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Report on the Phd thesis of Roman Gonin “Twisted representations of toroidal algebras and their applications”

Toroidal algebras are new interesting class of algebras. It appears as some sort of quantum group. Usually we start with Lie bialgebra and after that it is a canonical way to quantize and get family of algebras depending in one parameter. Toroidal algebras of type  $gl(n)$  are the result of such approach – we have to start with Lie algebra of matrix-valued difference operators (it depends in one parameter), fix bialgebra structure – there are many – and quantize after. We get family of algebras depending in two parameters. Toroidal algebras have the class of simplest representations which are called fundamental or Fock. They are building blocks – more involved representations appear as tensor products of Focks and taking the subquotients. Toroidal algebras are acting in such modules through the quotients – deformed  $W$ -algebras. We get by this way  $W$ -algebras corresponding to full linear group and supergroups and also generalisations. Modular group is acting in toroidal algebras and we can twist for example Fock module by automorphism. Romas object – such twisted representations – and also corresponding twisted  $W$ -algebras. Such objects are important from the pure algebraic point of view – and also Okunkov need it for some involve geometry.

Usual and deformed  $W$ -algebras appear as subalgebras in some simple algebras – bosons and fermions. Such imbedding is the most important tool for studying  $W$ -algebras and most applications are depend in such imbedding.

The main idea of Roman work – to find the analog of bosonization for twisted algebras.

Roma found it in the simplest case but the idea to get currents from  $W$ -algebra as combination of vertex operators looks very promising. I think I can be consider as specialist and I confess that I did not believe that Romas formulas can work. So I impressed very much. I also completely sure that similar constructions can be used in many situations. For example -quadratic combination of the components of vertex operators are related with  $q$ -characters formulas.

The last chapter of the text discussing the connection of twisted representations of toroidal algebras with Cherednik algebras. It looks also very promising and interesting.

Let us go to conclusion. The thesis contains new results, important, interesting and stimulating. The main results were published in good journals. Text is readable. I am completely sure that Roma deserve to get Phd degree in mathematics.

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 17.09.2021