

## Jury Member Report – Doctor of Philosophy thesis.

**Name of Candidate:** Andrii Liashyk


**PhD Program:** Mathematics and Mechanics

**Title of Thesis:** Bethe vectors and their scalar products in quantum integrable models

**Supervisor:** Professor Anton Zabrodin

**Date of Thesis Defense:** 20 January 2020

**Name of the Reviewer:**

<p>I confirm the absence of any conflict of interest</p> <p>(Alternatively, Reviewer can formulate a possible conflict)</p>	<p><b>Signature:</b></p>  <p><b>Date: DD-MM-YYYY</b></p> <p>04-01-2020</p>
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*The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.*

*If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.*

### Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

The report is in a separate file

**Provisional Recommendation**

I recommend that the candidate should defend the thesis by means of a formal thesis defense

I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report

The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense

Jury member report—Doctor of Philosophy Thesis;  
Candidate Andrii Liashyk.

The Report:

The thesis presents original research that was published in five papers. This is a modern research in the area of quantum integrable systems. The candidate has done an outstanding work. He obtained new important results in this competitive branch of mathematical physics. The first Chapter is an overview of main results with other Chapters focusing on details of specific results.

The main results can be described as follows. 1) The construction of current type generating functions for the double Yangian for  $gl(n|m)$  and, as a consequence the (universal) construction of Bethe vectors.

2) The derivation of the formula for scalar products of Bethe vectors in models with  $gl(n|m)$  symmetry. 3) Computation of norms of Bethe vectors in models with  $gl(n|m)$  symmetry.

4) The computation of scalar products of Bethe vectors for models based on quantum affine  $gl(N)$  algebras. 5) The description of symmetries of  $gl(N)$ -invariant Bethe vectors.

The topics described in the table of content and in the introduction correspond to the actual content of the dissertation and to published articles.

The methods used in the work are completely relevant. The candidate used a lot of representation theory, algebra and linear algebra, which is completely adequate regarding the nature of problems.

The results are important since they reveal certain fundamental properties of the spectrum of quantum integrable systems related to the Yangians. They will be used in a theoretical research both, in mathematics and in mathematical physics.

The thesis is well written, the existing literature is surveyed adequately and I like the results. From my perspective two results stand out. First, the determinantal formula for Bethe vectors related to various quantum groups related to  $gl(n)$ , second the results concerning the “reflection” symmetry of Bethe vectors.

This work is far beyond an average PhD thesis. Certainly, far beyond an average good work at the University of California at Berkeley where I work for the last 28 years. I have no doubts that the candidate deserves the title of a Doctor of Science based on the research presented in the thesis.

Nicolai Reshetikhin  
Professor of Mathematics  
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