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Skolkovo Institute of Science and Technology

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Andrii Liashyk

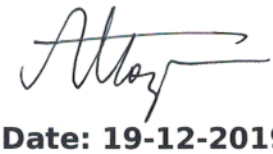
PhD Program: Mathematics and Mechanics

Title of Thesis: Bethe vectors and their scalar products in quantum integrable models

Supervisor: Professor Anton Zabrodin

Date of Thesis Defense: 20 January 2020

Name of the Reviewer: Professor Andrei Pogrebkov

I confirm the absence of any conflict of interest (Alternatively, Reviewer can formulate a possible conflict)	**Signature:**  **Date: 19-12-2019**

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

Brief evaluation of the thesis quality and overall structure of the dissertation.

Dissertation of Andrii Liashyk consists of Introduction, five chapters, concluding remarks and list of references. Introduction gives short description of the main notions of the algebraic Bethe anzatz. Here Gaussian decomposition of the quantum monodromy matrix is also introduced, as well as the current algebra related to it. Thesis quality is very high.

The main part of the thesis consists of five works, where A.Liashyk was one of coauthors. Introduction gives a list of results, presented for defense, together with perspectives of their applications and future development. List of references contains 125 titles.

The relevance of the topic of dissertation work to its actual content

Dissertation of A.Liashyk is dedicated to consideration of Bethe vectors and their scalar products in quantum integrable models with higher symmetry rang. Content of dissertation is in exact correspondence to its announced subject.

The relevance of the methods used in the dissertation

The main distinctive peculiarity of the thesis is alliance of two different approaches. The first of them uses traditional methods, common for the algebraic Bethe anzatz. The second approach is based on the Gaussian decomposition of the quantum monodromy matrix and the current algebra related to it.

Bethe vectors have very simple form for models with the symmetry rank equal 1. This enables calculation of their scalar products directly, by means of commutation relations in the Bethe algebra. But situation changes essentially for models with higher rank of symmetry. In this case vectors have very complicated structure being given in terms of action of matrix elements of the monodromy matrix on the vacuum vector. This is demonstrated in the second Chapter of the thesis. On the other hand, definition of Bethe vectors in terms of projections of the total currents is universal. It is applicable both to models with algebraic symmetry gl(N), and in their supersymmetric analogs and q-deformations.

Usage of the current algebra enables the author of thesis to discover formulas for action of matrix elements of the monodromy matrix on Bethe vectors. These action formulas can be separated in three groups. The first one includes actions of diagonal elements. Exactly these actions lead to Bethe equations defining spectra of the quantum Hamiltonians. The second group involves actions of upper triangular elements. They allow derivation of recursions for Bethe vectors, that in their turn give recursions for the scalar products. Finally, actions of the lower triangular elements are related to the third group. They are necessary for calculation of the scalar products of the Bethe vectors.

Another essential characteristics of Bethe vectors are their properties induced by means of coproduct. They are also established in the framework of the current approach.

Further investigation of the scalar products is performed by methods that were used traditionally in the algebraic Bethe anzatz. In particular, formula for the norm of eigen-vectors of the quantum Hamiltonian is derived by means of investigation of residues of the highest coefficients of scalar products.

The scientific significance of the results obtained and their compliance with the international level and current state of the art

Investigation performed in the thesis of A.Liashyk can be considered as a first step on the way of calculation of correlation functions in quantum integrable models with high rank of symmetry. This

problem is extremely complicated from technical point of view. For this reason up to resent times such models were either not investigated, or investigated under some simplifying hypothesis. Results derived by A.Liashyk are first and, correspondingly, new in this direction.

Main results derived in dissertation are:

a) formulas of action of matrix elements of monodromy matrix on Bethe vectors;

b) nontrivial mappings of Bethe vectors, that do not exist in models with symmetries of rank 1;

c) formula of coproduct of Bethe vectors;

d) formula for scalar products of Bethe vectors as sum by partitions of Bethe parameters;

e) formulas for norms of on-shell Bethe vectors in models with symmetries gl(N), gl(m|n), U_q(gl(n)).

All above results have no analogs in the world literature.

Not only separate results but also their relation deserves attention here. In particular, properties of the Bethe vectors with respect to coproduct were used for derivation of the formula for scalar products in terms of the higher coefficients. Such usage of the coproduct happens the first time. It reduces the problem of calculation of scalar products to trivial one, in fact.

In thesis it is also demonstrated that in models with higher rank of symmetry the approach based on the current algebra is much more effective with respect to the traditional methods. It is just this approach that enables effective handling in spite of numerous technical difficulties that appear in problems of such kind.

The relevance of the obtained results to applications (if applicable)

Dissertation of A.Liashyk has theoretical character. In this sense its main result consists in development of methods of investigation of quantum integrable models with higher rank of symmetry. In perspective the obtained results can be compared with experimental data in physics of condensed states and ultra cold atoms. This will give possibility to estimate adequacy of description of such physical systems in the framework of integrable models.

The quality of publications

The thesis is based on five articles being published in journals of categories Q1 and Q2, and citation bases WoS and Scopus. To my regret English presentation should be improved, while this remark does not diminish quality of thesis.

Provisional Recommendation

[X] *I recommend that the candidate should defend the thesis by means of a formal thesis defense*

[ ] *I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*

[ ] *The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*