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Skolkovo Institute of Science and Technology

**Thesis Changes Log**

**Name of Candidate:** Ilias Giannakopoulos

**PhD Program:** Computational and Data Science and Engineering

**Title of Thesis:** Memory compression of the Galerkin volume integral equations and coil modeling for the electrical property mapping of biological tissue

**Supervisor:** Professor Maxim Fedorov

*The thesis document includes the following changes in answer to the external review process.*

In answer to the external review process the following changes were included in the final version of my PhD thesis:

- A short paragraph was included at the end of the abstract highlighting the significance of the present thesis.
- Updated the list of abbreviations.
- Appropriate changes are included in the list of publications to clarify between conference and journal ones.
- The third paragraph of page 105 is adjusted.
- An explanation in the end of the second paragraph of section 5.2 is added.
- A summary paragraph is included in the end of the fourth and fifth chapter. The second and third chapter are mostly informative and do not offer any novel ideals.
- The list of abbreviations have been expanded with the missing ones.
- On pages 3 and 26, three phrases were adjusted for further clarification.
- Section 2.2.2 was rewritten
- Orthography issues found on Figures 3.7-3.10 and on the title of Section 4.3 were corrected
- Figure 3.15 was recreated in a logarithmic scale and figure 3.16 was changed to a table.
- Modified the references list to be consistent.

- Updated Figures 3.7-3.10 to vectorized format for higher quality and fixed the typos in the captions and titles.
- Equation 2.22 was removed as a duplciate of 2.20.
- A clarification was included in Section 4.1.2, regarding the rank of the CPD.
- Included the equality with zero in formula 4.3.
- Update labels in Figure 4.12, fixed the caption in Figure 4.10 and table 4.5.
- Figure 4.12 was placed after Figure 4.11.
- Included further clarifications for formula 5.5.

Sincerely,
Ilias Giannakopoulos

# Individual Replies

**Prof. Bastien Guerin**

GENERAL COMMENTS: I have read with great interest the thesis of Ilias, which describes in detail some important novel contributions to the field of computational electromagnetics (EM) with an particular emphasis on applications for magnetic resonance imaging (MRI, my field of specialization). The thesis is well written and presents clearly not only the methods developed by Ilias, but also the foundations upon which he built his work (EM theory and, especially, integral equation (IE) methods). For a reader like me, not specialist in IE methods, this was greatly appreciated! This work is very relevant for MRI, and I imagine for other areas (for example, the memory compression work using Tucker decomposition seems well suited to cell phone safety studies, which face similar concerns as MRI safety). It is now becoming clearer that fast safety computations for MRI are going to be essential for patient-specific evaluation of SAR and maybe even temperature. This is especially important as the main magnetic field (and therefore Larmor frequency) keeps increasing to improve the signal-to-noise ration but leads to greater energy deposition in the body (specific absorption rate problem) and non-uniform B1+. On this latter point, and although Ilias does not mention this explicitly in the thesis, it is likely that the fast computation methods presented here can help with the B1+ mitigation problem, for example by designing better coils for UHF applications or bypassing the B1+ mapping process (and calculating them instead). In other words, applications abound for Ilias' work on memory footprint reduction for SIE-VIE methods, and I am therefore confident that his work will receive a lot of interest.

The second part of the thesis is an application of Ilias' fast computation work, whereby the fast solver is used as the forward model (and derivative evaluation) for the so-called general Maxwell tomography (GMT) problem that aims to estimate the underlying permittivity and conductivity maps of a human being using as input 1) knowledge of Maxwell equations as encoded in the IE method and 2) measurement of an RF coil' B1+ maps. Ilias' work is a continuation of the initial idea by Serrales et al, and shows in simulation that GMT can work in principle using an 8-channel 7 Tesla MRI coil. The numerical results are good and it is comforting to see that the regularized method seems to work in principle in a human application (i.e., beyond the simple 4 compartment distribution modeled in Serrales et al). I was a bit disappointed not to see more experimental results however, given that some were presented in Serrales et al and that the coil actually exists. So one of my question for Ilias is: Why was this experimental evaluation not done? I don't suggest doing it for the defense (nor do I think this is a requirement for Ilias to graduate), but I sure would be curious to see how the method fares on a realistic physical phantom and/or a human head. Ilias elegantly shows that B1+ maps can in principle be used to estimate $\epsilon$ and $\sigma$, I expect the unavoidable (and likely large, i.e. 10-20%) discrepancies between simulated and measured B1+ maps to propagate into unavoidable and large $\epsilon$ and $\sigma$ estimation errors. Even if this proves true, this would not necessarily be a death blow of the method, since it may be the case that a combination of e, s and possibly other parameters can be more easily estimated and yield adequate contrast in cancer and other diseases. In other words, the ultimate test of the method is not (in my mind) accurate estimation of $\epsilon$ and $\sigma$, but whether or not the method yields good and reliable image contrast in the presence of some disease. I would be interested in hearing Ilias' thought on this matter, maybe we'll have this opportunity at the actual defense. All in all, it is clear that Ilias' work is state-of-the-art and represents the best methodology available to date to estimate electrical properties from MRI measurements – and for this, I congratulate him.

**In summary, Ilias' work is excellent and represents the state-of-the-art in both 1) fast computation of EM fields for biomedical applications and 2) estimation of electrical properties from MRI measurements. His peer-reviewed papers are of great quality and the work is eminently relevant. Therefore, I wholeheartedly recommend Ilias to go ahead with his defense, which I look forward to attending. I also look forward to hearing his thoughts on the questions and comments listed above.**

SPECIFIC COMMENTS: Below are some specific comments that may be good to discuss during the thesis defense:

- *LOW MEMORY FOOTPRINT*: The compression factors for the N and K operators are very impressive, even at GHz frequencies. It is impressive that you are able to fit the field computation problem for the entire human body model (head to foot) in a GPU, I feel that's an important result.

- *LMF*: I understand that you applied Tucker and CP compression to the PWL discretization problem, do the same results hold for the PWC discretization?
- *GMT*: How do you think the method would fare as the number of coils is increased? Would this help the conditioning of the method, and therefore help control noise better?
- *GMT*: In your paper you write "the combination of transmit and receive field information is sufficient to resolve fundamental indeterminacies related to absolute field phase". What do you mean by this?
- *GMT*: You also write "the memory required to store the N and K operators increases with the number of voxels and could rapidly become prohibitive even for modern GPU's." Is it possible to recontract only the brain? Would that improve the conditioning of the method and the speed?
- *GMT*: "In this work, we used the true EP of the head models to calibrate the current distribution on the coil conductors, which is used to compute the incident fields (see Eq. 11). This step consists of a surface-volume integral equation (VSIE) coupling, which was performed before the GMT optimization." I was a bit confused by this statement, this seems a bit circular... In reality, you don't know the currents. You do mention that in reality, one would have to estimate both the EP and the coil current. Is there enough information content in the B1+ map to achieve this though? As you point out, the recent development by Georgy would allow doing this quickly.
- *GMT*: Your point about the importance of initial guess is well taken. In practice, you could start with a FreeSurfer segmentation that would likely give you an excellent estimate of $\epsilon$ and $\sigma$. This could save you a lot of iterations and improve the chance of convergence to the true solution.

**Replies**

Dear prof. Bastien,

I would like to thank you very much for carefully reviewing my thesis and for your positive feedback. Your general comments are on point. To clarify a few things, indeed Tucker decomposition is already started being used in the same manner for other IE-based applications, such as in capacitance extraction simulators, among others (references 69-72). Moreover, I hope that the new method will be used for better coil design since the achieved speed-up through GPU programming in tandem with the usage of higher-order basis functions can offer more accurate results very fast.

Regarding the question in the second part of the general comments, the initial version of GMT was using an unrealistic electromagnetic basis for excitation, to resemble a transmit-receive coil array. This basis provided an excellent set of incident fields, but was hard to design in reality. Therefore, we focused on finding a problem-dedicated coil design, optimal for GMT, and investigate on excitation patterns to yield optimal results. From the results in Figure 5.11 it can be seen that a homogeneous $b_1^+$ map would not result to a good reconstruction, and "a one port at a time" excitation should be followed (This is happens because of the relative phase information, encoded in the "a one port at a time" approach). This was a necessary step that had to be completed before an in-vivo experiment, for which we are currenly working on, by bulding the head coil in the lab.

Your final comment about the ultimate test of GMT is right. Indeed one of GMT's goals is to detect a tumor inside the brain. Our simulation results, using the new coil desing, shown that this can be done with high accuracy (Figures 5.18-5.19), where a numerical tumor of homogeneous electrical properties (EP) was inserted in a healthy brain. The high conductivity value of the tumor, in respect to the neibhoring tissue, can be detected easily by a method such as GMT. However, I should note that the EP of an acutal tumor are not known with good accuracy (i.e., the tumor might be inhomogeneous), therefore it remains to be seen how good GMT will behave in an in-vivo tumor-detection application.

Please find below the responses for each of the specific comments.

- Solving a whole body problem in $\sim 15$ minutes is a really good outcome of the new memory-compression methods. The voxel isotropic resolution is 5 mm, but with the increase in GPU's memory in the last years, and in the future, we wil be able to perform simulations for the whole body at 2 mm. The highest number of unknowns that can fit in a 12 GB GPU is 84 millions, which are enough to for the EM simulation of a human head model with 1mm voxel isortropic resolution.

- The first, fifth and ninth term of the PWL basis function correspond to the PWC terms, thus the memory footprint reduction is the same. Similar results also hold for other VIE formulations (i.e., the flux-based). However, because the required memory for PWC is much smaller, we can already fit EM simulations for models discretized with fine resolutions in GPU.

- As we increase the numbers of coils, we encode additional information for the relative phase of $b_1^+$, therefore GMT will be able to reconstruct the EP with higher accuracy. It is important for the coils to produce different $b_1^+$ maps, otherwise they are redundant. Another key component for noise control is the regularizer. The match regularizer proposed by Jose is good at this point, but we need to optimize its parameters for optimal performance.

- This statement is related to Local Maxwell Tomography (LMT), an older method, which motivated GMT. LMT is free of assumptions regarding the RF phase, the coil, and field-magnetization structures. LMT solves simultaneously for key functions of the missing RF phase distribution along with unknown electrical properties, using complementary information from the transmit and receive sensitivity distributions of multiple coils to resolve these fundamental indeterminacies related to the unknown absolute phase (reference 372). However, because GMT is coil-dependent, the best approach if we include the receive information as well, will be to solve for each port of the receive and transmit coils and use the MR-measurable quantities to construct a more robust cost-function. This is quite costly because (due to the additional forward problems), thus we focus only on the 8-port receive transmit array, which can be parallelized in a (at most) 8 GPU server.

- This a very good comment. Indeed we can focus only on the brain when comparing the $b_1^+$ in the cost function, but we have to solve for the whole head, due to the global nature of Maxwell's equations to retrieve the correct $b_1^+$ in the brain. Thus, the speed will not increase. However, the conditioning might greatly benefit in an in-vivo experiment, because we will neglect the $b_1^+$ values in the skull region which are not very accurate. Actually, in the experiment with the cylindrical phantom, we neglected the surrounding plastic, because the $b_1^+$ values were very noisy there, thus improved the conditioning and were able to perform the reconstruction. Finally, the speed will greatly increase using the methods from the first part of the thesis, if we solve for finer resolutions.

- I apologize, my comment is not entirely clear. The initial version of GMT is using only VIE and not the coupled VSIE. Therefore, GMT needs a set of incident fields and the $b_1^+$ to work. This is straightforward when using an EM basis because the incident fields are independent of the EP of the scatterer. However, when using an actual coil, the coil currents change depending on the EP of the load, thus the incident fields produced from these currents to the load.

  To compute these incidents fields correctly we need to use the VSIE solver once before GMT, to incorporate the coil currents' alternations that appear due to the presence of the body. By considering the EP of the body known, we solve the coupled problem and compute the correct coil currents, and with them the resulting incident fields. It is evident that this method, although good and quick for simulation studies, cannot be used for in-vivo experiments, where the ground truth properties are not available.

In the new (under-work) version of GMT, the VSIE solver will be used, where only the voltage at each coil port, and the $b_1^+$ are needed. Specifically, in each iteration, after the update of the EP, the coil currents will also be updated. A simpler way to think about it is, multiple VIE solver solutions, with different incident fields each time. The reason we didn't use the VSIE solver approach yet, was because it was quite slow, with the available method in MARIE, to run hundreds of times. Thankfully, with Georgy's input, the pFFT projection would allow for much quicker use of the VSIE solver, since the precorrection and projection are only geometrical, and have to be computed only once for each individual inverse problem, while the time footprint of the forward problem is similar to the pure VIE one.

- Thank you very much for this input. I did not know about FreeSurfer. It seems that is well-suited for a good initial guess generator and can help accelerate GMT significantly. Alternatively, we were thinking of deep-learning approaches in a tensor-to-tensor translation fashion ($b_1^+$ to EP). During the thesis pre-defense, I had a discussion with prof. Ivan about such techniques where he suggested a very interesting approach. Basically, to learn the inverse problem's behavior by using values of gradient after a small number of iterations of GMT, instead of attempting to learn the highly non-linear inverse form of Maxwell's equations. The deep learning methods in this area are very new and we are currently investigating, in parallel with the in-vivo experiment setup.

I appreciate your recommendation for a formal thesis defense. Your comments and assistance provides significant help and great motivation for this research.

Sincerely,
Ilias Giannakopoulos

**Prof. Shaoying Huang**

The thesis is well structured and carried good scientific quality. The main contributions are a memory foot print reduction for FFT-based volume integral equation method (VIE) based on piecewise linear basis function (PWL) for magnetic resonance imaging (MRI) safety, and RF coils designs that favors one type of the algorithms for magnetic resonance electrical property tomography (MREPT) called global Maxwell tomography (GMT).

The topic exactly reflects the actual content of the dissertation

Based on the presented data, the methods employed to attach the problems are effective.

For the two problems this dissertation is tackling, one is a fast calculation of electromagnet (EM) field in an MRI setup to evaluate MRI safety and the other is the design of a more realistic RF coil for the MREPT method, GMT, they are both important problems in MRI. In terms of the solution, for the first problem, using tensor decomposition for compression has shown the effectiveness to reduce the memory footprint of a VIE with a higher order, which accelerate the calculation of EM fields in an MRI setup for MRI safety. For the solution of the second problem, the coi design is both practical and favors the GMT algorithm for MREPT.

The publications are in good international conferences and journals.

1. **Abstract**: The significance of the thesis can be added as a third paragraph to the abstract
2. List of publications: separate conference papers and journal papers; highlight your name
3. In the introduction of chapter 5, adding more specific characteristics of excitation that will favor GMT, i.e. the characteristics of the field distribution of the RF coil, will make the introduction clearer. The location could be paragraph 3 on page 105.
4. In chapter 5, I understand that a cylindrical 8-channel transmit-receive triangular loop array is designed, based on a previous design [119, 120]. However, the reason of choosing this design is not clear to me. Please add an explanation on this at the beginning of section 5.2.
5. Add a summary to each chapter will make the thesis easier to read.

**Replies**

Dear prof. Shaoying,

I wholeheartedly thank you for carefully reviewing my thesis and for your positive feedback. Please find below the responses for each of the issues you found on my thesis.

1. A short paragraph was included at the end of the abstract highlighting the significance of the present thesis.
2. The appropriate changes are included in the list of publications
3. The mentioned paragraph is adjusted accordingly. Our focus resolves around coils that can produce distinguishable (orthogonal) $b_1^+$ maps from each channel since they favor GMT's cost function, thus the triangular coil designs.
4. I apologize. The main reason behind such coil design is because they can be

highly decoupled and produce strongly diverse $b_1^+$, favorable for GMT's cost function. I have included this explanation in the end of the second paragraph of section 5.2

5. Thank you very much for this input. I included a summary paragraph in the end of the fourth and fifth chapter. The second and third chapter are mostly informative and do not offer any novel ideals. I kept this paragraphs short, since a more clear description of the contributions of the thesis is provided in the sixth chapter, thus, an extended description will be redundant.

I appreciate your recommendation for a formal thesis defense. Your evaluation provides great motivation for my research.

Sincerely,
Ilias Giannakopoulos

**Prof. Maxim Panov**

The considered thesis targets the important area of developing computationally efficient methods for solution of integral equations with applications to MRI. Overall the thesis is very well-written. However, I find that while the amazing work was done on covering the necessary basics on physics and computational math, the structure can be improved by explicitly highlighting parts which constitute author's personal contribution. Currently, it is relatively hard to distinguish between existing developments, own experiments with standard methods and actually novel results. It doesn't diminish the significance of the main results on speeding-up solving integral equations with tensor decompositions and modern computational architectures.

The thesis topic is very relevant as it targets the important area of biomedical engineering and the results are very practical. The content well corresponds to the title, in particular Chapter 4 summarizes some most important results on memory compression. I appreciate usage modern and efficient methods for the tensor decomposition such as Tucker and CP decompositions. They are very natural solution for the considered problem. Also, the usage of GPUs for speeding up the computations is an essential part of the developed approaches and highlights the nice interplay of this research between computational techniques and their parallelization on modern hardware.

Finally, the results of the thesis research are important both from the perspective of the computational science and for the considered applications in MRI diagnostics. The publications summarizing the results were published in the well-reputed journals and conference proceedings including 2 publications in Q1 journals. Thus, the quality of the publications well supports the overall good scientific quality of Ilias's thesis research.

While I have overall positive opinion about the research contents of the thesis I think that the text deserves some improvement. The list of suggestions which should be incorporated in the final version of the manuscript:

1. The Section 2.2.2 is slightly sloppy in terms of mathematics, confusing functions and distances, not defining the objects, ...
2. Some of the figures have low resolution, for example figures 3.3? 3.7-3.10. It would be great if it is improved. The best way is to do vectorized pictures where possible.
3. [page 22] Equations 2.20 and 2.22 duplicate each other, only one should be left.
4. [Section 4.1.2] The accuracy epsilon depends on the rank r. From the text one may deduce that for any r arbitrary epsilon can be achieved.
5. Misprint in the title of Section 4.3.

To sum up, I think that the issues found do not decrease the scientific quality of the thesis and Ilias Giannakopoulos deserves to be awarded with Skoltech PhD degree.

**Replies**

Dear prof. Maxim,

I wholeheartedly thank you for carefully reading and reviewing my thesis and for your positive feedback.

Regarding your first comment, “the structure can be improved by explicitly highlighting parts which constitute the author’s personal contribution.”. All results in chapter 4 and chapter 5 are my own and novel because they are based on the new tensor decomposition-based approaches (Ch.4) and the new RF coil designs for GMT (Ch.5). Results in Chapter 3 are also mine, in the sense that I coded the relevant integral equation formulations and designed the experiments myself. However, they do not contribute anything novel in the scientific community, because the methods have been developed by others in the past. They are included for information and understanding purposes, i.e., results in the SIE section are useful to understand the behavior of the RF coil used later for GMT, while results in the VIE section are key to understand the novel tensor decomposition-based methods. An exception is the comparison result presented in section 3.2.4.

I understand that this might be confusing when first reading the thesis, therefore I changed section 1.2 “thesis structure and contribution section” slightly, to explicitly mention which of these results are novel contributions and which are not.

Please find below the responses for each of the issues you found on my thesis.

1. Indeed the section 2.2.2 was sloppy and had serious mistakes. Prof. Sergey also mentioned that. I corrected it and tried to clarify what was missing.

2. Thanks for pointing this out. Figures like 3.7-3.10 were meant to be vectorized, but it seems that I changed their analysis while editing them. I have updated them to .eps format and they look much better now. For figures like 3.3, because they are hand-crafted by me with an external program, it was not possible to vectorize them.

3. Yes, the equations replicate each other. I have deleted equation 2.22 and changed the relevant text accordingly.

4. This is indeed confusing. I rewrote the text to make it clear that the rank depends on the prescribed accuracy.

5. Thank you for noticing this. The title is fixed now.

I appreciate your recommendation for a formal thesis defense. Your evaluation provides great motivation for my research.

Sincerely,
Ilias Giannakopoulos

**Prof. Nikolay Koshev**

Thesis background

Magnetic Resonance Imaging (MRI) is a powerful and very elegant technique of tomography applied to organic matter (human body tissues). The MRI technique use the fact that organic tissues are transparent with respect to magnetic field, which allows usage of rather strong magnetic fields (1-7 Tesla) in order to obtain a good resolution of tomography. The medical research needs, however, better resolution, which can be reached by application of the greater magnetic fields. Application of the field with the magnitude greater than 7 Tesla, however, has some issues both in terms of safety and computational needs. First of all, the greater field needs application of high frequency RF fields, which can lead to damage of the tissues. The design of the coils demands new techniques for precise mathematical modelling (simulation; of EM fields. Secondly, the greater resolution needs too much memory for processing the data, needs application of high frequency RF fields, which can lead to damage of the tissues. The design of the coils demands new techniques for precise mathematical modelling (simulation) of | EM fields. Secondly, the greater resolution needs too much memory for processing the data, which makes it hardly usable with

GPUs. Thus, greater resolution demands also new techniques of compression and fast and economic computational algorithms.

The thesis is devoted to two main problems, arising in high-resolution MRI:

- development of anew memory-efficient algorithms for the execution of the electromagnetic simulations via FFT-based volume integral equations method;
- Investigation of the problem- and person-dedicated RF coil design for efficient Global Maxwell Tomography (GMT).

Both problems are undoubtedly actual and highly needed by both medical and scientific societies.

Structure of the thesis

In terms of the structure, the thesis is divided into six chapters including the conclusion. The first chapter is an introductory chapter and contains a very global description of the problems being under consideration in the thesis, the review of the literature and state-of-the-art of the area being under consideration. The review of the literature and state of the art are given in a very sophisticated and clear manner. The text is a good causal structure and given in a form which is very easy and interesting to read.

The second chapter gives a structured consideration of the electromagnetism theory related to the research of the thesis. It starts with the Maxwell system, its forms and some fundamental : theorems of electromagnetism. After that, the wave equations and the surface and volume equivalence principles are being under consideration. The chapter finishing with a brief review of some computational methods and techniques taking part in the EM simulations. In my opinion, the chapter is very good although it does not contain the application of the considered — theoretical base to the MRI in order to show reader the physics of MRI in a sophisticated way. Despite that fact, the chapter is very accurate, mathematically strict and gives information ina clear and understandable way.

The third chapter is devoted to full review of the Integral Equations method, its specificity and particularity in application of it to EM simulations. The chapter starts with Surface Integral Equations (SIE) and continues with Volume Integral Equations (VIE) and Volume-Surface Integral Equations. The descriptions of SIE and VIE methods take the main part of the current chapter and are considered in a very detailed way. Some examples of VIE simulations and its analysis finish the chapter. The chapter is made with high accuracy and is easy-understandable.

The fourth chapter is devoted to the review of methods for the memory footprint reduction of the tensors appearing in FFT-VIE methods, and to the presentation of a set of novel matrix- vector product implementations. All implementations are given in form of algorithms, which is useful and helps to understand the methods better. The chapter finishes with a set of numerical experiments demonstrating pros and cons of proposed methods.

The Fifth Chapter is dedicated to the investigation of GMT's performance with realistic RF coils, in simulation, for the EP reconstruction of tissue-mimicking phantoms and more complex structures such as highly inhomogeneous human head models. The chapter starts with consideration of GMT and continues with the statement of the optimization problem for design of RF coils. The chapter also shows a big number of numerical experiments and their results showing applicability and usefulness of the proposed methods.

The last chapter gives the conclusion, some discussion and claim of the future research. The conclusion is clear and relevant to the research and results of the thesis.

Specific comments

The objective of this section is to provide several specific comments and recommendations on how to improve the quality of the presentation. The provided comments are not critical and mostly related to some inaccuracies in the thesis appearance.

1. The formula (4.3): orthogonality commonly means equality of the inner product to zero.

2. Some inaccuracy in Figures presentation: 4.5 — the time should be given in seconds, as caption says; 4.10 — description states comparison with traditional approach (FFT-JVIE): while the labels on Figure are “Tucker” and “HOSVD”; 4.12 is hardly understandable because of absence of the labels near each picture; also, 4.12 presented before fig. 4.11; it's also hard to find any difference on the figure 4.11. Obviously, it should be like that but it might be better to present, for example, logarithm of difference in modelling with different algorithms.

3. At the start of the section 5.1 author says that since GMT is noised inverse problem and thus, the regularization should be applied. It is not true: a lot of inverse problems with noisy input can be solved without regularization. For example, the inverse Radon transform does not need any regularization. In order to claim the need for regularization, the ill-posedness should be proven or cited.

4. The formula (5.5) is, first of all, not the equation, and, secondly, should be clarified. Does it use the finite-difference Laplacian ($\Delta_q$)?

Summary and conclusion

In my opinion, the thesis is a very sophisticated work which shows high level of the researcne-due to the following statements:

- The topic of the thesis is relevant to its actual content.
- The thesis contains a big and accurate research on the state of the art methods enc algorithms, which guaranties that all methods used in the thesis are fresh and relevan- to the problems.
- The results of the thesis are actual and undoubtedly scientifically significant anc relevant to international level and current state of the art.
- The research is devoted to the applied problems and considered in realistic way with realistic parameters, which automatically makes the results applicable and highly demanded by scientific and medical societies.
- During the research, author of the thesis published several high-quality publications in known and highly regarded journals.

The thesis seems to present a novel work, which value for scientific and industrial modelling is doubtless high. The research presented in the thesis is definitely actual and refers to contemporary results in the area. Despite some insignificant inaccuracies in the thesis appearance, the research is interesting. Adthor of the thesis definitely proved his ability to perform a research and to achieve high-rated scientific results. I am glad to recommend the thesis to the defense.

**Replies**

Dear prof. Nikolay,

Thank you for your very precise and careful review of my thesis and your positive feedback. Please find below my responses for each of your specific comments.

1. Thank you for the precise comment. Indeed the equality with zero is missing in 4.3. I have included it.
2. Thank you for all these detailed comments. I have tried to fix most of them in the revised thesis.
    - The caption in Table 4.5 was changed to be consistent with the rest of the table.
    - The caption is changed to HOSVD and HOSVD+CP. Figure 1 below shows the relative error between the traditional approach and the tensor decomposition-based methods, namely HOSVD or HOSVD+CP decomposition.

- Labels are included now in Figure 4.12.
- The numbering is now fixed.
- Figure 4.11 aims to give a qualitative comparison between the tensor decomposition-based methods and the traditional approach. For a quantitative comparison, Figure 4.10 gives the relative error. I am including here a picture of the logarithm of the absolute difference for the sagittal cut and the absorbed power, but it does not offer any additional conclusion to those that we can extract from Figure 4.10.

3. I apologize. Indeed the wording in not proper in this form. The reconstruction of the EP, from noise-free $b_1^+$ maps is possible without the regularizer. However, when we corrupt the $b_1^+$ measurements with additive noise, GMT starts fitting the noise after a few iterations and converges to a noisy map of electrical properties (Fig. 9 of reference 4). Therefore, the usage of a regularization term is key for reconstructing the electrical properties from noisy inputs.

4. Thank you for this comment. The "equation" term was removed and replaced with "formula". $\Delta_q\epsilon$ represents the absolute difference of the complex electrical properties between neighboring voxels, and $q$ is the direction (either $x, y$ or $z$). It is not related to the Laplacian. I am including a more detailed explanation for the regularizer here, and I am referring to reference 4 for an even more precise explanation.

   The regularizer acts on the finite-difference approximation of the electrical properties along each axis $|\Delta_q\epsilon|$ for every voxel:

   $$f_M(\Delta_q\epsilon) = f_S \circ f_{TV}(\Delta_q\epsilon) = 1 - e^{\beta\left(c - \sqrt{c^2 + |\Delta_q\epsilon|^2}\right)} \tag{1}$$

   where $q = x, y, z$ and $\circ$ is the composition of the following

   $$f_{TV}(x) = \sqrt{c^2 + |x|^2} - c, \quad f_S(y) = 1 - e^{-\beta y} \tag{2}$$

   The first function is a continuous relaxation of $L_1$-based total variation. The second function is a saturation function that causes the output to be bounded between 0 and 1. This is a key component of the regularizer since for large jumps between neighboring electrical properties, the saturation causes the differential dependence of the regularizer on the magnitude of the jump in EP to vanish. Simply, the regularizer leaves large transitions in EP unperturbed. The "large" jump is chosen by picking the constants $c$ appropriately. Moreover, $\beta$ is used to specify the desired sensitivity to changes in EP, by defining

   $$\beta = \frac{c^2}{v^2\sqrt{c^2 + v^2}} \tag{3}$$

Here, $v$ is the smallest possible jump in the electrical properties that is expected to be clarified by GMT during the reconstruction and is set for each scatterer accordingly. The final form of the regularizer is

$$f_r(\epsilon) = \frac{1}{3N_v^{\frac{2}{3}}} \sum_{\mathbf{n}\in N_v} \sum_{q\in\{x,y,z\}} f_M(|\Delta_q \epsilon_n|), \tag{4}$$

where $N_v$ denotes the voxels of the scatterer. This additive term to the cost function is the average over all voxels after the nonlinear transformation $f_M$ is applied to each voxel and along each axis.

I note that I used the regularizer as a black-box, and therefore I thought it is better to refer to the original article for additional information on it and focus mostly on the coil designs in this chapter.

I appreciate your recommendation for a formal thesis defense. Your report provides great motivation for the continuation of my research.

Sincerely,
Ilias Giannakopoulos

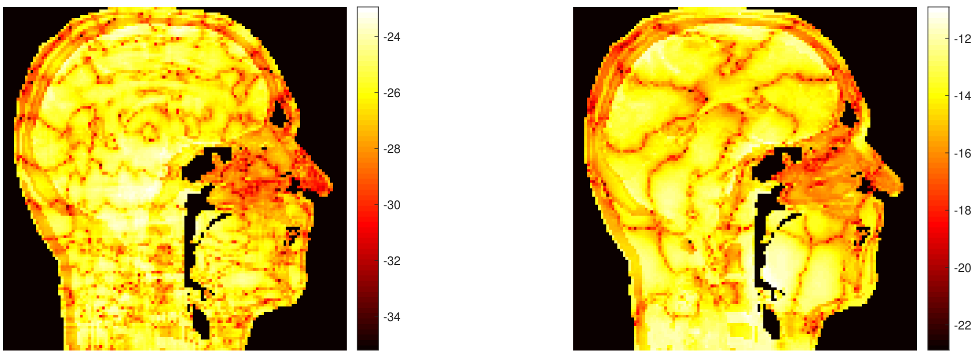


Figure 1: The logarithm of the absolute difference of the absorbed power, between the Tucker decomposition-based approaches (left HOSVD, right HOSVD+CP) and the traditional one. Results are shown for a sagittal cut, and refer to the simulations of Figure 4.11

**Prof. Sergey Rykovanov**

Thesis by Ilias Giannakopoulos is devoted to developing of novel methods for numerical modeling of electromagnetic phenomena for biomedical applications. The candidate shows excellent knowledge of the topic with an extensive and appropriate review given in the overall introduction and in the beginning of each chapter. Novel data compression methods based on tensor decomposition allowed the candidate to significantly speed up the calculation of the inverse electromagnetic problems using both CPUs and GPUs. The structure is well assembled, the thesis is well written, I only have minor comments and some additional questions more for my own education.

Biomedical applications using electromagnetic waves (for example, body screening that can help cancer detection) are of great importance for the humankind. The computational problems that arise are very challenging due to several reasons: numerical volume (i.e. massive computational grids), ill-posed problems. Having a tool that can do the calculations fast, correct and effective is definitely an important development. This is exactly what is performed in research done throughout Ilias's thesis. The topic of dissertation is actual, interesting, challenging and definitely deserves to be reviewed and (successfully) defended.

Main problems of the numerical solution of electromagnetic scattering are well identified already in the introduction of the thesis and later elaborated in details during the main part. Due to the single magnetic field frequency, one can use Helmholtz equation approximation to the full set of Maxwell's equations. In turn, one can then use the known Green's functions of the Helmholtz equation to turn the PDEs into the integral form and use methods like JVIE, DVIE etc. Later, in Chapter 4 where one of the main results are presented, Ilias applies tensor decomposition methods to speed up the VIE method. Methods used throughout the thesis are definitely relevant, not to say the only possible ones for the linear situation where magnetic field has only one frequency. I especially appreciated the usage of compression methods and transferring the simulations to GPUs.

Healthcare is one of the most important areas to be considered. Siemens has recently upgraded their machines to have magnetic field with the strength of 7 Tesla which makes full body screening much faster and more precise. Results of the thesis will further help the scientist to make biomedical analysis faster and more precise. I think the results are significant, definitely deserving a PhD after successful defense. New numerical methods (especially compression of data, GPU usage and deep learning approach) presented in the thesis also have the potential to become commercial and licensed.

As mentioned before, results obtained within this thesis are applicable in healthcare, which is very important and relevant in the modern days.

As far as I can tell, Ilias has 8 publications in peer-reviewed international journals, quality is definitely high.

The summary of issues to be addressed before/during the thesis defense:

1. List of abbreviations and notations is not complete, there are some abbreviations in the text that are not within this list. Moreover, there is such a number

of abbreviations that it is really hard to read the thesis for the unprepared reader – you have to constantly go back and forth to the list to check what things mean. In future, I would advise to make texts more user friendly. This is a comment, which does not need to be addressed.

2. On page 3, there is a claim that "... MARIE and the MARIE 2.0 can produce reasonable EM field estimations in minutes." At this point it is not clear about the size of the object under discussion and I believe the time estimate will change whether this is a finger of a full body. This can be adjusted.

3. On page 26, there is a sentence: "Regrettably, accurate solutions exist for perfect spheres..." This is a very bold statement and I would avoid such statement in texts. For example, without any comments given in the thesis I can immediately contradict this statement by saying that there are exact solutions for the case of a cylinder. This statement should be adjusted.

4. On page 26, there is a sentence about FDTD: "... solution is obtained with some preprocessing." Having been working with FDTD for years I do not really understand this statement yet. This should be extended and clarified.

5. Page 28, equation 2.38, does norm of 'a's have to be strictly larger than 0?

6. On the same page not all quantors are explained, for example in eq. 2.40.

7. Figures 3.7-3.10 there is a typo in Figures caption "Radiattion" and typos in the text: "Radiattion" and "azimuthail". "Azimuthail" also appears in text on page 48.

8. Figure 3.15, text is not readable at all, font size has to increase or some of the curves have to be removed. Caption and axes labels are also not readable.

9. Same for Figure 3.16

10. Merely nitpicking to make thesis perfect: In the reference list, the style of citations is not constant. Sometimes there is "Nature" and sometime "nature" (i.e. ref 121). Ref. 155 refers to a paper in German, so german spelling rules should be applied: " ... zur Optik ... Medien, ... Metalloesungen", Annalen der Physik. This is a very minor comment that can be applied or not upon decision of the candidate.

**Replies**

Dear prof. Sergey,

I wholeheartedly thank you for your very precise and careful review of my thesis and for your positive feedback. Please find below the responses for each of the issues you found on my thesis. I am including the exact adjustments I did on my thesis as well, where needed, with *italic* font, below the response.

1. I have filled the list of abbreviations with the missing ones. I apologize for the inconvenience that the long list caused. I will keep your comment in mind for future publications and scientific reports.

2. The claim is indeed misleading. It is aimed at head models of clinical resolutions ($5\text{mm}^3$ - $2\text{mm}^3$). I have adjusted it accordingly.

   *(MARIE) [22-24] and the MARIE 2.0 (soon the be released) [2,6] can produce reasonable EM field estimations in minutes for the analysis of a realistic human head model of clinical voxel resolution* ($5\text{mm}^3$ *up to* $2\text{mm}^3$)

3. Indeed the statement is partially correct. I modified it to include the case of cylinders and changed the wording. I am not entirely sure, but, I think the electrostatic problem is also solved analytically for infinitely long cones.

   *Regrettably, analytical solutions exist only for perfect spheres with stratified media layers and infinitely long cylinders, based on the Mie Series approach [138].*

4. I apologize, the statement is oversimplified. The postprocessing term was meant for open scattering problems, where specific absorption boundary and radiation conditions should be included in the computations.

   *These time-domain methods (i.e., finite difference time domain method, FDTD [142-144]) mimic the spatio-temporal behavior of the EM fields. Specifically, the EM fields are computed and stored in the 3D space and the respective computational space should be terminated due to the limited capability of storage in the computer's memory. As a result, these methods are optimal by definition for closed problems, while for the case of open scattering problems, specific absorption boundary and radiation conditions should be included.*

5. If the $L_1$ norm of $\mathbf{a}$ is equal to zero, then all the terms of the vector would have to be zero. This case appears if $g$ is zero, meaning that we do not have any excitation for the problem of interest or a zero right-hand-side for the system, and practically there is no reason to solve it since there won't be any scattering field. I do not see a reason why equality with zero cannot be included. Some textbooks are strict on inequality, while others do not include it. I will follow the notations of ref. 154 and include equality as well.

6. I have filled the missing information in the equation and added a description for the inner product below it.

*Any function $p \in \mathbb{D}(\mathcal{L})$ can be expressed, using the projection theorem, as*

$$p = P_{\mathbb{W}^n(\mathcal{L})}\{p\} + h, \quad \langle h, \mathbf{w}_i \rangle_{\mathbb{W}^n(\mathcal{L})} = 0, \quad \forall \mathbf{w}_i \in \mathbb{W}^n(\mathcal{L}), \tag{5}$$

*where $h$ is the minimum distance of $p$* from $\mathbb{W}^n(\mathcal{L})$, *and $\langle \cdot, \cdot \rangle_{\mathbb{X}}$ is the inner product on $\mathbb{X}$*

7. Thank you for pointing these out. I have corrected all the mentioned typos.

8. Indeed both Figures (3.15,3.16) are not acceptable to be presented like this in a scientific report. In Figure 3.15, the font was increased, the legend was simplified to avoid redundant information and the scale was shifted to logarithmic, where the curves can be detected more easily now. Figure 3.16 was changed to a table since the fruitful conclusion is only the number of iterations. Minor adjustments were also applied in the text and captions.

9. See the previous comment.

10. Thanks for locating these out. I have corrected the form of some references as suggested.

I deeply appreciate your recommendation for a formal thesis defense. Your report and evaluation provide great motivation for the continuation of my research.

Sincerely,
Ilias Giannakopoulos