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Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Andrii Liashyk

PhD Program: Mathematics and Mechanics

Title of Thesis: Bethe vectors and their scalar products in quantum integrable models

Supervisor: Professor Anton Zabrodin

Date of Thesis Defense: 20 January 2020

Name of the Reviewer: Evgeny Feigin

I confirm the absence of any conflict of interest (Alternatively, Reviewer can formulate a possible conflict)	**Signature:**  **Date: DD-MM-YYYY** 19.12.2019

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

The PhD thesis of Andrii Liashik is devoted to the study of Bethe vectors in quantum integrable systems with various algebras of symmetries. This includes superalgebras of type A, quantum affine algebras and quantum Yangians. The thesis contains many new, important and interesting results and is based on five papers of Andrii Liashik published in reputable international peer-reviewed journals. There are six chapters in the thesis: the first one is introductory giving an overview of the whole picture, while each of the five following chapters treats a separate question on Bethe vectors in certain class of quantum integrable systems. In short, the presented thesis is of exceptional quality and I do recommend to grant Liashik the PhD degree.

Quantum integrable systems play a central role in modern mathematical physics. From one hand, they are expected to describe the behavior of various physical systems. From the other hand, deep mathematical structures are needed for their study, linking the theory of quantum integrable systems to such fields of mathematics as representation theory, algebraic geometry, the theory of special functions, etc. One of the

central ideas in the theory is to use the algebras if symmetries of a system in order to construct various important objects and to compute main quantities of the system, such as correlation functions and form factors.

The thesis under review is mainly devoted to the study of Bethe vectors. In short, the problem is to diagonalize commuting Hamiltonians of a quantum integrable system, coming from the transfer matrix of the system. There are two basic questions to answer: what are the joint eigenvalues and what are the joint eigenvectors of the Hamiltonians? The algebraic Bethe ansatz approach is a procedure to find the eigenvalues, while in order to find eigenvectors one needs to use the action of the operators from the algebras of symmetries of the system. The main results of the thesis of Andrii Liashik are explicit formulas for the Bethe vectors in several models and explicit formulas for the scalar products of Bethe vectors, crucial for the computation of correlation functions and form factors.

Let us briefly describe the content of the thesis. Chapter 1 contains the introduction. The main objects of study are introduced, the problems and questions are formulated and all the main theorems are stated.

Chapter 2 treats the case of quantum integrable systems with the algebra of symmetries formed by the double super Yangian for the algebra gl(m|n). Explicit formula for the Bethe vectors is given in terms of the generators of the double Yangian. Using Gauss decomposition, two formulas for the Bethe vectors are given. The Bethe vectors are proved to satisfy certain recursion relations.

In Chapter 3 the author studies the scalar product of Bethe vectors for the gl(m|n) model. A sum formula for the scalar products is obtained; the formula describes the scalar products in terms of partitions of Bethe parameters. A recursion for the highest coefficient of the scalar product is found.

Chapter 4 is devoted to the computation of the norms of Bethe vectors in the gl(m|n) models. A collection of axioms is formulated fixing the norms of Bethe vectors. A determinant formula for the norm is presented, It is worth mentioning that the results of this chapter prove a generalized Gaudin hypothesis for the norm of the Hamiltonian eigenstates.

In Chapter 5 Andrii Liashik computes the scalar products and norms of the Bethe vectors for the quantum integrable models based on the quantum affine gl(n) algebra. A recursion formulas for the Bethe vectors of models with periodic boundary conditions is obtained. A sum formula for the scalar products of Bethe vectors is presented in terms of a sum over partitions of the Bethe parameters. It is shown that in certain cases the norms take the form of a Gaudin determinant.

Chapter 6 deals with the gl(n) based model. A new formula for the Bethe vectors in terms of the matrix coefficients of the inverse monodromy matrix is proved. A relation to the usual presentation is given. An explicit combinatorial formula for the highest coefficients of the scalar products is found. The thesis is closed with the Conclusions section, where the main results of the thesis are collected, the main topics are outlined and further directions are described.

Summarizing, the thesis of Andrii Liashik contains important, new and interesting results from the theory of quantum integrable systems. The results are published in the reputable international peer-reviewed journals and will be influential for the further studies in the field of quantum integrable systems. The thesis is of the very high quality and I do recommend to grant Andrii Lisahik the PhD degree.

Provisional Recommendation

☒ *I recommend that the candidate should defend the thesis by means of a formal thesis defense*

☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*

☐ *The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*