

Report

on the PhD thesis

“Twisted representations of toroidal algebras and their applications”

by Roman Gonin.

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In the thesis entitled “Twisted representations of toroidal algebras and their applications” Roman Gonin presents an extensive and systematic investigation concerning the twisted representations of the toroidal algebras. As is explained in the introduction of the thesis, his motivations come from various directions, such as the double affine Hecke algebra, the Macdonald polynomials, the deformed W -algebras, the representation theories based on bosonizations, the K -theory of moduli spaces, and so on. The reason for a serious need for the twisted version of the representations of toroidal algebras consists (at least) of the following two: K -theoretic stable envelopes (the Gorsky-Negut conjecture), q -analogues of the Whittaker vectors and the conformal blocks (the q -isomonodromic tau function).

Roman Gonin, with M. Bréshtein, have constructed three types of representations (fermionic, bosonic, and *strange* bosonic) for the Schur specialization case ($q = t$ case). In Chapter 1 is presented all the details of such constructions, towards the deep analysis of the Whittaker vectors. Chapter 2 deals with the twisted and non-twisted Virasoro algebras from the point of view of the level one representations of the quantum affine algebra $U_q(\mathfrak{sl}_2)$ (XXZ model à la Jimbo-Miwa). It is quite unexpected that the twisted and non-twisted Virasoro algebras are acting on the space of XXZ spin chain in such a natural way, indicating a possibility to study the stable envelopes from some other directions. The semi-infinite construction is given in Chapter 3 (towards the main statement in Theorem 3.5.3.).

I found the present thesis extremely interesting and important containing various nontrivial and new results. Hence I conclude that Roman Gonin deserves the academic degree Doctor of Philosophy in Mathematics.

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