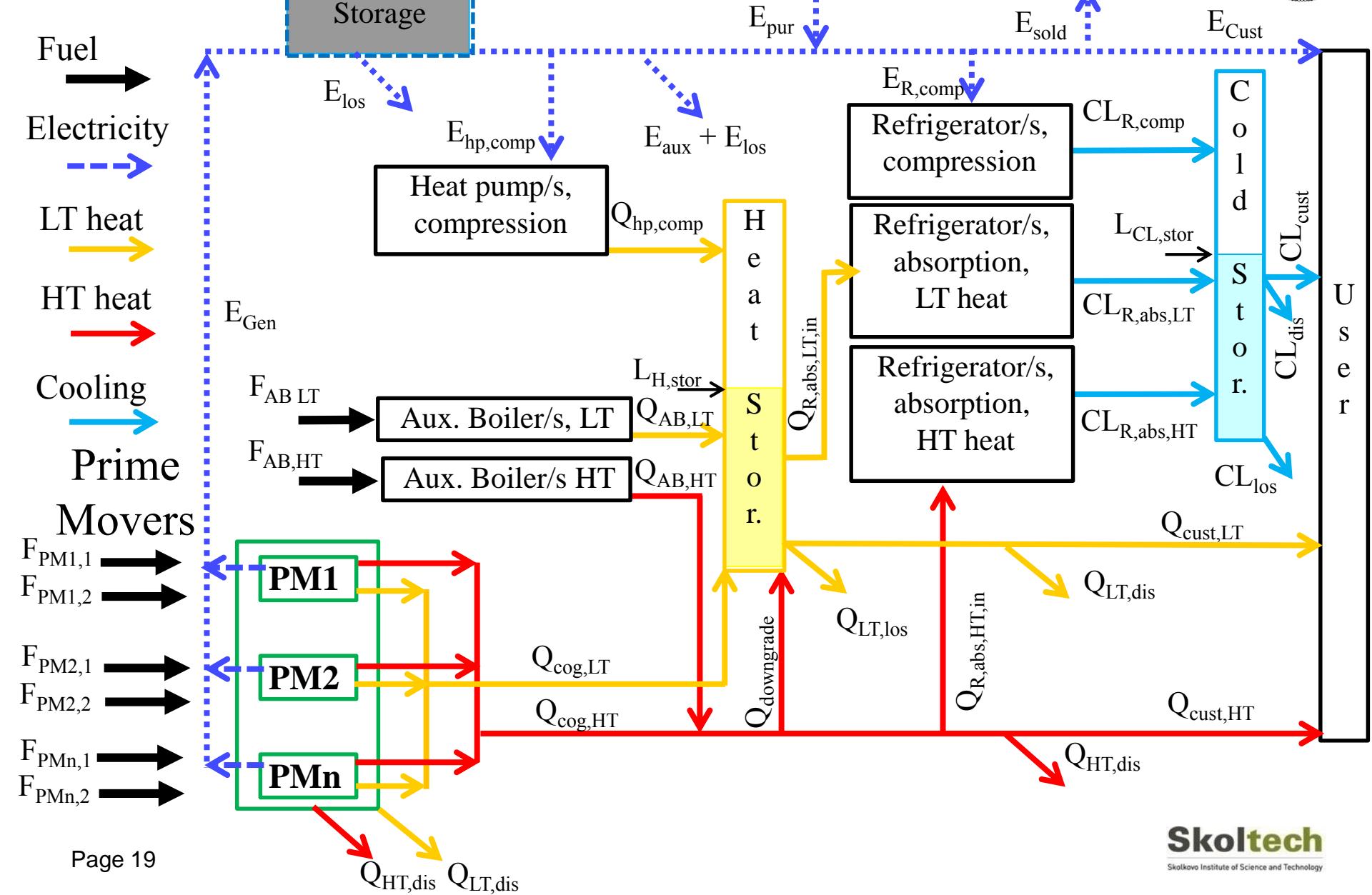
- < cycle efficiency
- < mass flow compressor

# Performance Strong Temperature Dependency

→ Temperature as a “second-degree-of-freedom”



# Integrated Energy Infrastructures Conceptual Layout



# Problem Statement:

## Optimized Management of a CCHP System

### Given the Customer demands:

- Time-dependent Electrical Load
- Time-dependent Cooling, High and Low Temperature Heating Load
- ...

### Other known parameters:

- Time-dependent ambient Temperature
- Time-dependent Price of Electricity (Sold and Purchased)
- Units (engines, boilers, chillers, heat pumps...) performance curves
- ...

### Given the set of Equipment units:

- $n_1$  Prime Movers (Gas Turbines, Internal Combustion Engines, etc)
- $n_2$  Auxiliary Boilers,  $n_3$  Heat Pumps,  $n_4$  Compression Chillers,  $n_5$  Absorption Chillers
- Low temperature storage tank with fixed capacity
- ...

### Objective: minimization of Daily/Weekly Costs of Operation

$$\sum_{t=1}^{24 \cdot 7} C_{F_{\text{tot},t}} + \sum_{t=1}^{24 \cdot 7} C_{O\&M_{\text{tot},t}} + \sum_{t=1}^{24 \cdot 7} C_{\text{On/Off},t} \mp \sum_{t=1}^{24 \cdot 7} E_{\text{tot},t}$$

Fuel Consumed      Operation & Maintenance (Hours, Energy input, Start-up)      Extra Fuel/Electricity required for start-up      Electricity Sold/Purchased

# Problem Statement:

## Optimized Management of a CCHP System

**Objective:** Minimization of Daily/Weekly Cost of Operation (linear)

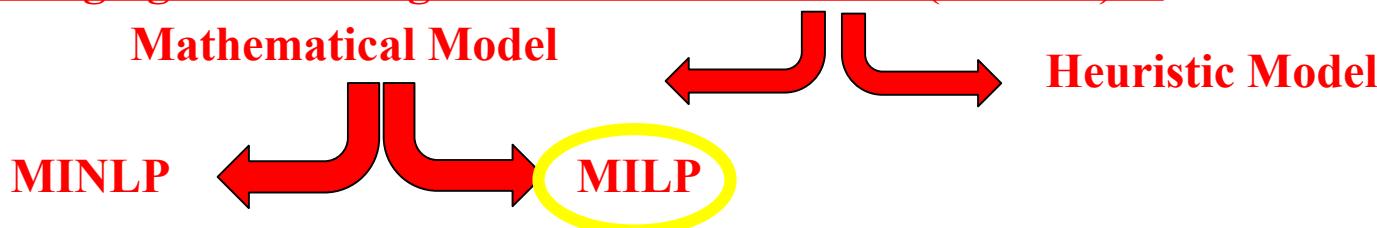
**Decision Variables,** each time period:

Units Operative Variables, Units On/Off status (Binary Variables), Heat Storage Level, Thermal Power Downgraded, Sold/Purchased Electricity

**Main Constraints:**

- Satisfaction of heat and cooling load demands (linear)
- Max and Min load of each unit (linear)
- Max number of start-ups per day (linear)
- Energy balance of heat storage tank (linear)
- Performance curves of equipment units (Non-Linear, typically Non-Convex)

**Challenging Mixed Integer Non-Linear Problem (MINLP)!!!**



## → Heuristic/Multi-Step

**Set of pre-defined Plant Operating Modes**, Nested “For-cycles” saving best hourly results  
Challenging with respect to several CHP units & NO accurate Storage (too many combinations)

[A.Bischi](#), S.Campanari, A.Castiglioni, G.Manzolini, E.Martelli, P.Silva, E.Macchi, “Tri-Generation systems optimization: comparison of heuristic and mixed integer linear programming approaches”. **ASME Turbo Expo 2014**

[A.Bischi](#), E.Pérez-Iribarren, S.Campanari, G.Manzolini, E.Martelli, P.Silva, E.Macchi, J.M.P.Sala-Lizarraga. “Cogeneration Systems Optimization: Comparison of Multi-Step and Mixed Integer Linear Programming Approaches”, **International Journal of Green Energy (2016)**

## → MINLP

**lack of**

- **Guarantees to find the global optimum**
- **Effective large scale solvers,**

L.Taccari, E.Amaldi, [A.Bischi](#), E.Martelli (2015). “Short-term planning of cogeneration energy systems via MINLP”, **book chapter** from ”Advances and Trends in Optimization with Engineering Applications”. **Society for Industrial and Applied Mathematics (SIAM)**

L.Taccari, E.Amaldi, [A.Bischi](#), E.Martelli. ”Short-term planning of cogeneration power plants: a comparison between MINLP and piecewise-linear MILP formulations”. **Computer Aided Chemical Engineering**, Volume 37, 2015, Pages 2429-2434

## → MILP

**LINEARIZE it into a Mixed Integer Linear Program (MILP)**

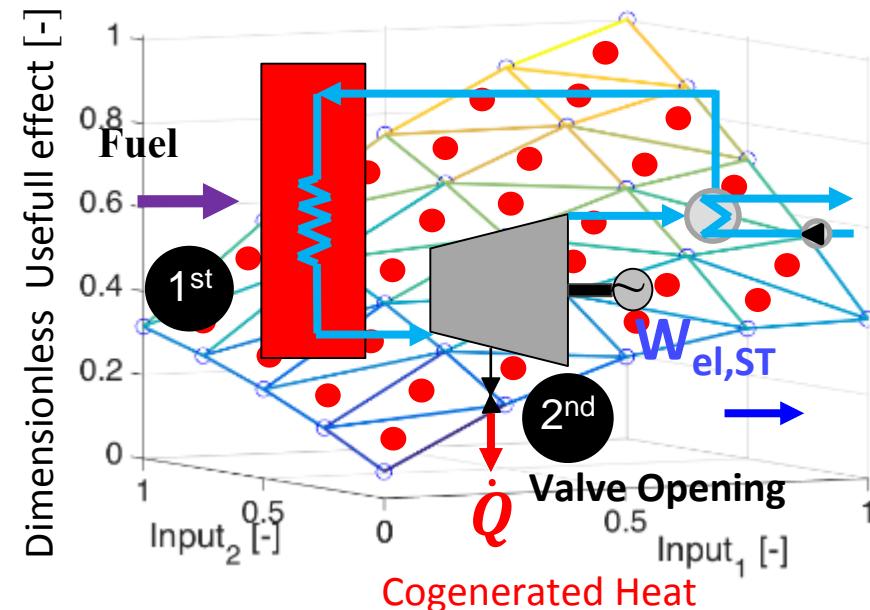
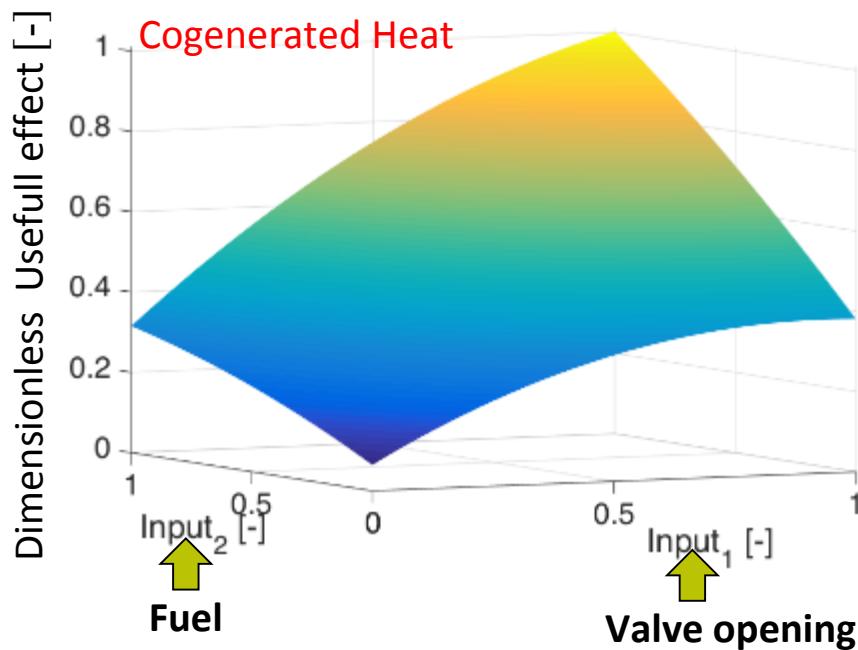
**So as to use more robust and effective MILP solvers (e.g., CPLEX, GUROBI)**

[A.Bischi](#), L.Taccari, E.Martelli, E.Amaldi, G.Manzolini, P.Silva, S.Campanari, E.Macchi (2014). “A detailed MILP optimization model for combined cooling, heat and power system operation planning”. **Energy**, Volume 74, Issue C, 2014, Pages 12-26

# PieceWise Linear (PWL) Approximation

## Mixed Integer Non Linear Problem (MINLP)

“Two-degrees-of-freedom” units: Output & Performance depend on two operative variables (e.g., extraction condensing steam turbine)



For Each Temperature!

Highly NON linear & Highly dependent on both variables interconnected

\* C. D'Ambrosio, A. Lodi, S. Martello, "Piecewise linear approximation of functions of two variables in MILP models", Operational Research Letters, 38 (2010) 39-46

# PWL Computational Performance

---

- Weekly problems with one hour time step i.e. 168
  - e.g. six “one-degree-of-freedom” units + thermal storage ranges **1000÷5000 seconds**, with 5000 as time limit & 0,01% MILP-gap!

- Heat storage up to **2000%** higher computational time!

- Number of variables;
  - e.g. 6000 Integer, 14000 real and 20000 constraints:

$$\prod_{k=1}^n (i_k + 1) \quad \text{Integer per each time-step}$$

... $n$  degrees of freedom and  $i$  intervals

**Weekly below/about 1% MILP-gap in one minute**  
**....good but not enough!**

# Superposition Principle:

## Integer variables reduction & “N-degrees-of-freedom”

From

$$\prod_{k=1}^n (i_k + 1)$$

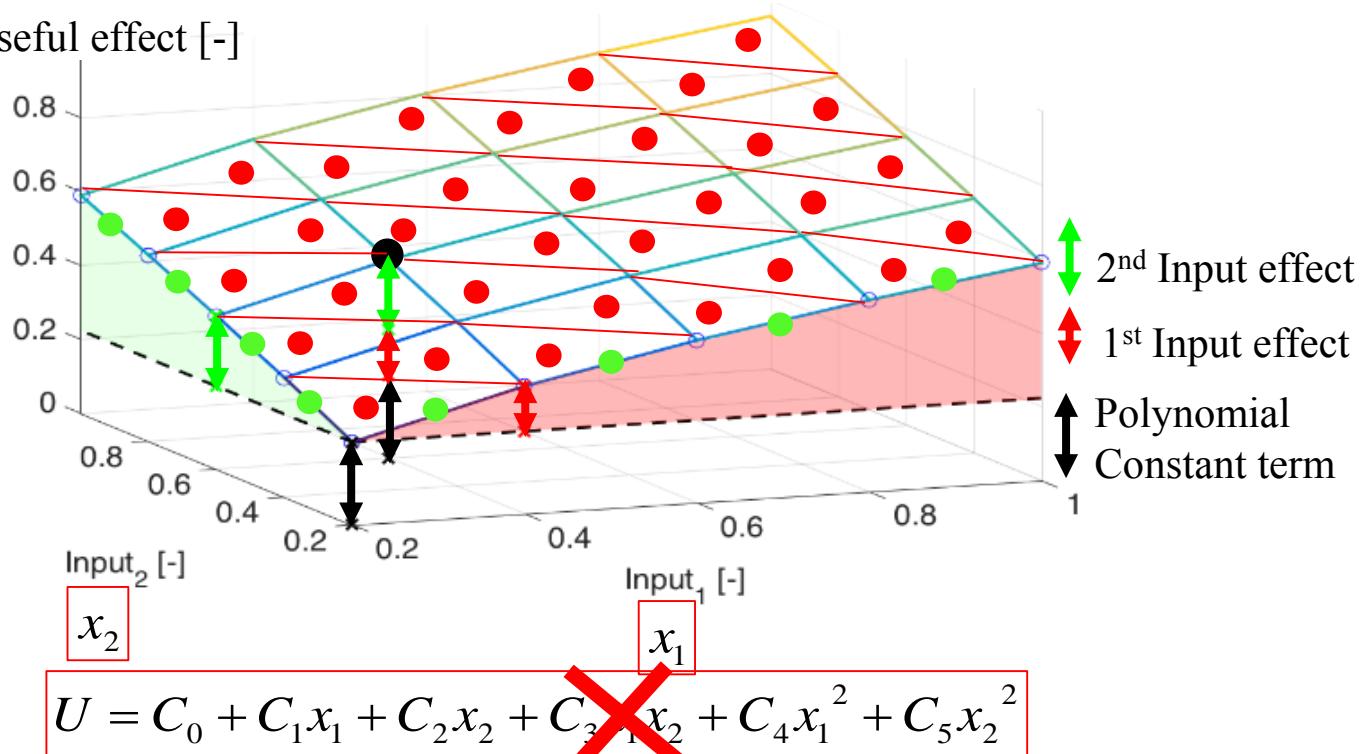
$$\sum_{k=1}^n (i_k + 1)$$

Integer variables

... $n$  degrees of freedom and  $i$  intervals

$U$

Useful effect [-]



# Superposition Principle:

## Integer variables reduction

---

**Eliminate the mixed term by linearly transforming the control variables!**

$$U = C_0 + C_1x_1 + C_2x_2 + C_3x_1x_2 + C_4x_1^2 + C_5x_2^2 = x^T Q x + b^T x + c$$

Q, Polynomial Quadratic Term Matrix:

- Symmetric
- Diagonalizable to avoid mixed term

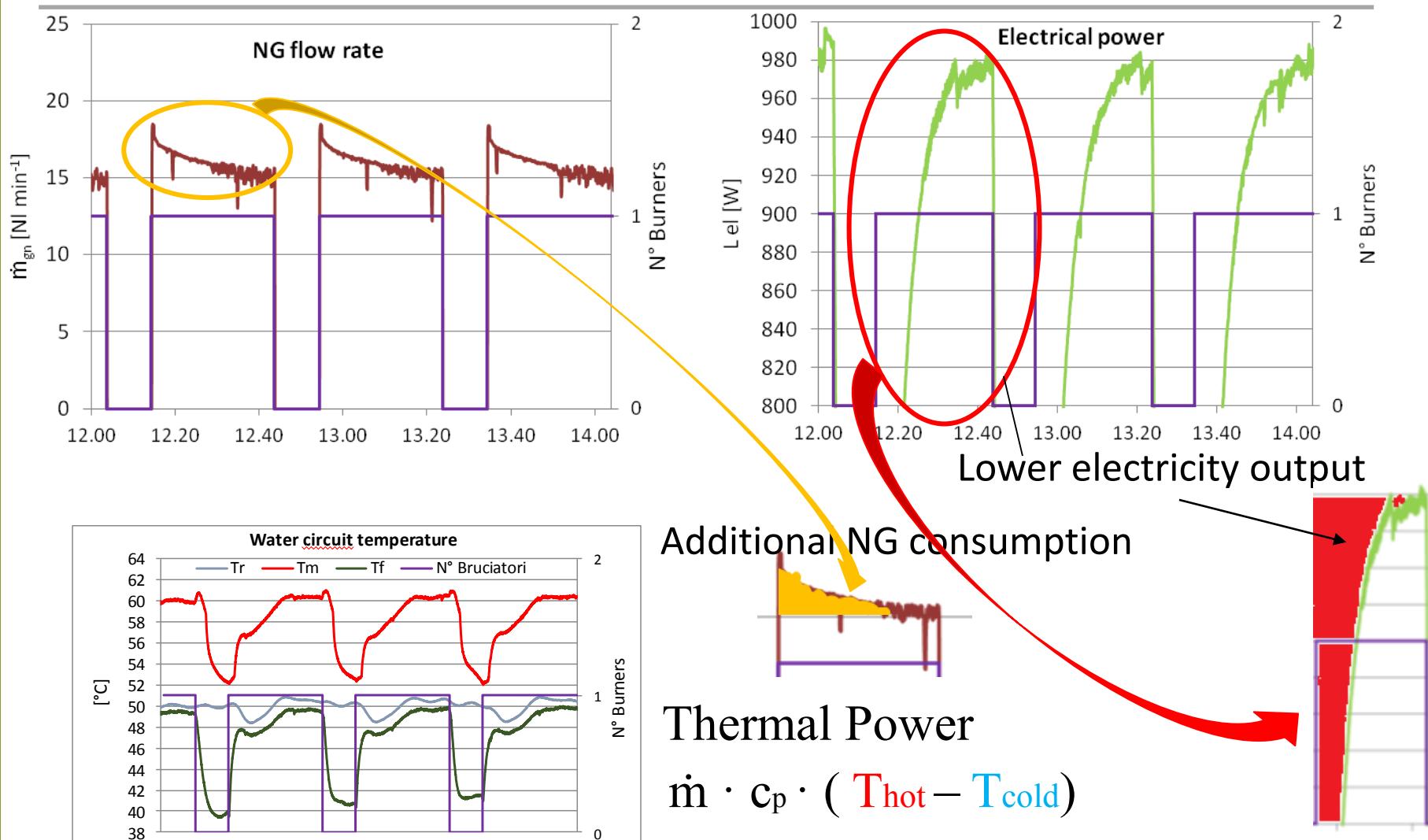
By means of the orthogonal Q eigenvectors matrix M

$$M = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \quad \begin{cases} x_{1T} = k_{11}x_1 + k_{12}x_2 \\ x_{2T} = k_{21}x_1 + k_{22}x_2 \end{cases}$$

$$U = c_0 + c_1x_{1T} + c_2x_{2T} + c_3x_{1T}^2 + c_4x_{2T}^2$$

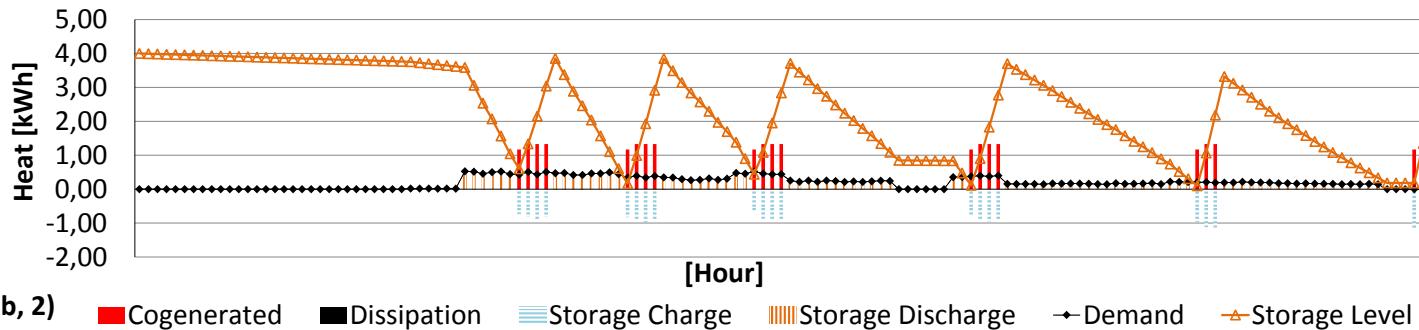
# Startup & Shutdown Constraints

## / Commercial Stirling Engine – Laboratory



# Startup & Shutdown Constraints / Optimal Scheduling Impact

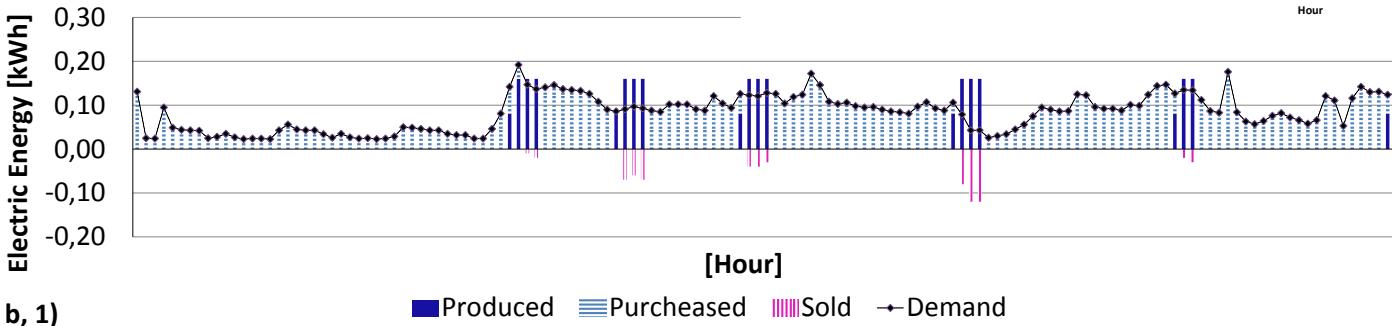
10 minutes time step



b, 2)

30 minutes time step

10 minutes time step



b, 1)

30 minutes time step

G.Valenti, S.Campanari, A.Bischi, P.Silva, A.Ravidà, E.Macchi (2016), "Experimental and numerical study of a micro-cogeneration Stirling unit under on-off cycling operation", under preparation.

# Yearly problem & Performance-dependent parameters

## Objective: minimization of Yearly Costs of Operation

$$\sum_{t=1}^{24365} C_{F,tot,t} + \sum_{t=1}^{24365} C_{O\&M,tot,t} + \sum_{t=1}^{24365} C_{on/off,tot,t} + - \sum_{t=1}^{24365} El_{tot,t} + \sum_{t=1}^{24365} Ex_{h,t} + Ex_{forf} - Inc$$

Hourly taxation on the electric energy consumption (excise)      Periodic taxation on the electric energy consumption (excise)      Periodic Incentives on the primary energy savings (“white certificates”)

**Monthly Threshold!**      **Monthly Threshold!**      **Yearly Bases!**

Yearly constraints, e.g. Primary Energy Savings (PES) & first principle efficiency!

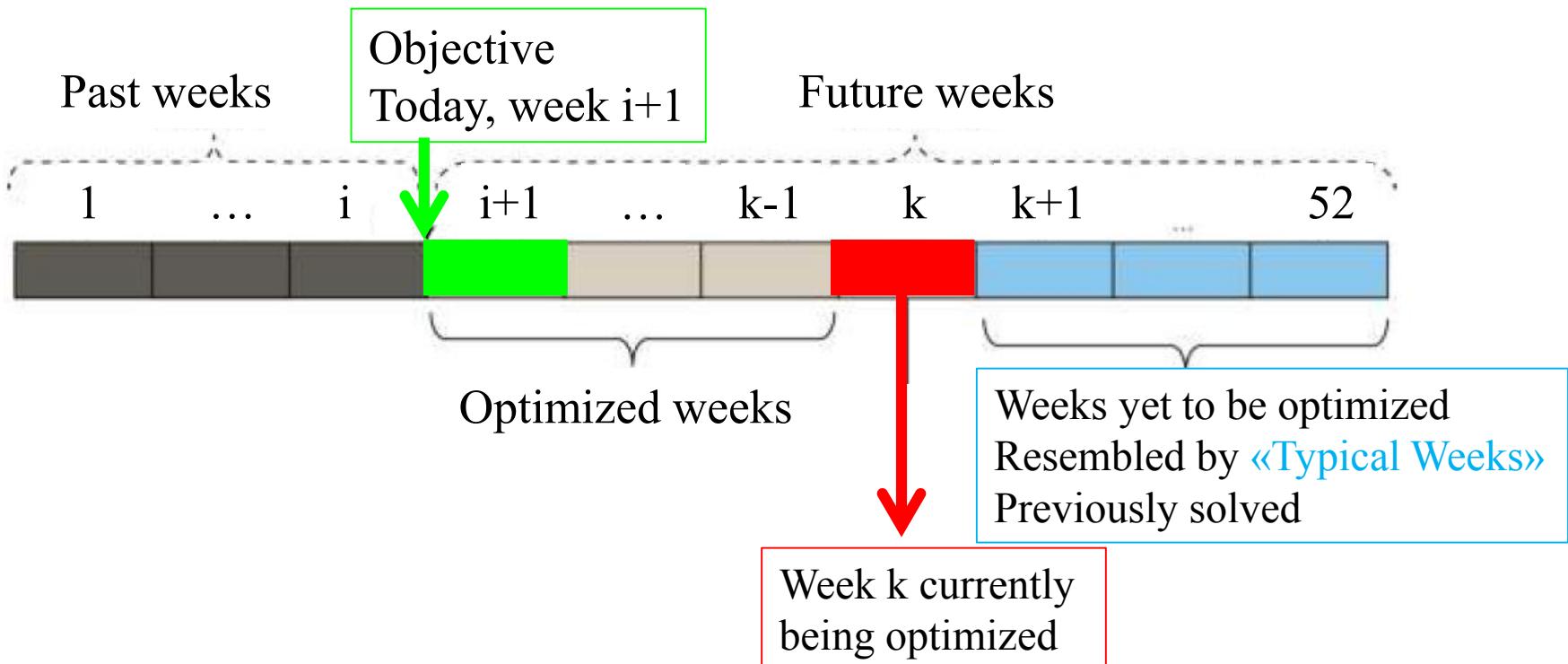
Extremely Challenging Mixed Integer Non-Linear Program (MINLP)

Due to Non-Linearity & High number of variables solving at once the whole problem

«ad hoc» Rolling Horizon heuristic for the yearly schedule

# Rolling Horizon heuristic optimization method

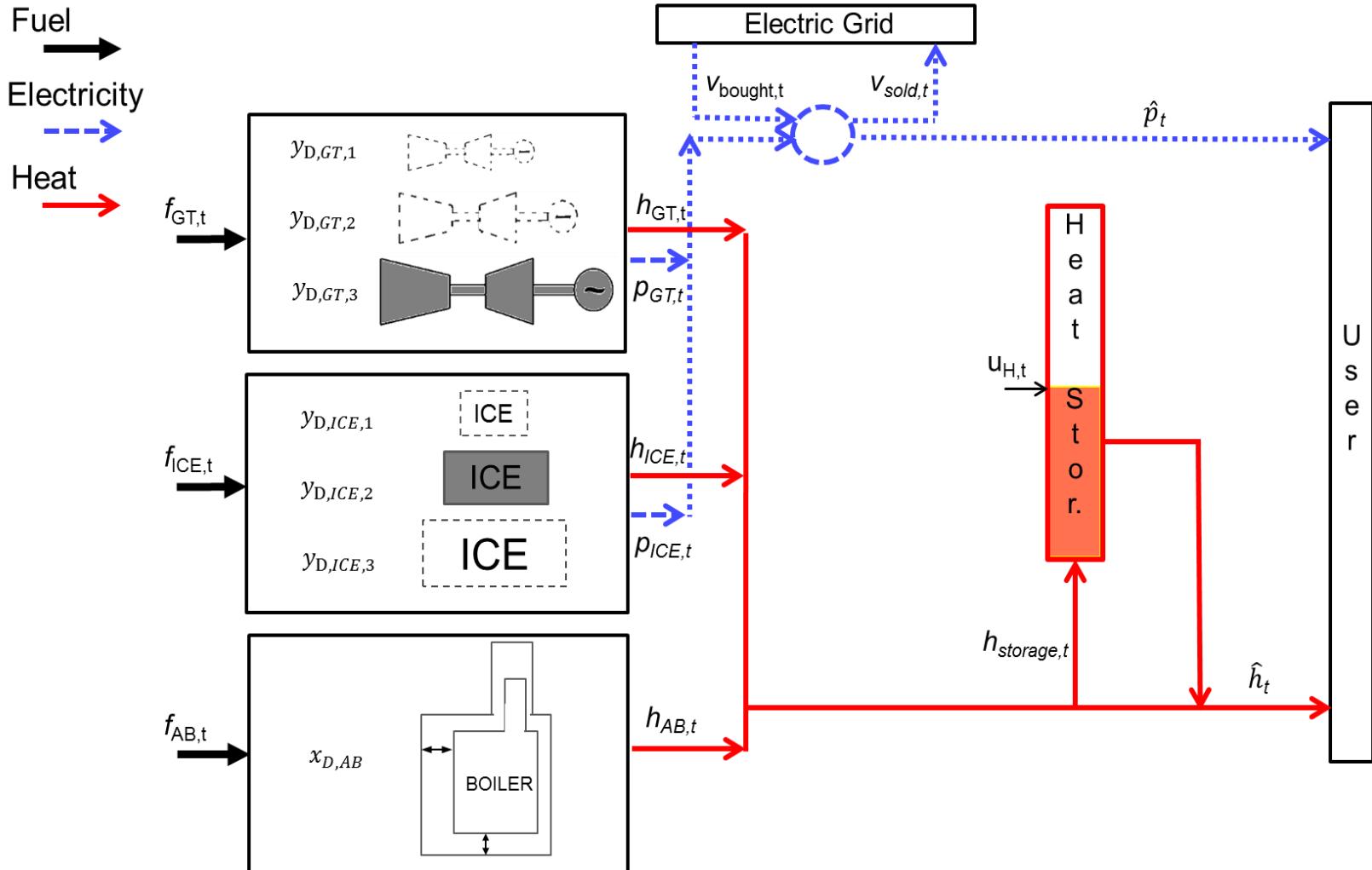
Overall problem subdivided into a sequence of smaller (weekly) sub-problems:



Iteratively Optimize only the variables corresponding to a **subproblem** while fixing the variables of all the other subproblems based on aggregated information  
(Past weeks, already optimized weeks, **future weeks based on typical weeks**)

# Combined Heat and Power (CHP) plants,

## Networks of CHP units feeding district heating



# Non linear effects of the size on the performance

## The larger the size...

---

**1) Lower fluid friction losses for isentropic efficiencies of machineries  
(Pumps, Compressors, Turbines)**

**2) Positive effects on specific costs**  
(larger heat transfer areas, better materials and technologies)

- **Steam Turbine**

Isentropic efficiency of turbine stages from 60% till 92%

- **Medium-small Gas Turbines plant (< 100 MWel):**  
electric efficiency from 28,5% (5MW) till 41% (100 MW):

- **Large Gas Turbines & Natural Gas Combined Cycles plants**  
Lower BUT Not-Negligible

- **Gas-Fired Industrial Boilers**

Thermal efficiency from 90% (1MW) till 93% (20MW)

# Two stages optimization algorithm

## Decomposition Method

2



Metaheuristic,  
e.g. genetic algorithm, particle  
swarm, etc.

$$\underline{x}_D_{OI}$$

$\underline{x}_D$ : design variab  
choice and size)

1

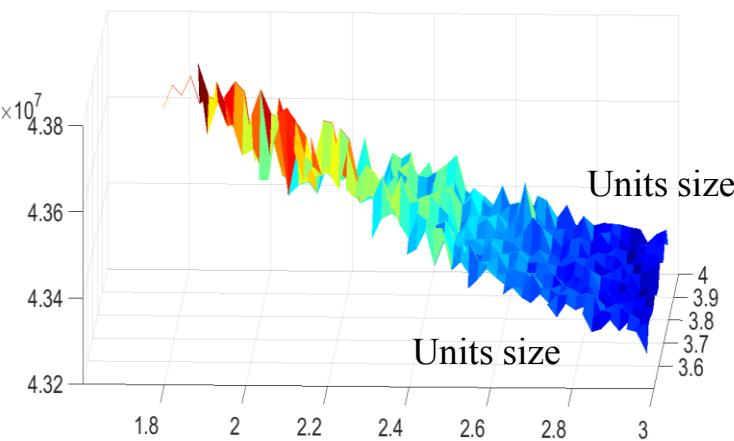
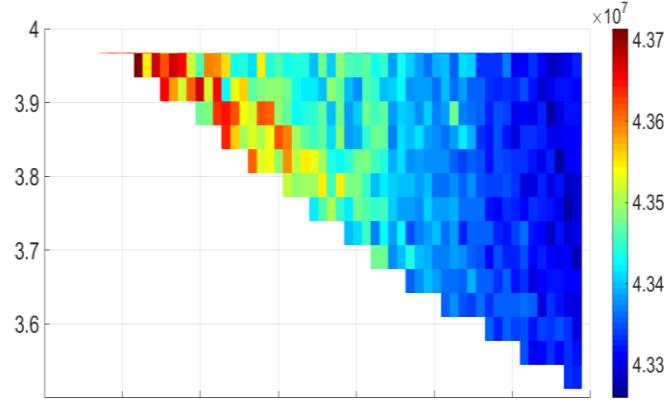


$$\underline{x}_D$$

$$o.f.(\underline{x}_D)$$

$\underline{x}_S$ : sc  
(on/off  
variables:  
manage

$$\underline{x}_S_O$$



# Typical Weeks decomposition based on the optimization results!

---

**Reduce computational load by an accurate selection of typical periods,**  
e.g. Clustering techniques (K-means).

**Weights calculation for input data ( $x_a$ )** like Thermal load, Electric load,  
Temperature to reduce the approximation error

- 1) Base Typical weeks determination via standard methods
- 2) Optimize the Total Operating Costs for the Base typical weeks ( $TOC_0$ )
- 3) Each input data variation ( $\Delta x_a$ )....keeping constant the others
- 4) For Each of the input variation optimize Total Operating Costs ( $TOC_a$ )
- 5) Compute the weight ( $w_a$ ) for the input  $x_a$

$$w_a = \frac{\frac{TOC_0 - TOC_a}{TOC_0}}{\Delta x_a} \quad \dots \text{for } N \text{ design configurations}$$

Approximation error from 10 (standard typical Weeks) to 3%!

# Achievements

---

- Development of an accurate MILP model for CCHP scheduling
  - Daily/Weekly basis ..... heat storage
  - Validation ..... Qualitative, Quantitative with Heuristic
- Extension up to Yearly Simulations

## Iterative Simulations to Adjust Assumptions avoiding Non Linearity

- Natural Gas / Electricity price according to monthly-yearly consumed volumes
- Incentive calculation based on annual energy indexes, Primary Energy Savings
- Extension up to:
  - Higher number of degrees of freedom > 2,  
by superposition principle & domain change for N variables (degrees of freedom)
  - Design, by two stage optimization algorithm,
- Future plans:
  - Load Forecast & Integration with Building physics model
  - Topology & Transport: Integration with Electric, Gas & Thermal grid
  - Stochastic & Robust optimization

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*Thank you for your attention.*

**a.bischi@skoltech.ru**

PoliMI Colleagues:

E.Martelli, G.Manzolini, P.Silva, S.Campanari, E.Macchi

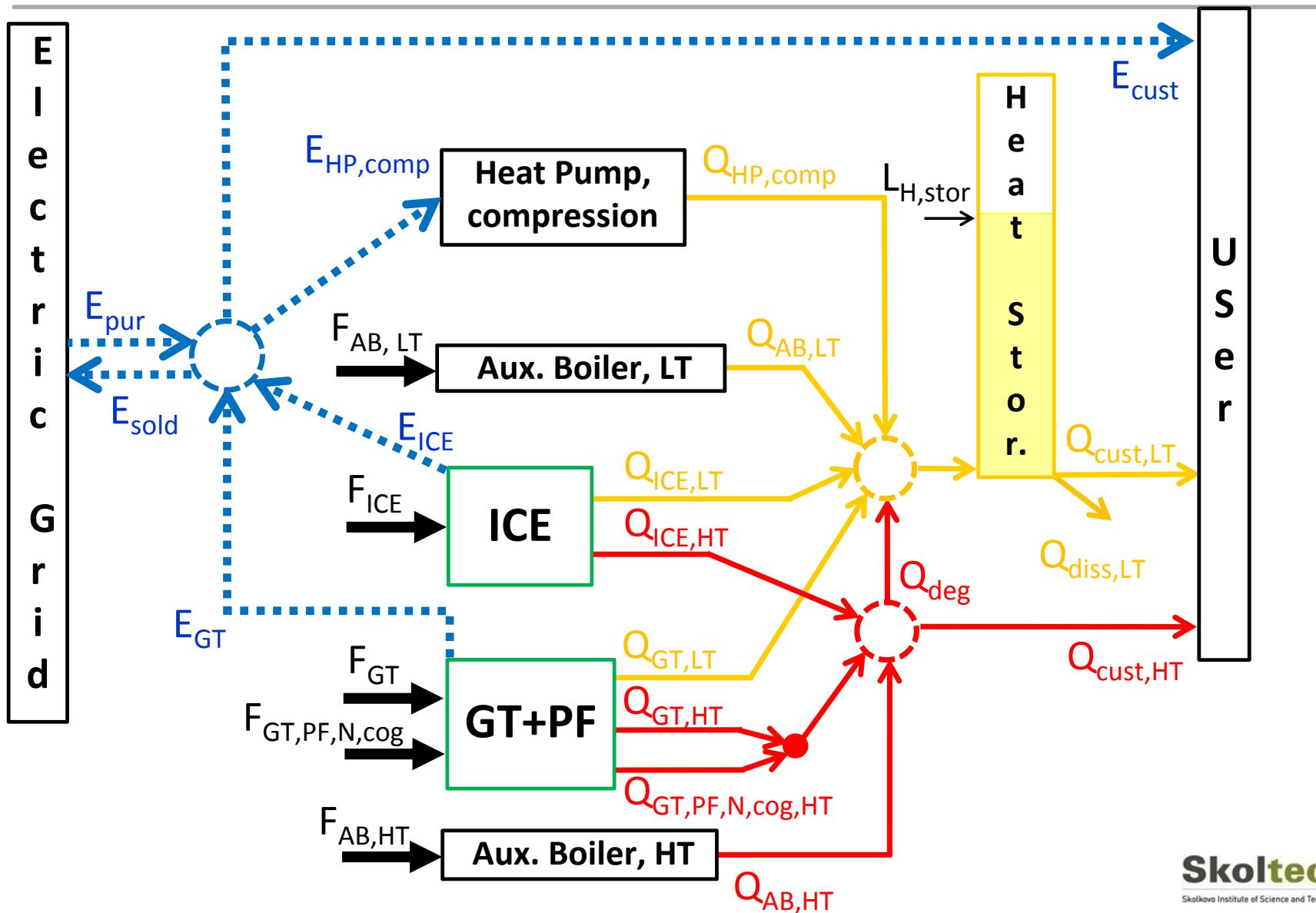
Group Energy COnversion Systems – **GECOS**, <http://www.gecos.polimi.it/>



PoliMI Students:

S.Lico, T. Cortigiani, C. Elsido, A. Emondi, G. Gentilini,  
A.M. Castiglioni, D. Rossin, P. Colombo, E. Perez-Iribarren

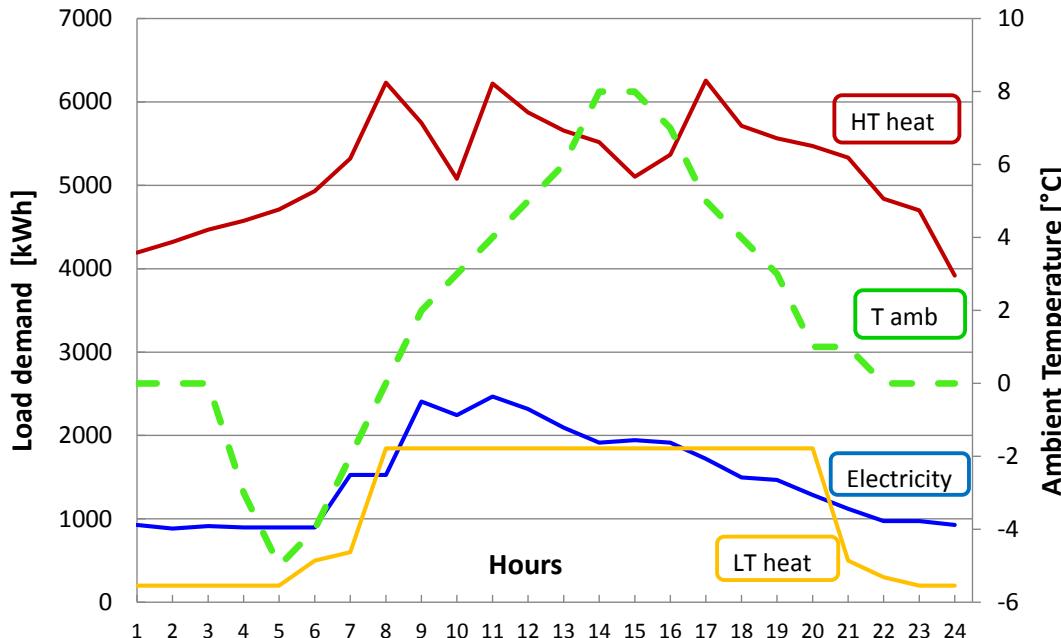
# Test Case – Energy Fluxes



# Test Case – Units & Load Characterization

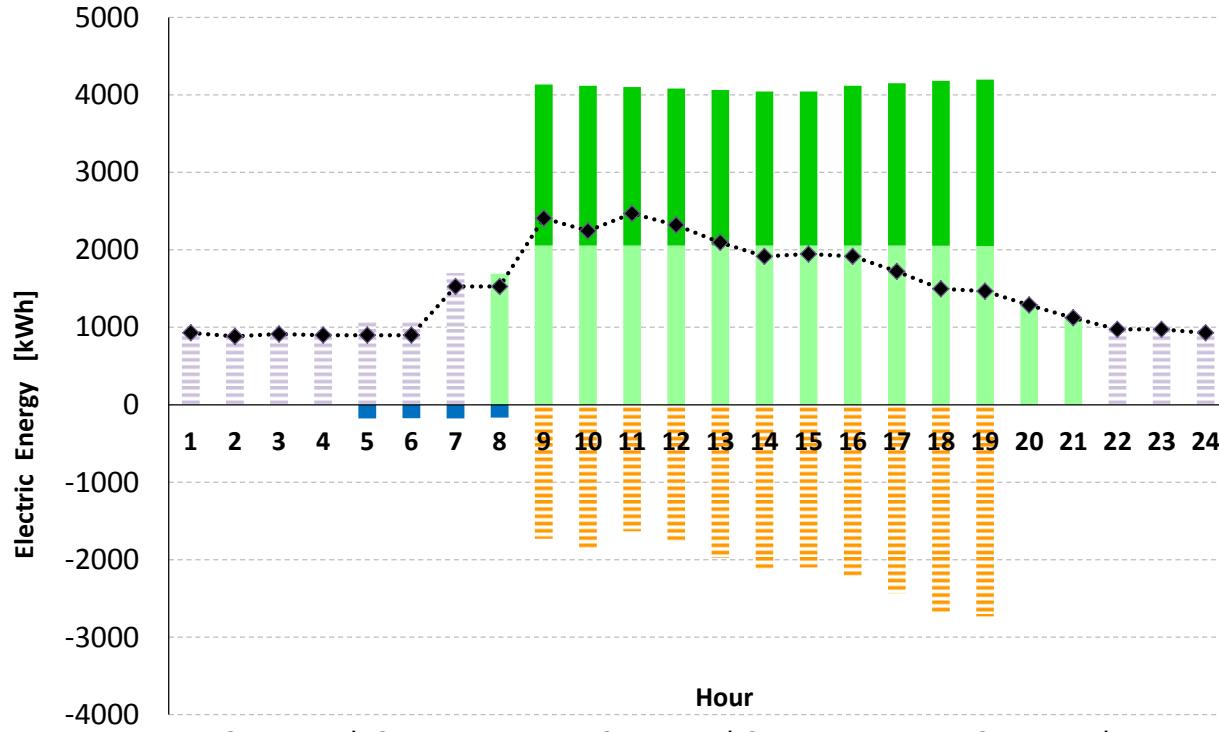
Unit Performance* [kW]	Input	Electric Power	Heat, HT	Heat, LT
Gas Turbine + Post Firing	6765 +2500	2029	2380 +2350	944
Internal Combustion Engine	5198	2048	1483	988
Heat Pump, LT	280			1316
Auxiliary Boiler, HT	6818		6000	
Auxiliary Boiler, LT	2666			2400

\*Nominal Conditions, Temperature will change the units performance

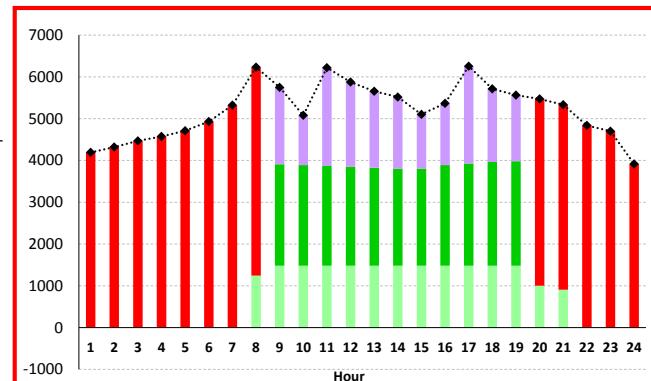


El.Cost [€/kWh]	9 <sup>th</sup> 19 <sup>th</sup>	8 <sup>th</sup> ; 20 <sup>th</sup> 23 <sup>th</sup>	24 <sup>th</sup> 7 <sup>th</sup>
Purchase	0,158	0,116	0,088
Sale	0,126	0,093	0,070
Fuel Cost [€/kWh]			0-24h
Prime Movers			0,06
Boilers			0,06

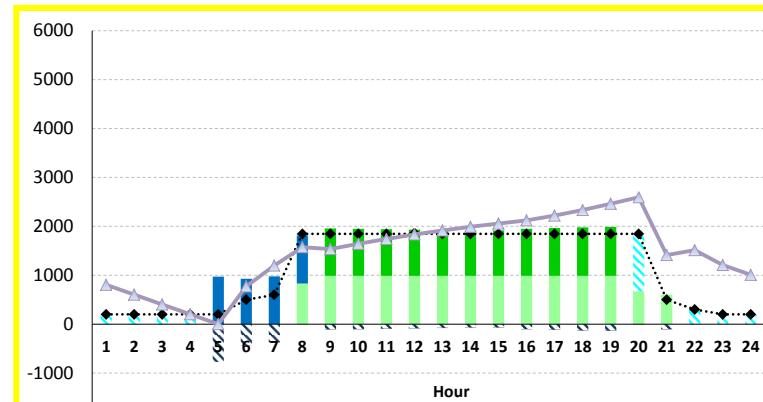
# Test Case, 20 intervals – Electric Energy [kWh]



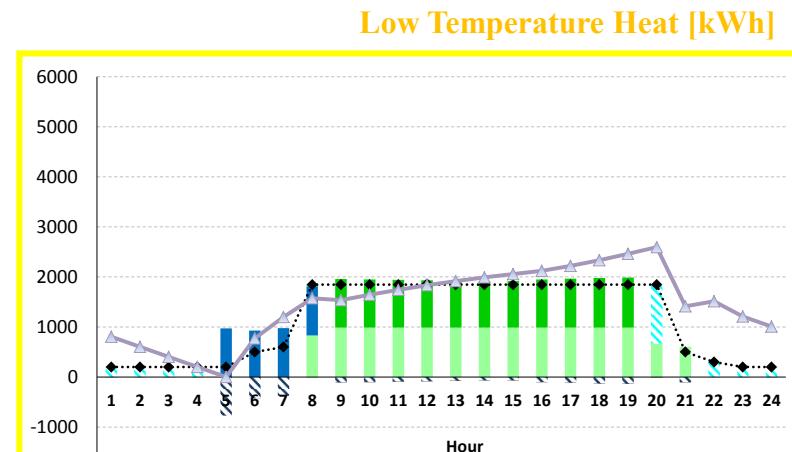
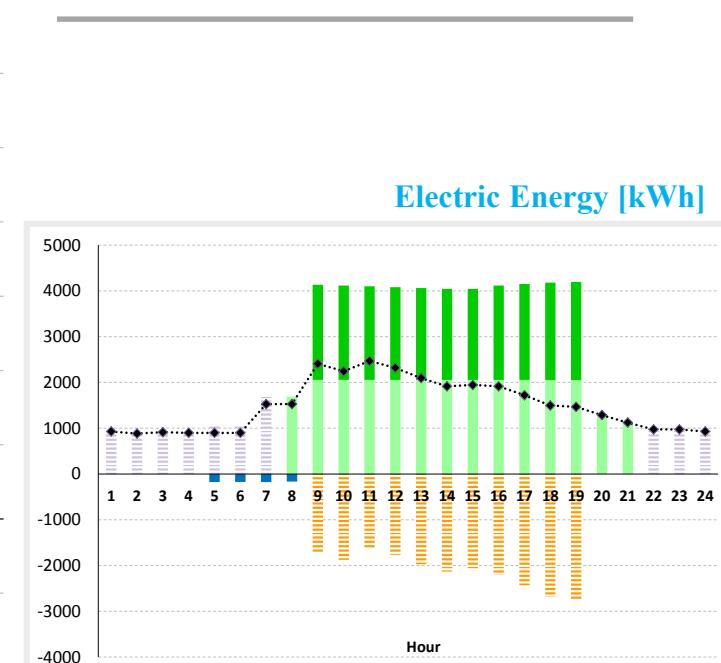
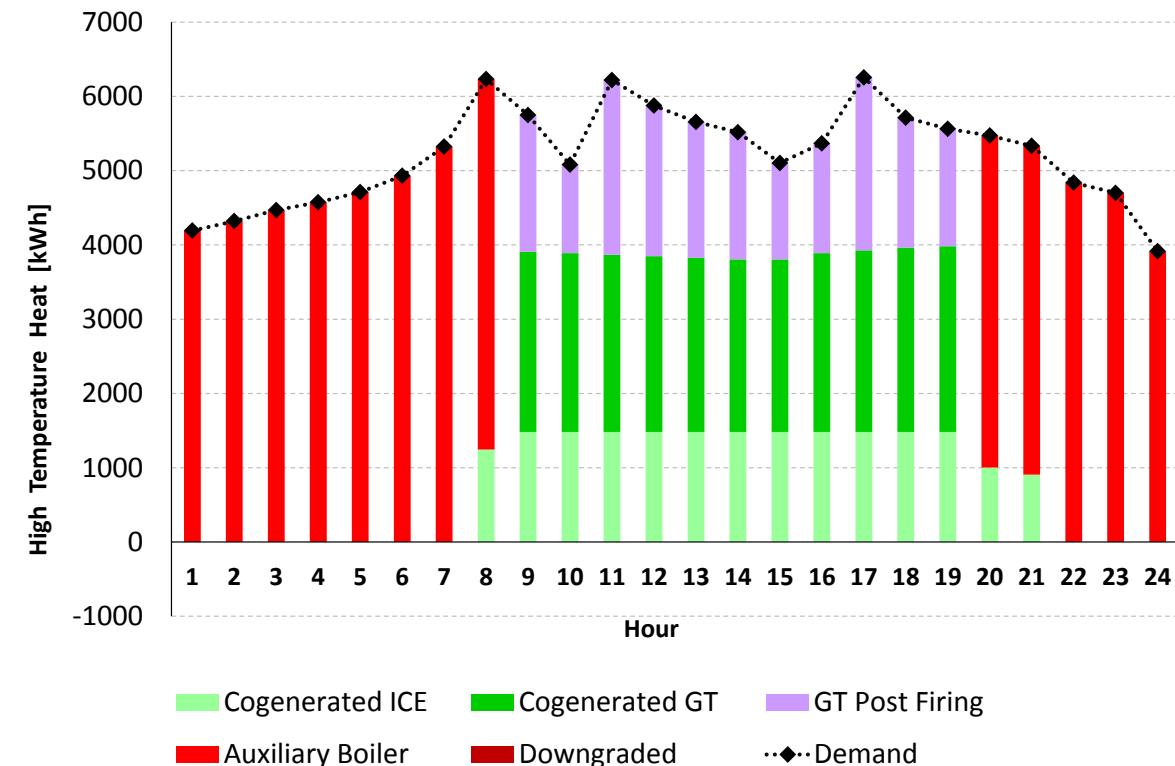
High Temperature Heat [kWh]



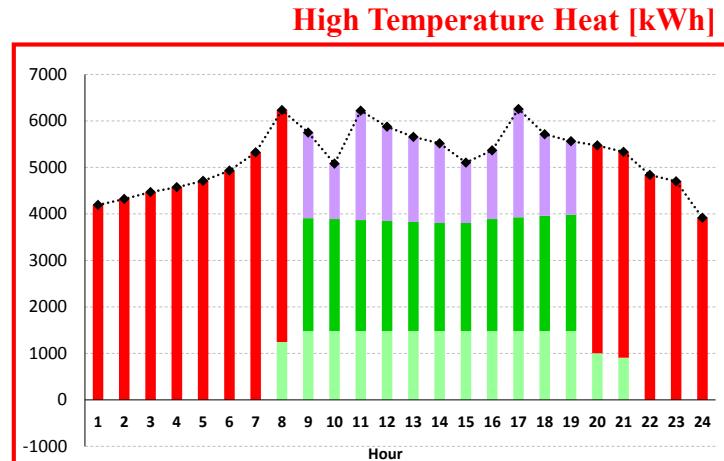
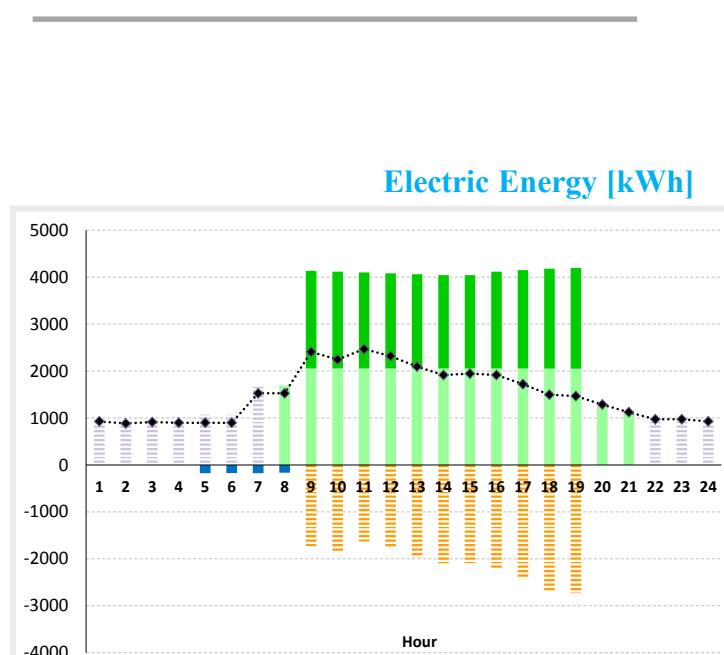
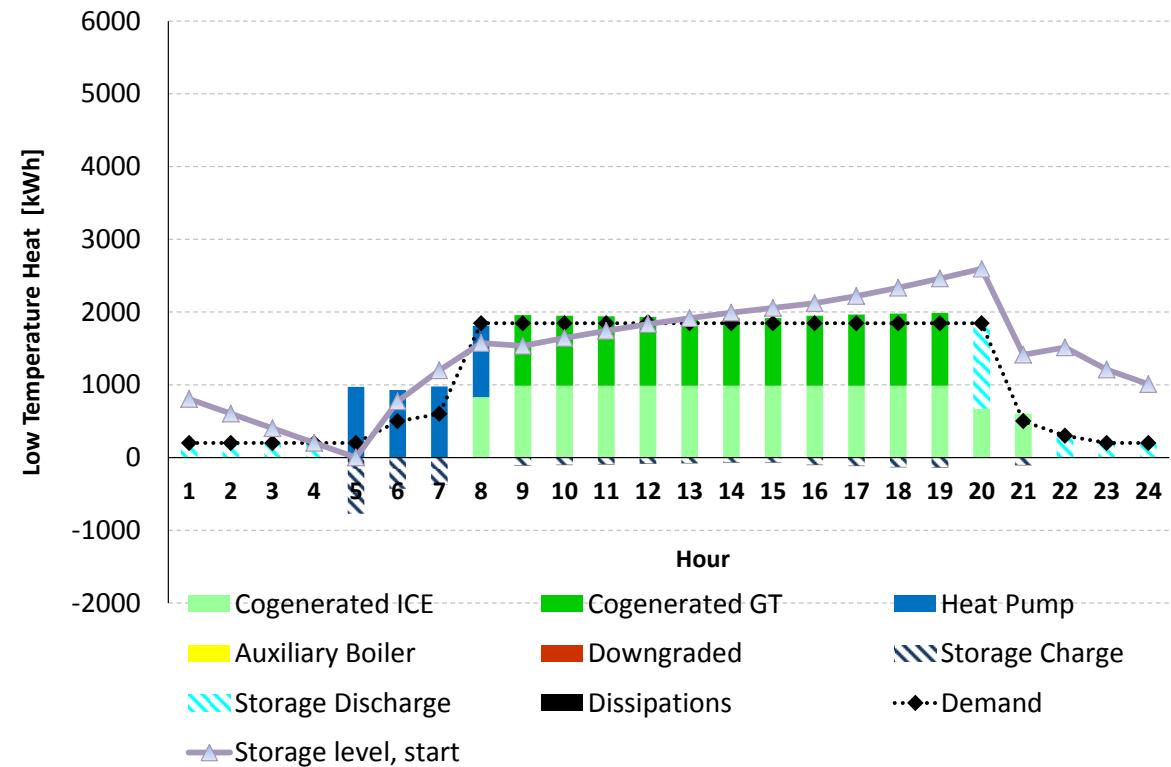
Low Temperature Heat [kWh]



# Test Case, 20 intervals – High Temperature Heat [kWh]



# Test Case, 20 intervals – Low Temperature Heat [kWh]



# Results summary

Discretization Intervals►	5 intervals	10 intervals	20 intervals
Number of binary variables*	1944	6144	21744
Total number of variables	5712	13512	39912
Number of constraints	4685	8285	19085
Computational time (s)	22.69	68.28	293.86
Relative MILP gap (%)	9E-3	9E-3	9E-3
Objective function value (€)	14059.19	14055.43	14054.55

0,009% gap from the global optimum  
corresponds to ~ 1,2€ out of 14060€ in  
the worse case scenario

**Excluding “macro mistakes”!!**

\*Without the «second-degree-of-freedom», post firing, the total number of variables goes down to about 2400 (5 intervals) ÷ 6000 (20 intervals)!!

# Short term differences, 20 intervals vs. 5 intervals

