

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Andrii Liashyk


PhD Program: Mathematics and Mechanics

Title of Thesis: Bethe vectors and their scalar products in quantum integrable models

Supervisor: Professor Anton Zabrodin

Date of Thesis Defense: 20 January 2020

Name of the Reviewer: *Sergey Denkadov*

<p>I confirm the absence of any conflict of interest</p> <p>(Alternatively, Reviewer can formulate a possible conflict)</p>	<p>Signature:</p>  <p>Date: 20-12-2019</p>
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The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

St. Petersburg, December 20, 2019

Sergey Derkachov

Leading Researcher, Laboratory of Mathematical Problems of Physics,

St. Petersburg Department of Steklov Mathematical Institute

of Russian Academy of Sciences,

Fontanka 27, 191023, St. Petersburg, Russia

A report on the PhD thesis

*"Bethe vectors and their scalar products
in quantum integrable models"*

by Andrii Liashyk

The thesis deals with the quantum integrable systems based on high rank Yangian and quantum affine algebras. More concretely it is devoted to the study of Bethe vectors and their scalar products in quantum integrable models with high rank symmetry.

This area is currently extremely important and active, in particular, due to the possible applications to the AdS/CFT integrability. It should be mentioned that recently there was a significant progress in constructing the SoV representation for compact chains of higher ranks and several new approaches to the description of Bethe vectors was obtained.

The first Chapter is devoted to the general overview of the results of the thesis and I should note that it is written very clearly.

In the second Chapter Bethe vectors are found for quantum integrable models associated with the super-symmetric Yangians $Y(g|mn)$ in terms of the current generators of the Yangian double $DY(g|mn)$. The method of projections onto intersections of different types of Borel subalgebras of this infinite-dimensional algebra is used to construct the Bethe vectors. Calculation of these projections makes it possible to express the super-symmetric Bethe vectors in terms of the matrix elements of the universal monodromy matrix. These Bethe vectors are also shown to obey certain recursion relations which prove their equivalence.

Third Chapter is devoted to the study of the scalar products of Bethe vectors in the models solvable by the nested algebraic Bethe ansatz and described by $g|mn$ super-algebra. Using co-product properties of the Bethe vectors we obtain a sum

formula for their scalar products. This formula describes the scalar product in terms of a sum over partitions of Bethe parameters. The recursions for the Bethe vectors are obtained and this allows to find recursions for the highest coefficient of the scalar product.

In the fourth Chapter the Gaudin hypothesis for integrable models with $gl(mn)$ symmetry described by the super-Yangian $Y(gl(mn))$ is proven and in the next Chapter the previous results are generalized to the case of quantum affine algebra $Uq(gln)$.

In the last Chapter a new representation for the Bethe vectors in terms of inverse monodromy matrix is proposed. This gives an important formula describing the symmetry of the highest coefficient in the scalar product.

This work contains many very important and interesting results. The most impressive results of the thesis are the sum formula for the scalar product of Bethe vectors and its generalization of the determinant formula for norm of eigenvectors for super-Yangian $Y(gl(nm))$ and quantum affine $Uq(gl(n))$ algebra. Andrii proposed a new representation of Bethe vectors in terms of inverse monodromy matrix. It appears that the new representation is related to the standard ones, but with the converted parameters. This relation gives important formula describing symmetry of the highest coefficient in the scalar product.

I should to note the extremely high technical level of all results which requires mastery of computing technology.

This thesis based on five recent papers of Andrii Liashyk with co-authors.

They are published in the high quality international journals. Novelty and importance all the results of the thesis make the thesis fully satisfying all the criteria of PhD. I am pleased to recommend to award Andrii Liashyk with the PhD degree.

Provisional Recommendation

I recommend that the candidate should defend the thesis by means of a formal thesis defense

I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report

The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense