Skoltech
Skolkovo Institute of Science and Technology

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Polovnikov Kirill

PhD Program: Physics

Title of Thesis: On connection between sparse graphs and hyperbolic geometry

Supervisor: Professor Mikhail Gelfand, Skoltech

Professor Sergey Nechaev, Interdisciplinary Scientific Center Poncelet

Name of the Reviewer: Raphael Voituriez

I confirm the absence of any conflict of interest (Alternatively, Reviewer can formulate a possible conflict)	**Signature:**  **Date: 20-08-2020**

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

This thesis focuses on different properties of the spectrum of sparse matrices. This theoretical question emerges in various contexts, in physics and beyond, in which systems or data of interest can be represented by networks where, qualitatively, the number of existing links is significantly smaller than the number of possible links. This question has raised a growing interest over the last decade with the bloom of data science; it has applications in biology, finance and sociology and has deep connections with fundamental problems of theoretical physics. This thesis is therefore very well motivated and timely.

More precisely, this thesis focuses on the spectral density of random sparse matrices, and on the distribution of their maximal eigenvalue. The thesis has several aims: (i) establish connections between the spectral density of random sparse matrices and special functions that emerge naturally in the study of the modular group, (ii) make use of these connection to analyze examples of sparse (hyperbolic) networks with applications to plant morphogenesis (iii) develop new theoretical tools to identify structures (communities) in real (biological) networks. To successfully accomplish this program, K. Polovnikov had to make use of a variety of elaborate tools and methods ranging from theoretical and mathematical physics to data analysis and biology. Overall, this work of very high quality has led to 5 publications accepted in world class journal.

The manuscript starts with an introduction devoted to the definition of the conceptual tools and methods that have been used throughout the thesis; while it is rather technical, it is clear and synthetic and provides a useful state of the art of random matrix theory in the dense and sparse regimes. This first chapter also introduces the key theoretical tools that can be used for community detection in networks (in particular the stochastic block model), and defines the detectability transition.

Chapter 2 focuses on the analysis of the spectral density of a specific class of sparse matrices, which can be viewed as the adjacency matrix of a graph made of independent linear chains. The spectral density of this class of matrices is determined and is showed to be expressed in terms of the so-called pop-corn

function of number theory; in turn, it is suggested that it can be re expressed as a limit of the Dedekind function, which appears in the analysis of the modular group.

Chapter 3 focuses on another example of sparse graph, namely the Cayley tree, and discusses its embedding in the 3D Euclidean space. Conformal analysis shows that such embedding can be realized by making use of the Dedekind function. Applications to plant morphogenesis, and more generally to inhomogeneous tissue growth are discussed, suggesting a geometric mechanism for the emergence of hierarchical structures.

Chapter 4 is rather independent, and studies the fluctuations of examples of random walks under strong geometric constraints. This work shows analytically and numerically that geometric constraints, such as confinement along a circular or triangular wall, can modify scaling exponents of different observables.

Chapters 5,6 and 7 aim at applying different tools of network theory and spectral analysis to the analysis of real data set. The outstanding question is then to determine, if any, communities in such real networks. Different tools are applied to examples of data set from finance and biology (DNA contact maps in cells). In particular, it is showed that a specific choice of operator (with non backtracking property) can be used to efficiently detect communities.

Overall, the manuscript is clear, and the introduction and embedded papers are well presented. The thesis covers rather different fields, which are all based on common tools and concepts. It shows undoubtedly the very high quality of the work performed by Kirill Polovnikov, who fully deserves to be awarded a phD

Provisional Recommendation

X☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense*

☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*
☐ *The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*