Design, Modeling, and Control of Cable-suspended Aerial Manipulator

Doctoral Thesis
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I hereby declare that the work presented in this thesis was carried out by myself at DLR Institute of Robotics and Mechatronics (Wessling) and Skolkovo Institute of Science and Technology (Moscow), except where due acknowledgment is made, and has not been submitted for any other degree.

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Abstract

Aerial manipulation is a modern and prospective field in interaction robotics with many industrial applications in remotely located and dangerous environments. The typical aerial manipulator is a flying system; the manipulation is conducted by utilizing a robotic arm, while the translation of the arm’s base in the space is done by an aerial vehicle. Such systems have found utilization in the inspection of different structures, e.g., bridges, electric lines, and pipelines, assembly/repair of remote constructions, and various operations in conditions hazardous or dangerous for human safety, e.g., decommissioning damaged nuclear power plants.

Despite the functional benefits, the manipulator serves as an additional payload, requiring more powerful actuation and, consequently, a bigger flying vehicle. Thus, the high risk of a collision between the aerial vehicle and the obstacles in a complex environment imposes restrictions on aerial manipulation in industry. To overcome this issue and achieve higher safety, a novel approach, a cable-suspended aerial manipulator, is recently proposed. Instead of attaching a robotic manipulator directly to an aerial carrier, it can be mounted on a compact actuated platform, which is suspended by a cable on the external mobile crane and responsible for the system stabilization and non-vertical motion. However, due to the physics of the suspended aerial manipulator, static and dynamics disturbances occur during manipulation that affect the system: pendulum-like cable oscillations due to external perturbation or the platform tilt because of the robotic arm weight.

To this end, this thesis brings three major contributions that jointly aim at the development of the control framework and the extensive investigation of the described cable-suspended aerial manipulation concept.

We first present an approach for oscillation damping of the critical pendulum-like motion caused by suspension cable. It turns out that the considered concept of the aerial manipulation might be modeled by a double pendulum with a first bob corresponding to the mobile crane’s hook and the second - to the platform itself. The main challenge is the presence of only one onboard Inertial Measurement Unit sensor, which does not provide complete information on the system state, i.e., crane’s chain motion remains unknown. Moreover, common onboard actuation for aerial vehicles, propeller-based actuation, is integrated at the platform, so we cannot affect pendulum joints directly. To cope with these difficulties, we design a controller motivated by a simplified model. The proposed controller is very simple...
yet robust to uncertainties. Moreover, we propose a gain tuning rule by formulating the proposed controller in the form of output feedback Linear Quadratic Regulator problem. Consequently, it is possible to dampen oscillations with minimal energy consumption quickly. The proposed approach is validated through simulations and experiments aimed at the robustness investigation with respect to the influence of the unmodeled aspects such as wind, robotic arm motion, suspension point motion, and others.

Additionally, to achieve smooth manipulation of the aerial system, we introduce a winch-based actuation for the cable-suspended aerial manipulator. Three controllable rigging cables link the suspension point (the crane’s hook) with the platform and allow to change its translational pose. Such an actuation approach reduces the effect of disturbing gravitational torque caused by the robotic arm weigh distribution on the aerial base. In order to coordinate robotic arm and winch dynamics, a Hierarchical impedance-based Whole-Body Controller is elaborated. It resolves two tasks: keeping the robotic arm end effector at the desired pose and shifting the platform Center Of Mass to the location with zero torque due to Gravity. Additionally, in order to pass torque commands to the position-controlled winch motors, the admittance interface is accommodated. The performance of the introduced actuation system under the considered control strategy is validated through experimental studies.

Finally, it is worth highlighting that research work in the scope of this thesis was conducted within the H2020 AEROARMS project, which addresses the development of the aerial manipulation technologies for industrial inspection and maintenance. Therefore, the last contribution of this thesis is devoted to the extensive field investigation of the system demonstrator under the developed controllers in industrial-like conditions. It should allow validating the performance, safety, and robustness of the overall framework. The prepared industrial-like environment is complex due to various obstacles in close proximity and challenging due to wind and absence of a direct line of sight. Three industrial scenarios are implemented for investigation of the concept and controllers efficiency: deployment of a mobile inspection robot at the remotely-located pipe, peg-in-hole assembly, and turning a valve.

The research results significantly update the state of the art in the aerial manipulation field and facilitate the topic toward the enhanced technology readiness level.
Publications

Lead author


Co-author


4. Chiara Gabellieri, Yuri S Sarkisov, Andre Coelho, Lucia Pallottino, Konstantin Kondak, and Min Jun Kim. Compliance control of cable-suspended aerial ma-

Side topic


To my beloved family and friends
Acknowledgments

Every adventure has its end, they said. So, mine led me to the doctorate accomplishment at the most innovative and technological Russian university, Skolkovo Institute of Science and Technology (Skoltech). It was full of excitement, exploration, joy, and, especially luck which gave me a golden opportunity to perform the research in the dreamland of every roboticist, Institute of Robotics and Mechatronics of the German Aerospace Center (DLR). Being a part of such unique and vibrant environments allowed me not only to work with cutting-edge robotic systems and technologies but, most notably, to meet the incredible and talented people to whom I owe for the final result of my journey.

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Yura
"Mark my words! This drill will open a hole in the universe! ... Drilling a path towards tomorrow!"

Simon

(Tengen Toppa Gurren Lagann, 2007)
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List of Symbols

In this thesis, all scalar quantities are denoted by a plain font (e.g., $l_1$, $n$). Vectors are described by bold lower case letters (e.g., $\mathbf{\lambda}$), and matrices are printed in bold upper case (e.g., $\mathbf{M}$, $\mathbf{A}$). Total derivatives with respect to time are abbreviated by dots (e.g., $\dot{x} = \frac{d}{dt} x$). We denote a $n$-order unit matrix as $\mathbf{I}_n$ and a vector of all ones in $\mathbb{R}^n$ as $\mathbf{1}_{n \times 1}$.

The various subscripts are used with quantities through the thesis. The most important among them are $b$ (body), $w$ (world), $a$ (actuator), $m$ (manipulator), $p$ (platform), and $e$ (end effector). Moreover, for the rotation matrices the subscript and superscript are used simultaneously as $\mathbf{R}^{p}_{\text{from}}$ to denote the initial (from) and transformed (to) frames of the vector $\mathbf{v}$.

Here, only key quantities are listed without or with certain subscripts that are of the prominent importance. Specific meaning of the same parameters with different subscripts as well as non-defined mathematical operations in the thesis become clear when they are introduced in the text.

**Kinematics notation**

\[ \mathbf{R}_b^w \] Rotation matrix representing the orientation of the body frame with respect to the world frame

\[ \phi = [\varphi, \theta, \psi]^T \] Roll, pitch, and yaw angles

\[ \mathbf{v} \] Vector of the linear velocities

\[ \mathbf{\omega}_b = [p, q, r]^T \] Vector of the angular velocities expressed in the body frame

\[ \mathbf{V} = [\mathbf{v}, \mathbf{\omega}]^T \] Twist coordinates representing the rigid body velocity

**Aerial platform modeling**

\[ t_i \] Thrust force generated by $i$–th rotor group

\[ \mathbf{t} = [t_1, ..., t_n]^T \] Vector of thrust forces generated by $n$ rotor groups

\[ \mathbf{t}_i \] Unit thrust vector defined in $\mathcal{F}_b$

\[ \alpha \] Roll angle with respect to the actuator frame $\mathcal{F}_a$

\[ \beta \] Pitch angle with respect to the actuator frame $\mathcal{F}_a$
A Allocation matrix
\[ \mathbf{f} = [f_x, f_y, f_z]^T \] Three dimensional force vector
\[ \mathbf{\tau} = [\tau_x, \tau_y, \tau_z]^T \] Three dimensional torque vector
\[ \mathbf{w} = [f, \tau]^T \] Wrench

Dynamics notation
\[ \mathbf{q} \] Vector of generalized coordinates (joint displacements for the robotic system)
\[ \mathbf{x}(\mathbf{q}) \] Vector of task space coordinates (Position and orientation of the end effector for the robotic system)
\[ \mathbf{M}(\mathbf{q}) \] Symmetric and positive definite inertia matrix
\[ \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \] Matrix containing centrifugal and Coriolis terms
\[ \mathbf{g}(\mathbf{q}) \] Gravity vector
\[ \mathbf{\tau} \] Vector of generalized torques
\[ \mathbf{J}(\dot{\mathbf{q}}) \] Jacobian matrix defining relation between joint space and task space velocities

Others
\[ \mathcal{F}_i : \{O_i, x_i, y_i, z_i\} \] Orthogonal reference frame centered at \( O_i \) with axis \( x_i, y_i, \) and \( z_i \) for \( i \in \{w, b, h, a, p, \text{com}, t\} \), where \( w \) stands for world, \( b \) - for body, \( h \) - for hook, \( a \) - for actuator, \( p \) - for platform, \( \text{com} \) - for COM, \( t \) - for tool
\[ \mathcal{PD}(.) \] Proportional-derivative operator applied to the value \( (.) \)
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Chapter 1

Introduction

1.1 Motivation

Aerial robotics has significantly impacted human lives in the past decades. The rapid growth in this field is driven by the technology development, maturing of the open-source, scientific, and industrial communities, and an availability of the wide range of important applications where such highly mobile robots can contribute [Siciliano and Khatib, 2007, Valavanis and Vachtsevanos, 2014]. Among the most common applications, it is essential to highlight the following:

- mapping, surveillance, and monitoring of the remotely-located areas and objects such as agricultural fields, international borders, urban traffics, or epidemic spots [Irizarry et al., 2012, Kaleem and Rehmani, 2018, Mogili and Deepak, 2018, Ventura et al., 2016],

- search and rescue for missing people or other objects of interest [Mishra et al., 2020, Tomic et al., 2012],

- transportation of the various payloads, e.g., shipment of medical supplies, spraying fertilizer for crops, package delivery in an urban environment [Balasingam, 2017, Villa et al., 2019],

- deployment of communication services, e.g., internet or radio broadcasting in disaster areas [Mayor et al., 2019, Naqvi et al., 2018],

- filming/photography for cinematography and entertainment [Bonatti et al., 2018, Kim et al., 2020],

- various military applications [Asaro, 2013, Hartmann and Giles, 2016].
1.1. Motivation

(a) Inspection of the George Washington Bridge [Stantec, 2017]
(b) Maintenance of the wind turbine blade by a technician [Altitec, 2019]
(c) Window cleaning in the Burj Khalifa tower by climbers [Marmet, 2019]
(d) Replacement of the light bulb at the 457 meter TV tower [Prairie Aerial, 2014]
(e) Inspection of high-power lines by linemen in Faraday suits [World Channel, 2016]
(f) Taking of the soil samples in the toxic environment by the researcher [Gandy, 2020]

Figure 1-1: Application scenarios for aerial manipulation.

All applications above are linked by the same work principle, in which the free-flying Unmanned Aerial Vehicle (UAV) operates within an open environment without direct interaction. However, a significant number of potential applications is required physical interaction with surroundings. The unaccomplished need has led researchers to an investigation toward such a capability within last years. The field of direct interaction with the environment performed by UAV is commonly known as the aerial manipulation.

The aerial manipulator can generally be defined as the aerial base with an endowed manipulation device, e.g., a robotic manipulator or gripper. Developing a system combining robotic manipulator capabilities and UAV dexterity is complex and requires in-depth research, analysis, and significant experience. Among many potential applications for the aerial manipulators, it is vital to list the operations which involve significant human risks and time costs, i.e., the inspection and maintenance of various remotely located structures (e.g., bridges, electric lines, or pipelines, wind turbine blades, communication towers, skyscraper windows) and any operations in hazardous/dangerous conditions for human safety (e.g., decommissioning of damaged nuclear power plants, installation sensors, taking samples), see Figure 1-1.
The most relevant and promising industrial domains for the aerial manipulators in the Russian Federation are the following:

- **Wind turbines.** The most promising and profitable renewable energy market supported by Russian Government programs is wind power generation. According to experts, the economic potential of wind generation in Russia is approximately 17,100 billion kWh/year [Ermolenko et al., 2017], while the total output of the existing wind power plants barely exceeds 1.5 billion kWh/year.

- **Pipelines.** Trunk and industrial pipelines are used to transport a variety of substances, including flammable liquids and gases. According to the EMERCOM of Russia², the total length of trunk pipelines in Russia is 217,000 km, some of which are installed above the ground level on separate supports and overpasses through water and relief obstacles.

- **Bridges and overpasses.** According to Rosstat³, there are about 42,000 bridges and overpasses in Russia with a total length of 2.1 million meters. Inspection and repair of such structures is a challenging task. Thus, in 2020, 60 climbers cleaned and repaired the cables of the cable-stayed bridge, the Russky Bridge, in Vladivostok in extreme conditions: at an altitude of more than 300 meters with wind gusts reaching 20 m/s.

- **Power lines.** According to the Great Russian Encyclopedia⁵, the total length of the active power lines is about 3 mln km.

- **Cell and TV towers.** According to Roskomnadzor data for 2019⁶, the total number of TV and cell towers in Russia is about 800,000.

- **Offshore drilling rigs.** The share of offshore drilling is growing every year. By 2030, Russia should double the number of drilling rigs according to CDU TEK⁷.

More examples of the aerial manipulation applications with detailed description can be found in [Khamseh et al., 2018, Korpela et al., 2012, Ladig et al., 2021, Mohiuddin et al., 2020, Ollero et al., 2018, Ollero et al., 2021, Orsag et al., 2018, Ruggiero](https://www.cdu.ru)
et al., 2018]. Nevertheless, despite the growing interest in aerial contact interaction, the tremendous amount of research in the field, dozens of experimental setups globally, only a few research projects have resulted in industry-level technology ready for the market.

## 1.2 Literature overview

### 1.2.1 Aerial robotics

According to the [Ben-Ari and Mondada, 2018], all robots can be classified by environment and interaction mechanism in the following manner. There are two big groups: fixed-base and mobile robots. The first one includes robotic manipulators, cable-driven robots, and other representatives with a constantly fixed point in the space. In contrast, the mobile robots can move without any connection to a particular environment. Further, they can be divided into three subgroups depending on the environment properties, i.e., aerial, aquatic, and terrestrial robots. Let us consider the aerial type in more detail.

The aerial robot is a mobile robot that can fly, i.e., it can change its pose in three-dimensional aerial space. Such a robot type can also be referred in the literature as UAV, UAS (Unmanned Aerial System), or simply drone. In general, all aerial robots can be modeled as a floating platforms. Depending on the aerodynamic configuration [Belmonte et al., 2019, Lozano, 2010], four classes with particular characteristics can be distinguished, see Figure 1-2. The first one is a fixed-wing UAV. This class is

![Figure 1-2: Robotics taxonomy in terms of environment and interaction mechanism.](image)
featured by the capability to fly long distances with high efficiency. The second class includes rotary-wing UAVs (or rotorcrafts), which are highly maneuverable in hovering and short-distance movements during relatively short flight time. The next class is a flapping-wing UAV which is inspired by nature: it reproduces the flight of insects or birds. This class can be characterized by low power consumption, the best potential toward miniaturization, and an extremely low-weight payload. The last class is devoted to aerostatic aircraft, i.e., airships. They have low power consumption, a low-weight payload, and big size. By investigating the advantages and disadvantages of the classes mentioned above, humanity has started to mix them into separate systems. For example, convertiplane and gyrodyne are aerial systems that combine flight endurance of the fixed-wing class and capability for the hovering and VTOL (Vertical Take-Off and Landing) of the rotorcrafts [Robb, 2006].

One of the main aspects of the rotorcrafts is the number of rotors that generate the thrust. There are single-rotor and multirotor types. The typical representative of the former kind is the helicopter. The latter type includes various configurations, e.g., well-known quadcopter, hexacopter, or octocopter [Basset et al., 2014].

In the scope of this thesis, multirotor type is of most significant interest. Let us consider the related technical challenges, which reflect recent research activity. One of the most critical among them is the development of robust perception, navigation, and control methods which provide UAV with efficient and intelligent capability to operate in various environments. The primary sensors which help with three-dimensional state estimation and mapping for the aerial robots include Global Navigation Satellite Systems (Global Positioning System (GPS), Global Navigation Satellite System (GLONASS), Galileo, BeiDou), Inertial Measurement Unit (IMU), stereo and mono cameras, and Light Detection and Ranging (LiDAR) scanner [Sanfourche et al., 2012]. In terms of control algorithms, various nonlinear methods such as sliding mode control, model predictive control, and adaptive techniques reinforced by data-driven approaches are exploited [Luukkonen, 2011, Mo and Farid, 2019].

Another challenge in this field is the human-machine interface. Researchers have designed and investigated different tools for intuitive and reliable drone control by human operators. As a control source, not only the joystick or touchscreen commands but also the gestures, speech, or even brain signals are utilized [Cauchard et al., 2015, Peschel and Murphy, 2012, Tezza and Andujar, 2019]. The last ones become especially practical in swarm coordination when one human operator should control multiple UAVs simultaneously. To facilitate novel human-machine interface,
different approaches, e.g., leader-following or single-controlled methods [Bashyal and Venayagamoorthy, 2008, Nagi et al., 2014], and embedded technologies such as augmented reality or tactile wearable interfaces [Hoenig et al., 2015, Tsykunov et al., 2019, Yashin et al., 2019] have been recently developed.

The last challenge that we list here is the interaction with the surrounding environment, such as landing or manipulation. During contact tasks, the dynamics of the drone evolves because of the appeared physical constraints. It significantly complicates the control of the drone. To overcome this issue during landing, extended dynamics in control algorithms or auxiliary mechanical systems are developed, e.g., robotic landing gear [Sarkisov et al., 2018, Shi et al., 2019]. In general, the physical interaction of the drone with the environment is quite broad research direction, which will be covered in more detail in the following subsection.

### 1.2.2 Aerial interaction and manipulation

Aerial interaction is a modern and prospective field in robotics with a significant number of industrial applications. Two main branches can be distinguished in this area. The first one is the use of the specific mechanisms (e.g., gripper) for a particular type of aerial interaction with an environment. For example, in [Thomas et al., 2013], an avian-inspired aerial vehicle capable of grasping and transporting different objects was demonstrated. The UAV equipped with a brush for cleaning of the vertical surfaces was proposed in [Albers et al., 2010]. In [Bernard and Kondak, 2009], slung-load transportation and deployment by single and multiple UAVs utilizing slung-load state observer for disturbance reduction were demonstrated. In [Raz et al., 1989], two damping wings were installed directly on the slung-load in order to further facilitate damping of the suspended load. Mechanisms that enable compliant interaction with the environment by utilizing passive springs were shown in [Bartelds et al., 2016, Keemink et al., 2012].

Another important branch is integrating a robotic arm (or even multiple arms) into the UAV. Thus, in [Kim et al., 2013, Korpela et al., 2012, Suárez et al., 2016], light-weight 2-4 Degree Of Freedom (DOF)’s manipulators were used to perform grasping and surface detection tasks. Furthermore, in [Caraballo et al., 2017, Jimenez-Cano et al., 2013], aerial manipulators were exploited to perform assembly tasks. In order to explore further capabilities of aerial manipulation, a 7-DOF torque-controlled KUKA LWR robot was integrated into a UAV [Huber et al., 2013, Kim et al., 2018, Kondak et al., 2014]. Although 7-DOF robotic arm provides ability
to operate with heavy payload and advanced manipulation capabilities (kinematic redundancy [Kondak et al., 2014, Lippiello and Ruggiero, 2012] and full task space formulation [Kim et al., 2018]), due to the heavy weight (roughly 16 kg), the manipulator had to be mounted on the autonomous high-payload helicopter system with 3.7-meter diameter rotor blades. With such dimensions, approaching to a target object might be challenging and unsafe in a complex environment. Moreover, dynamic turbulence caused by ground effect near wide surfaces might serve as an additional source of danger for the helicopter and makes it even harder to operate. Another approach for the aerial manipulation is utilizing aerial articulated manipulator [Endo et al., 2018, Zhao et al., 2018] which represents a multi-DOF manipulator, each link of which is a small-scale UAV.

Following the definition presented in section 1.1, aerial manipulator represents the coupled robotic system containing the aerial base and manipulation device, see Figure 1-3. Both elements are subject to various disturbances that have to be compensated for precise manipulation. Most of the disturbances during an operation can be grouped into dynamic and static by the nature of the source. The dynamic disturbances include wind, signal noise, inertia couplings [Mebarki et al., 2014, Zhang et al., 2019], while examples of static disturbances are internal displacement of the system Center Of Mass (COM) and external wrench induced to the system during interaction with environment [Bodie et al., 2019, Gardecki et al., 2017, Heredia et al., 2014]. The most efficient way to deal with the first group of disturbances is to use the standard actuation system for the aerial base, propeller-based propulsion [Cunio et al., 2009, Tran, 2016, Zhang et al., 2016]. However, the static perturbations bring the most troubles for precise and robust aerial manipulation. To cope with them, the researchers have proposed and investigated the direct weight sliding at the aerial base during the operation, e.g., the motion of additional masses or bat-
tery [Haus et al., 2016, 2017, Kim et al., 2018, Lanteigne et al., 2016]. In the case of cooperative aerial manipulation, where a swarm of drones carries the common manipulation device or tool [Gioioso et al., 2014, Kim et al., 2018, Michael et al., 2011], the static perturbations can be compensated by centralized control strategy for the whole UAV formation [Aghdam et al., 2016, Chung et al., 2018].

The use of aerial manipulators in the industry has been the subject of interest for many research groups and companies over the past decade [Ollero and Siciliano, 2019]. As a result, many international consortia have been established to facilitate the topic progress toward increasing the technology readiness level. The European consortium has achieved the most significant results during several projects such as Aerial Robotics Cooperative Assembly System (ARCAS)\(^8\), AERial RObotic system integrating multiple ARMS and advanced manipulation capabilities for inspection and maintenance (H2020 AEROARMS)\(^9\), and H2020 collaborative AERial RObotic WORKerS (AEROWORKS)\(^10\). In the scope of these projects, applied industrial tasks have been performed, including pipeline inspection, deployment of mobile robots at a remote location, and the use of two robotic arms for carrying objects. In addition, different research groups worldwide have demonstrated the possibility of performing other complex tasks, such as a valve turn [Korpela et al., 2014], the weaving of a rope bridge by a swarm of drones [Augugliaro et al., 2013], inspecting complex surfaces [Jimenez-Cano et al., 2015, Nayak et al., 2018], and joint cooperation of the industrial robot and aerial manipulator [Staub et al., 2018].

### 1.2.3 Long reach aerial manipulation

Recently, a new branch in the field of aerial manipulation has begun to emerge, it is entitled as long reach aerial manipulation. Long reach implies that the robotic manipulator is decoupled with the aerial base by a long link or cable instead of being mounted directly [Caballero et al., 2017, Sarkisov et al., 2019, Shimonomura, 2019, Suárez et al., 2020]. The idea of the long reach aerial manipulation is motivated by applications from the different fields: remediation of the waste storage tanks [Burks et al., 1991], inspection of the underground closed-space places [Mavroidis et al., 1995], and manipulation in space [Yoshida et al., 1996]. This approach brings to the aerial manipulation certain advantages including:

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\(^8\)https://www.arcas-project.eu  
\(^9\)https://www.aeroarms-project.eu  
\(^10\)https://www.aeroworks2020.eu
• ability to operate in a narrow and complex environment at which UAV cannot be placed, e.g., because of the size,

• improving manipulation performance:
  – by increased manipulation workspace without any UAV-related constraints such as landing gear,
  – by reduced influence of the aerial carrier on the environment and the manipulation device, e.g., UAVs downwash or ground effect,

• ensuring safety distance between the operator, object of manipulation, environment, and aerial base,

• increasing the safety of the aerial base by removing rigid coupling with manipulation device and thereby preventing propagation of the wrenches generated from the interaction of robotic arm with the environment.

Despite the benefits mentioned above, there are a number of disadvantages that long reach configuration possesses. The most critical among them is increased weight due to an additional element adding and resulted underactuation of the system, which complicates the control problem.

1.2.3.1 Link-decoupled aerial manipulators

The first group of the long reach aerial manipulation systems exploits the passive link to decouple the robotic arm from the aerial base, see Figure 1-4. It is worth

(a) Rigid link-decoupled aerial manipulator [Suárez et al., 2018]

(b) Flexible link-decoupled aerial manipulator [Suárez et al., 2018]

Figure 1-4: Link-decoupled aerial manipulation systems.
mentioning that dual-arm manipulation is also realized using this approach [Caballero et al., 2018]. The most significant contribution to this direction is provided by GRVC Robotics Labs\textsuperscript{11} of the University of Seville. In [Suárez et al., 2018], rigid one-meter link separates hexacopter with a 2-DOF robotic arm for pipe inspection. This link contains the passive rotational joint in the base, so it behaves as a passive pendulum state of which is measured and utilized in the control law formulation. On the other hand, in [Suárez et al., 2018], flexible link rigidly attached to the both, aerial base and manipulator, is exploited. For such a system, undesired oscillations of the flexible link are suppressed by a coordinated motion of the dual-arm manipulator. It is worth mentioning that the use of the non-controllable (passive) links, which serve as an extension, requires constant and direct contact with the environment in order to operate, while during the flight, it becomes a source of oscillations which implies limitations on the aerial base trajectory.

\subsection*{1.2.3.2 Cable-decoupled aerial manipulators}

In pursuit of increased performance and safety in aerial manipulation, cable-suspended aerial manipulators have been recently proposed [Miyazaki et al., 2020, Sarkisov et al., 2019, Yiğit et al., 2021]. Decoupling of the UAV and manipulation device is performed using a length-controllable cable. Although such a modification presents an additional benefit via increased workspace via changing the cable length during the mission, utilizing such a solution in real world scenarios is also challenging because of the pendulum motion caused by the cable suspension.

It is critical to dampen the oscillation of the cable as quickly as possible when it occurs due to any disturbances such as the motion of the aerial base, robotic arm’s activity, or wind gust. To this end, one may control the aerial carrier itself [Lee et al., 2015, Yoshikawa et al., 2017] to cancel out the payload oscillations. However, these methods can provide only indirect damping.

To damp out the oscillations directly, we can consider having an auxiliary small-scale actuated platform attached to the manipulation device. In [Kim et al., 2018], Min Jun Kim proposed a concept of the UAV carrying a slung-load equipped with moving masses to damp out the oscillations caused by the pendulum configuration. Although the generated by moving masses torques can dampen oscillations of the suspended load, this type of actuation is not practical in interaction problem. Additionally, the moving masses unnecessarily increase the total system weight.

\textsuperscript{11}https://grvc.us.es
Chapter 1. Introduction

(a) Cable-suspended aerial manipulator SAM [Sarkisov et al., 2019]
(b) Long reach aerial manipulator with wire-suspension [Miyazaki et al., 2019]
(c) Spring suspended aerial manipulator dextAIR [Yiğit et al., 2020]

Figure 1-5: Cable-suspended aerial manipulation systems.

To this end, researchers from German Aerospace Center (DLR) Robotics and Mechatronics Center (RMC)\(^{12}\) within the H2020 AEROARMS project have developed the so-called cable-Suspended Aerial Manipulator (SAM) platform which is suspended to the mobile crane and equipped with dual actuation: winches and omnidirectional propeller-based actuation. The former allows compensating for the static disturbances while the latter is capable of generating three-dimensional wrench, which is beneficial during interaction with the environment as well as for the oscillation damping, see Figure 1-5a. One may refer to chapter 3, [Coelho et al., 2020, Gabellieri et al., 2020, Lee et al., 2020, Sarkisov et al., 2019] for more details about design and application of the SAM. The main results of this thesis, including design investigation, modeling, and development of control strategies, are validated on the demonstrator SAM and will be considered in more detail in the following chapters.

Long reach aerial manipulator with wire-suspension and swing-suppression system is proposed by Ryo Miyazaki et al. from the Integrated Sensors and Intelligence Lab of Ritsumeikan University\(^{13}\) [Miyazaki et al., 2019]. The main UAV carries the wire-suspended device equipped with a 1-DOF gripper and the four ducted fans to dampen pendulum-like oscillations in the roll and pitch directions, see Figure 1-5b. The unique feature of this system is the suspension by two wires, which allows to avoid twisting around the yaw axis [Miyazaki et al., 2018].

Recently, a novel suspended aerial manipulator dextAIR has been presented by Arda Yiğit et al. from the ICube Laboratory of the University of Strasbourg\(^{14}\) [Yiğit et al., 2020]. In contrast to the previously described systems, dextAir is suspended

\(^{12}\)https://www.dlr.de/rmc
\(^{13}\)http://www.ritsumei.ac.jp/~skazu/
\(^{14}\)http://www.avr.icube.unistra.fr
1.3 Main contribution

to a carrier by a low-stiffness spring with a long elastic range, see Figure 1-5c. It compensates for gravity and reduces a power consumption by avoiding the use of a winch-controllable cable. Also, due to the spring the vertical dynamics of the aerial robot is fully decoupled from the carrier dynamics. The system is actuated by six pairs of contra-rotating propellers generating a 6-DOF wrench and can be equipped with any gripper.

To the best of the author knowledge, there are no more systems related to the cable-suspended aerial manipulation so far.

1.3 Main contribution

It could be seen from the previous sections that the research in the aerial manipulation field is growing rapidly. Nevertheless, the existing technology level is not mature enough to be exploited in the industrial environment mainly because of the low safety and unsatisfactory performance.

To address these challenges, at the beginning of the research path behind this thesis, we have formulated the main Research Question (RQ) reflected through the SMART (Specific, Measurable, Attainable, Relevant, and Time-Bound) framework [Simmons, 2015] as follows:

**RQ:** How to enhance **manipulation performance** and **operational safety** for the aerial manipulation in industrial applications, e.g., valve turn, peg-in-hole assembly?

The RQ contains two key factors: manipulation performance and operational safety. Let us provide some clarifications on these terms. Manipulation performance is defined by accuracy and level of manipulation capabilities. High-level implies advanced manipulation skills [Khatib, 2005], including full task formulation, kinematic redundancy, dexterous manipulation, robustness to the external constraints, and operation with a heavy payload. Operational safety implies preserving all participants during the process, i.e., operator, environment, an object of manipulation, and aerial manipulator. Deeper investigation of state of the art led us to the novel concept for the aerial manipulation, namely, cable-suspended aerial manipulator, which laid the foundation of our primary hypothesis:
Hypothesis: Adequately controlled cable-suspended stabilizing platform equipped with a robotic arm can provide high performance and safety in aerial robotic manipulation.

Hypothesis illustration is shown in Figure 1-6. As seen in the schematic diagram, the robotic manipulator is decoupled from a carrier by utilizing a cable. The cable-suspended stabilizing platform is equipped with actuation means to counter various disturbances and the robotic arm to perform manipulation. At the same time, the carrier at which the platform is suspended has high payload capability and compensates for the whole system gravity, so actuation efforts at the platform for stabilization should not be high. It allows to make the platform compact and safe. Such a concept completely meets two key factors established in the RQ. Indeed, due to heavy payload capable carrier, one or several redundant robotic arms with full task formulation and high operating weight can be installed at the stabilizing platform. Moreover, the absence of gravitational effect allows to use small-scale and safe platform. Additionally, the cable decoupling ensures the safe distance between the environment and carrier.

Based on the preceding discussion, let us define the three research objectives and corresponding results which will form the main scientific contribution of this thesis:

- To develop a robust control approach that can compensate for dynamic disturbances during transportation and manipulation of the cable-suspended platform without access to the system entire state and characterize its performance.

A research gap for defined objective can be formulated as follows: the long reach configuration is affected by dynamic disturbances, which lead to the pendulum-like oscillations and restrict the exploitation of the system. Dynamic disturbances are expected to alter with high frequency. To this end, fully-actuated propeller-based propulsion, which can generate omnidirectional wrench, is utilized. Since the carrier for the cable-suspended platform is not unique, the system controller should not rely on the specific model. There-
1.3. Main contribution

Therefore, it is natural to consider model-free approaches which are robust to the uncertainties and disturbances [Damak et al., 2020, Iskandar et al., 2019, Ma and Cai, 2011]. As a result, onboard IMU-based Optimal Oscillation Damping Controller (OODC) for confronting dynamic disturbances by virtue of the propeller-based actuation is developed. The controller provides high performance in disturbance countering and low power consumption during the operation. It resolves three main challenges. The first one is the cable partial state estimation exploiting single onboard IMU data. This task is complicated by the fact that the overall system behaves as double pendulum [Singhose et al., 2008, Vaughan et al., 2010]. The second issue is related to the transformation of the onboard generated omnidirectional wrench to the double pendulum passive joints. Lastly, the strategy for the optimal selection of the control gains is proposed based on the minimization of a quadratic cost function balancing energy consumption and system performance. To validate the controller performance, simulative and experimental studies are conducted. Moreover, stability analysis is performed.

- To develop a control approach to facilitate a manipulation performance of cable-suspended platform endowed with kinematically redundant robotic manipulator by compensation for static disturbances and evaluate its performance.

The research gap here is the following: weight of the heavy robotic arm causes the static disturbances in the system. Namely, a shift of the system COM results in undesirable platform tilt. To compensate for this type of disturbances, exploitation of a slow winch-based actuation is proposed. It allows regulating the translational DOFs of the platform relative to the suspension point. Considering that the static disturbances are caused by known (measured) robotic arm motion, the total system (robotic arm and winches) is self-contained in terms of the internal dynamics for the case when the carrier is motionless and propellers do not exert any wrench. To this end, model-based control approach might be utilized [Brosilow and Joseph, 2002, Kool et al., 2016, Todorov et al., 2012]. Based on the state-of-the-art methods [Henze et al., 2015, Sentis and Khatib, 2005], a Hierarchical impedance-based Whole-Body Controller (HWBC) framework is designed using the winch suspension and robotic arm real-time simulated model. A number of challenges within this objective are resolved. First, the complex closed-chain winch cabling is modeled by Lagrangian constrained dynamics and mapped to the intuitive
quasi-state [Blajer, 1997] defined by equal open-chain coordinates for further control formulation. Secondly, task space definition is introduced. Namely, two tasks with different priorities are defined: regulation of the robotic arm in the task space toward desired pose and keeping system COM at zero location to compensate for the static disturbances. Both tasks are regulated under the impedance-based control. Lastly, the admittance interface is introduced and tuned to transform controller torque commands to the position-controlled winch servos. Resulted actuation and control approach is validated in various experimental studies.

- To assess manipulation performance and operational safety of the cable-suspended stabilizing platform through extensive experimental studies in a similar to the industrial environment.

To demonstrate and investigate the applicability of the proposed hypothesis and the performance of designed control approaches in terms of the key factors defined in RQ, manipulation performance and operational safety, extensive experimental studies are conducted under the effect of various disturbances such as external perturbations, the motion of the suspension point, and motion of the robotic arm, in indoor and outdoor conditions. Moreover, the proposed solution is tested in a similar to industrial environment to demonstrate efficiency of the approach in the real-case challenging tasks. In particular, deployment of the mobile inspection robot on the remotely located pipeline surrounded by fence, peg-in-hole assembly, and valve turn are performed in a complex environment with various obstacles.

We would like to highlight that most of the work in the scope of this thesis has been performed within the H2020 AEROARMS project. We are extremely grateful to all consortium partners for their valuable comments and feedback regarding the work. We believe that the proposed concept and thesis results will facilitate the aerial manipulation field in both research and development directions and bridge the gap between laboratory research and real world applications.

1.4 Thesis structure

This section provides a description of the thesis organization to help in navigation within the manuscript and facilitate a reading experience. The thesis contains in
1.4. Thesis structure

![Diagram of thesis structure]

**Chapter 1**: Introduction

**Chapter 2**: FUNdaMENTALS

**Chapter 3**: Physical system description

**Chapter 4 and 5**: Modelling and control

**Chapter 4**: Oscillation damping control with the propeller-based actuation

**Chapter 5**: Hierarchical whole-body control with the winch-based actuation and kinematically redundant manipulator

**Chapter 6**: Experimental investigation of the proposed control strategies

**Chapter 7**: Field investigation in the industrial-like environment

**Chapter 8**: Conclusion

The total eight chapters and has coherent information flow illustrated at the diagram depicted in Figure 1-7.

**Chapter 2 - Fundamentals** - recalls the basic mathematical and robotic principles which are utilized in the scope of this thesis. In particular, general approaches for the physical modeling and control are considered, main concepts of the aerial, cable-driven, and manipulation robotics are provided.

**Chapter 3 - Physical system description.**

In this chapter, we form the foundation and investigate the cable-suspended aerial manipulation concept. Specifically, we consider potential operational scenarios at which the system can operate, formulate the qualitative requirements to this type of system, and finally, we consider the detailed design of a specific representative, system demonstrator, the SAM, including main features, quantitative requirements, sensors, mechanical and electronic design.

**Chapter 4 - Oscillation damping control with the propeller-based actuation.**

This chapter is devoted to the development of the OODC, which relies on the

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15Spelling is inspired by http://pergatory.mit.edu/resources/fundamentals.html
propeller-based actuation. Modeling of the general and reduced cases is considered. Further, control goal, detailed controller derivation, gain tuning rule, stability analysis, and controller performance evaluation are established. Additionally, model behavior with regard to the variation of system parameters is investigated. In the end, the developed approach is validated on the toy problem.

Chapter 5 - Hierarchical whole-body control with the winch-based actuation and kinematically redundant manipulator.

The chapter presents the HWBC framework for the winch actuated platform with the attached redundant robotic arm. Original constrained dynamics of the winch cabling is derived and further presented in the reduced form by introducing assumptions to formulate control law. A whole-body control framework with included system inverse kinematics, task definition, and admittance interface is described. In the end, performance evaluation on the toy problem is presented.

Chapter 6 - Experimental investigation of the proposed control strategies.

Within this chapter, validation and experimental investigation of designed controllers developed in the scope of the chapter 4 and chapter 5 on the system demonstrator are conducted. At the beginning of the chapter, we describe the experimental environments in which validation is performed. Further, OODC performance is analysed through experiments for various situations including the robotic arm’s motion, external disturbances, and changes in the operational environment. Next, we investigate the functionality of the elaborated controller with common operational tasks, namely, robotic arm motion and pick and place. In the end, both controllers are integrated and tested.

Chapter 7 - Field investigation in the industrial-like environment - considers and evaluates an operation of the hypothesis-based system demonstrator under the developed controllers in outdoor similar to the industrial environment for three proposed missions: deployment of the mobile inspection robot, peg-in-hole assembly, and the valve turn at the remotely located pipeline.

Chapter 8 - Conclusion - focuses on the discussion of the obtained results, learned lessons, limitations, and future potential of the cable-suspended aerial manipulation technology.
Chapter 2

Fundamentals

In chapter 2, we briefly review the basic foundations exploited in the scope of this thesis. At first, we define the general terms from the robot kinematics and dynamics. Secondly, the modeling approaches applied to the aerial and cable-driven robots are discussed. Lastly, we explain control methodologies on which we relied in our developments. More complete and elaborated discussions on the topics can be found in the commonly used literature [Asada and Slotine, 1986, Corke, 2017, Craig, 2009, Murray, 1994, Siciliano and Khatib, 2007, Siciliano et al., 2010].

2.1 Modeling of physical systems

The physical systems with which we deal in the scope of this thesis consist of single or multiple rigid bodies (links) connected to the unified chain through joints. Each joint provides one DOF motion for pair of surfaces. Two main joint types for our applications can be distinguished. Namely, prismatic joint provides translational motion of two surfaces relative to each other, and a revolute type allows to rotate one surface around another. Further, we will introduce the kinematic and dynamic modeling basics for the single rigid body, and then briefly provide some insights on modeling of multirotor, cable-driven, and serial manipulation robotic systems.

2.1.1 Notes on kinematics and dynamics

2.1.1.1 Rigid body transformation

The location of any rigid body is fully described by its position and orientation (in brief, a pose) with respect to a certain reference frame.
Chapter 2. Fundamentals

Reference frames

The pose of the rigid body can be defined relative to the reference frame. Let us denote by \( \mathcal{F}_w : \{O_w, x_w, y_w, z_w\} \) the world (inertial or fixed) frame placed in inertial space and by \( \mathcal{F}_b : \{O_b, x_b, y_b, z_b\} \) the body frame attached to the COM of the rigid body and aligned with its principal axes, see Figure 2-1. Both frames are right-handed and with orthogonal axes. Then, the position of the \( O_b \) with respect to the world reference frame can be defined by a vector \( p_w^b = [x_{O_b}, y_{O_b}, z_{O_b}]^T \in \mathbb{R}^3 \). At the same time, the rotation of the rigid body can be expressed in multiple ways.

![Figure 2-1: Reference frames for the rigid body.](image)

Rotational motion

Let us consider the case when only the rotational motion of the rigid body is performed. There are different strategies to represent the rotation of one coordinate frame with respect to another. All of them are interchangeable and have their own advantages and disadvantages. Let us introduce the most common:

- A rotation matrix expresses the orthonormal basis vectors of the body frame, i.e., unit vectors \( \mathbf{i}_b, \mathbf{j}_b, \mathbf{k}_b \), with respect to the world frame:

\[
R_w^b = \begin{bmatrix} \mathbf{i}_b & \mathbf{j}_b & \mathbf{k}_b \end{bmatrix}.
\]

The rotation matrix belongs to the special orthogonal group \( \text{SO}(3) \) of the real \( 3 \times 3 \) matrices with orthonormal columns and determinant equal to 1:

\[
\text{SO}(3) = \{ \mathbf{R} \in \mathbb{R}^{3 \times 3} : \mathbf{RR}^T = \mathbf{I}_3, \det(\mathbf{R}) = 1 \}. \quad (2.1)
\]
Due to properties of the SO(3) group, the composition rule for rotation matrices can be formulated: rotation matrices can be combined using matrix multiplication, i.e., $R_c = R_b R_a$, where $a, b, c$ represent some reference frames. It is worth mentioning that from (2.1) the interesting result for rotation matrix follows: $R^{-1} = R^T$.

- Roll, Pitch, Yaw (RPY) angles is the set of Euler angles $\phi = [\varphi, \theta, \psi]^T$ to describe the rigid body orientation around the world frame axes. It consists of 3 independent parameters and corresponds to the minimal representation of spatial rotation (in contrast to the rotation matrix with nine redundant elements).

One of the critical disadvantages of the representation through Euler angles is discontinuity for a singular configuration when two rotational axes are parallel. This effect is known as gimbal lock.

- A unit quaternion is a four-parameter rotation representation defined as:

$$q = \{\eta, \epsilon\},$$

where $\eta = \cos \frac{\xi}{2}$ and $\epsilon = [\epsilon_x, \epsilon_y, \epsilon_z]^T = r \sin \frac{\xi}{2}$ are the scalar and vector parts of the quaternion representation which decodes rotation of an angle $\xi$ around of rotation axis represented by unit vector $r$. Quaternion is constrained by the following condition that explains the title unit:

$$\eta^2 + \epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2 = 1.$$

### Rigid motion

As we mentioned above, the rigid body transformation is composed from the rotational and translational motions. Thus, configuration of the rigid body with respect to certain reference frame is completely described by pair $g = (p, R)$. The product of $\mathbb{R}^3$ and SO(3) is called special Euclidean group SE(3)$^1$:

$$SE(3) = \{g = (p, R) : p^w_b \in \mathbb{R}^3, R^w_b \in SO(3)\} = SO(3) \times \mathbb{R}^3.$$

---

$^1$In general, groups can be defined for any dimension $n$, i.e., SE($n$) or SO($n$). In three-dimensional space $n = 3$.  

20
Let us denote by $e_w$ and $e_b$ the coordinates of some point $E$ in the world and body frames, respectively. Then, we can formulate relation between these two sets of coordinates through $g^w_b$ configuration as follows:

$$e_w = p^w_b + R^w_b e_b,$$

or exploiting so-called *homogeneous transformation matrix*, $\bar{g}^w_b$:

$$
\begin{bmatrix}
    e_w \\
    1
\end{bmatrix} = 
\begin{bmatrix}
    R^w_b & p^w_b \\
    0_{3 \times 1} & 1
\end{bmatrix}
\begin{bmatrix}
    e_b \\
    1
\end{bmatrix}. 
$$

It is worth mentioning that similar to rotation matrices, the composition rule might be applied to homogeneous transformation, i.e., $\bar{g}^c_a = \bar{g}^c_b \bar{g}^b_a$, where $a, b, c$ denote some reference frames.

**Twist and wrench**

The two groups, special Orthogonal and special Euclidean, are of particular interest to us. Both of them are representatives of the Lie group, which is a group and a differentiable manifold\(^2\). First, we need to introduce a Lie algebra of the group as the tangent space at the identity element.

Let $\omega = [\omega_z, \omega_y, \omega_z]^T \in \mathbb{R}^3$ be an angular velocity vector specifying the direction of rotation in the world frame. Then, we can define Lie algebra of SO(3) group as:

$$\text{so}(3) = \{ \dot{\omega} \in \mathbb{R}^{3 \times 3} : \dot{\omega}^T = -\dot{\omega} \},$$

where $\wedge$ (wedge) sign maps the vector to the skew-symmetric matrix as follows:

$$\dot{\omega} = \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}.$$

A general element of Lie algebra for the SE(3) group is a partitioned matrix:

$$S = \begin{bmatrix}
\dot{\omega} & v \\
0 & 0
\end{bmatrix} \in \text{se}(3).$$

\(^2\)The manifold is a topological space that is locally Euclidean.
2.1. Modeling of physical systems

Here, \( \mathbf{v} = [v_x, v_y, v_z]^T \in \mathbb{R}^3 \) is the linear velocity vector. Then, Lie algebra of special Euclidean group is defined as follows:

\[
se(3) = \{ (\mathbf{v}, \mathbf{\omega}) : \mathbf{v} \in \mathbb{R}^3, \mathbf{\omega} \in so(3) \}.
\]

We call the element of Lie algebra \( se(3) \) as a twist. Such an element \( \mathbf{S} \) might be conveniently represented in the vector form of twist coordinates \( \mathbf{s} \) by utilizing \( \vee \) (vee) map as:

\[
\mathbf{s} = \begin{bmatrix}
\hat{\mathbf{\omega}} & \mathbf{v} \\
0 & 0
\end{bmatrix} \in \mathbb{R}^6.
\]

Similar to the \( SO(3) \) group, the inverse operator wedge, \( \wedge \), transforms twist coordinates \( \mathbf{s} \) to the twist \( \mathbf{S} \):

\[
\begin{bmatrix}
\mathbf{v} \\
\mathbf{\omega}
\end{bmatrix} = \begin{bmatrix}
\hat{\mathbf{\omega}} & \mathbf{v} \\
0 & 0
\end{bmatrix} \in \mathbb{R}^6.
\]

Let us define wrench as the six dimensional force-torque vector:

\[
\mathbf{w} = \begin{bmatrix}
f \\
\tau
\end{bmatrix} \in \mathbb{R}^6,
\]

where the first vector \( \mathbf{f} \) is the linear force, and the second one \( \mathbf{\tau} \) is the rotational torque vector. As a vector, the wrench can be expressed with respect to the different reference frames. Thus, the spatial wrench is represented in the world frame, and the body wrench is the wrench in the instantaneous body frame.

**Rigid body velocity**

Let us denote the configuration of the body frame with respect to the world frame by homogeneous transformation matrix \( \bar{\mathbf{g}}_b^w \):

\[
\bar{\mathbf{g}}_b^w = \begin{bmatrix}
\mathbf{R}_b^w & \mathbf{p}_b^w \\
0_{3 \times 1} & 1
\end{bmatrix}.
\]
Let us define the *spatial velocity* expressed in the world frame as the following twist:

\[
\dot{V}_w = \dot{g}_b^w (g_b^w)^{-1},
\]

and the *body velocity* expressed in instantaneous body coordinate frame as:

\[
\dot{V}_b = (g_b^w)^{-1} \dot{g}_b^w.
\] (2.2)

The spatial and body velocities are related as follows:

\[
\dot{V}_b = (g_b^w)^{-1} \dot{V}_w (g_b^w).
\]

Corresponding twist coordinates of these two frames are connected by \(6 \times 6\) matrix, *adjoint transformation* associated with \(g_b^w\):

\[
\begin{bmatrix}
v_w \\
\omega_w \\
v_b \\
\omega_b
\end{bmatrix} =
\begin{bmatrix}
R_b^w & p_b^w R_b^w \\
0^7 & R_b^w
\end{bmatrix}
\begin{bmatrix}
v_w \\
\omega_w \\
v_b \\
\omega_b
\end{bmatrix}.
\]

**Error elements**

Based on the composition rule, we can define error elements between two frames, i.e., desired and current pose of the rigid body in terms of:

- **homogeneous transformation**

  \[g_{error} = g_{des}^{-1} g_{real} = (R_{error}, p_{error}),\]

  here \(g_{des}\) is the homogeneous transformation that describes desired pose, \(g_{real}\) is the homogeneous transformation that describes current pose of the object.

- **unit quaternion**

  \[\varrho_{error} = \varrho_{des} \circ \varrho_{real}^{-1} = (\eta_{error}, \epsilon_{error}),\]

  here \(\varrho_{des}\) is the unit quaternion that indicates desired pose, \(\varrho_{real}\) is the unit quaternion that describes current pose of the object, \(\circ\) is the operator of quaternion multiplication.
Additionally, it is worth introducing a velocity error element as follows:

\[ V_{\text{error}} = V_{\text{real}} - \text{Ad}_{g_{\text{error}}}^{-1} V_{\text{des}}. \]

### 2.1.1.2 Robot kinematics

A regular robotic system contains several rigid bodies (links) and can be described by joint coordinates \( \mathbf{q} \in \mathbb{R}^n \), where \( n \) is the number of robot DOF. Here, \( \mathbf{q} \) is the vector of displacements in translational and rotational robot joints at which the system evolves naturally. It defines the joint (or configuration) space.

However, as a rule, we want to control certain DOF \( \mathbf{x}(\mathbf{q}) \in \mathbb{R}^m \) along desired direction instead of direct robot joints. For example, in robotic manipulation, we can control either joint displacements or end effector location. The values of vector \( \mathbf{x}(\mathbf{q}) \) establish the task (or Cartesian, operational) space. In the case of \( m < n \), the robot is kinematically redundant, i.e., while maintaining the main task in \( \mathbf{x}(\mathbf{q}) \) coordinates, the internal motion of the robot is possible by utilizing \((n - m)\) DOFs null space.

Two main kinematics problems can be distinguished. The forward kinematics solves the task of finding position and orientation of the end effector based on the joint displacements:

\[ \mathbf{x}(\mathbf{q}) = f(\mathbf{q}). \]

In contrast, an Inverse Kinematics (IK) solves the task of calculating the joint displacements for a certain end effector configuration:

\[ \mathbf{q} = f^{-1}(\mathbf{x}). \]

Then, let us define a linear map of the joint velocities space to the task velocities space as follows:

\[ \dot{\mathbf{x}}(\mathbf{q}) = \frac{\partial f(\mathbf{q})}{\partial \mathbf{q}} \dot{\mathbf{q}} = \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}. \]  

(2.3)

The matrix \( \mathbf{J}(\mathbf{q}) \in \mathbb{R}^{m \times n} \) which defines relation between velocities is referred to as the analytic Jacobian matrix. Differential kinematic map \( \mathbf{J}(\mathbf{q}) \) is illustrated in Figure 2-2. Two subspaces range \( \mathcal{R}(\mathbf{J}) \) and null \( \mathcal{N}(\mathbf{J}) \) can be distinguished. The range subspace of \( \mathbf{J}(\mathbf{q}) \) represents the set of all task velocities that can be achieved.
by the joint velocities for current robot configuration. The null subspace of $J(q)$ is the set of joint velocities that does not generate any motion in the task velocities space for given robot posture.

Due to the law of energy conservation, the power generated by joint motion should be equal to the power produced by task motion. It allows us to exploit the Jacobian matrix for establishing the map between an end effector wrench $F \in \mathbb{R}^6$ and vector of joint torques $\tau \in \mathbb{R}^n$ as well:

$$\tau = J(q)^T F.$$  \hspace{5cm} (2.4)

Additionally, the Jacobian matrix analysis can provide some valuable insights on the instantaneous robot configuration. Primarily, we are interested in the singularity and manipulability of the robot.

At the singular configuration, the Jacobian matrix is rank-deficient, i.e., the determinant of the Jacobian is 0. At such configurations, the singularity causes the loss of one or more DOF. Therefore the joint velocities required to maintain desired task velocities in specific directions become close to infinite [Williams II, 1999].

One of the possible metrics to evaluate robot proximity to the singular configuration is to estimate its manipulability by calculating manipulability ellipsoid volume [Vahrenkamp et al., 2012]:

$$EV(q) = \frac{4\pi}{3} \sqrt[3]{\det(J(q)J(q)^T)}.$$  \hspace{5cm} (2.5)

Here, $EV(q)$ is the volume of the manipulability ellipsoid, $J(q) \in \mathbb{R}^{m \times n}$ is the robot Jacobian defined by its instantaneous joints configuration.
2.1. Modeling of physical systems

2.1.1.3 Dynamics

Two main approaches for the derivation of the rigid body dynamic equations exist. In general, the equations of motion describe the robotic system dynamic response to input joint torques.

Newton-Euler formulation

The *Newton-Euler formulation* of equations of motion is based on the direct interpreting of the momentum conservation and angular momentum conservation laws. These two laws result in dynamic translational and rotational equations of motion, respectively. The first one is well-known as the Newton’s Second Law. Let us write it down in the world frame:

\[
\frac{dp}{dt} = m \frac{dv}{dt} = ma = f,
\]

where \( p = mv \) is the momentum of the system. Similarly, we can write down the *Euler’s equation* in the body frame as follows:

\[
\frac{dL}{dt} = Y \frac{d\omega_b}{dt} + \omega_b \times L = \tau,
\]

where \( L = Y \omega_b \) is the angular momentum, and \( Y \) is the diagonal inertia tensor with respect to the principal axes. We use the aerospace notation to denote the angular velocity expressed in the body frame \( \omega_b = [p, q, r]^T \).

Formulated equations form the Newton-Euler formulation and describe the translational and rotational motion of the rigid body COM under the applied forces and torques. Robot modeling utilizing this formulation is performed by generating and combining the equations of motions for every robot link.

Lagrangian formulation

The *Lagrangian formulation* describes the system behavior in terms of the stored energy. Let \( q \in \mathbb{R}^n \) be *generalized coordinates* that effectively describe the \( n \)-DOFs robot, e.g., for robotic manipulator this set of coordinates might be the vector of the joint angles. Then, the Lagrangian of the mechanical system can be defined as
the difference between system kinetic energy $\mathcal{T}(q, \dot{q})$ and potential energy $\mathcal{U}(q)$:

$$\mathcal{L}(q, \dot{q}) = \mathcal{T}(q, \dot{q}) - \mathcal{U}(q).$$

By utilising the Lagrangian, the Lagrange’s equations of motion in terms of generalized coordinates are expressed as follows:

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}} \right)^T - \frac{\partial \mathcal{L}}{\partial q}^T = Q,$$

where $Q$ contains generalized and external torques.

### Serial and closed chains

Regardless of the selected formulation, the resulted equation of motions for the robotic systems can be described in general by the following dynamic equations:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau + A^T\lambda + \tau_{ext},$$

$$A(q)\dot{q} = 0,$$

where $M$ is the symmetric and positive definite inertia matrix, $C$ contains the centrifugal/Coriolis terms, $g$ is the gravity vector, $\tau$ is the vector of the generalized torques, and $\tau_{ext}$ are external forces and torques.

Given the generalized torques and the initial system conditions, the forward dynamics equations evaluate the robot motion, expressed in terms of the generalized coordinates and their derivatives, i.e., $\dot{q}, \ddot{q}$, and $q$. So, the generalized torques serve as inputs, and kinematic parameters - as outputs. For the serial chain, the matrix $A$ is a zero matrix, while for the closed chains, it represents system constraints. There are two types of constraints, namely, holonomic and nonholonomic constraints. The former constrains the configuration and consequently reduces the configuration space, i.e., the DOF. The latter type puts constraints on the velocity and does not reduce the configuration space. Constrains in the form $A(q)\dot{q} = 0$ are called Pfaffian constraints, they might be both holonomic or nonholonomic.

### 2.1.2 Multirotor systems

Let us introduce the multirotor aerial robot. Such a system represents a single rigid body with several propeller-based actuators, each of which can produce a force.
2.1. Modeling of physical systems

(a) CCW propeller thrust force $t$, drag torque $q$, and angular velocity $\omega$ directions
(b) Orientation angles of the propeller relative to the frame arm

Figure 2-3: Propeller actuation.

2.1.2.1 Performance of the propeller

First of all, let us provide some details about the actuation. Namely, let us consider the motor with the attached propeller at the shaft, see Figure 2-3a.

While spinning, the propeller creates a reduced pressure zone on the top of the blade. Due to Bernoulli’s principle [Lindsay, 1952], the air molecules result in an upward-lifting force. Let us call it as the thrust force [Spakovszky et al., 2008] and define it for the environment with constant parameters and particular propeller geometry design as follows:

$$ t = f(D, \omega, \rho, u_0, K) = k_t \omega^2. $$

Here, $D$ is the propeller diameter, $\rho$ is the air density, $K$ is the fluid bulk elasticity modulus, $u_0$ is the flight velocity, $\omega$ is the propeller angular velocity in $[\text{rad s}^{-1}]$. While operating in the same conditions, most of those parameters remain constant and can be combined in one gain, thrust coefficient, $k_t$, such that the thrust force is proportional to the square of the propeller angular velocity.

Since the propeller spins in the air environment the drag torque, $q$, directed opposite to the angular velocity, is generated as well:

$$ q = f(D, \omega, \rho, u_0, K) = k_q \omega^2 = k_{ad} t. \quad (2.7) $$

Here, $k_q$ is the constant drag coefficient that depends on the propeller design and environment conditions, and $k_{ad}$ is the aerodynamic gain presenting ratio between drag torque and thrust force of the propeller.

It is worth noting here that there are two types of propellers: ClockWise (CW)
and CounterClockWise (CCW) propellers. The first one generates the vertical thrust and co-directional drag torque when the propeller performs clockwise motion, as seen from the tip of the thrust vector. The latter generates vertical thrust and contradi-rectional drag torque when the propeller spins in the counterclockwise direction. Both vectors, thrust and drag, are aligned with the motor spinning axis.

2.1.2.2 Unidirectional and multidirectional thrusts

First of all, let us define an actuator frame $F_a : \{O_a, x_a, y_a, z_a\}$ which is centered in the COM of the actuator and placed such that $x_a$ is aligned with the installation frame arm and pointed outward, $z_a$ is aligned with motor spinning axis, and $y_a$ complements the frame to the right-handed, see Figure 2-3b. Then, the propeller rotor group can be placed with various orientations with respect to the installation arm. Namely, two installation angles might be introduced, $\alpha$ and $\beta$ which represent the rotation around $x_a$ and $y_a$, respectively.

Now, we consider two representatives of the multicopters. The first one is a quadcopter with collinear propellers, that can generate unidirectional thrust. The second system is a hexacopter with tilted propellers, i.e., installed with certain $\alpha$ and $\beta$ angles, see Figure 2-4. The latter is able to produce the thrust in multiple directions.

Let us denote by $w_b$ a wrench that is produced in the body frame by thrust forces generated by propellers.

$$w_b = \begin{bmatrix} f_b \\ \tau_b \end{bmatrix} = At. \tag{2.8}$$

Here, $A$ is the allocation matrix that maps the thrust vector $t$ to the generated body wrench. Thus, the allocation matrix reflects the general design and depends on the geometric parameters (number of propellers, dimension of the arm frame, propeller installation angles) of the particular system.

Let us assume we want to generate the upward lifting force $f$ by introduced multirotor systems, see Figure 2-4a and Figure 2-4b. In this case, both systems can produce such an amount of thrust that can satisfy the task, i.e., provide zero torque and non-zero force:

$$w_b = \begin{bmatrix} f_b \\ \tau_b \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}.$$
2.1. Modeling of physical systems

(a) Quadcopter case. The desired wrench contains a vertical force and zero torque
(b) Hexacopter case. The desired wrench contains a vertical force and zero torque

c) Quadcopter case. The desired wrench contains a tilted force and zero torque
(d) Hexacopter case. The desired wrench contains a tilted force and zero torque

Figure 2-4: Wrench generation by the common quadcopter with collinear thrusts (left) and hexacopter with tilted thrusts (right).

Now, let us consider the case when we would like to generate slightly tilted force $f'$, see Figure 2-4c and Figure 2-4d. In the case of the quadcopter with collinear propellers, we need to tilt the whole system in order to align each thrust force with desired force $f'$. Thus, we have to generate additional torque $\tau$. On the other hand, hexacopter with tilted propellers can generate desired force with zero torque.

The first system is a representative of unidirectional thrust multirotor [Hamandi et al., 2020]. Such a system is underactuated and has coupled dynamics. The second system with multidirectional thrusts has partially decoupled dynamics. So, it could generate the desired force without exerting any torques. It is worth mentioning that to fully decouple translational and rotational dynamics, at least 7 unidirectional thrusts are required, as was shown by Marco Tognon in [Tognon and Franchi, 2018].

The linear map diagram shown in Figure 2-5 can help interpret the equation (2.8)
Chapter 2. Fundamentals

(a) Allocation matrix map for the quadcopter with collinear thrusts. All configurations of the $t$ space generate the coupled wrench.

(b) Allocation matrix map for the hexacopter with tilted thrusts. Certain configurations in the $w_b$ space result in the decoupled wrench.

Figure 2-5: The allocation matrix map between the thrust space $t$ and the body wrench space $w_b$.

more rigorously. Let us define two spaces, namely the space of thrust vector and the generated body wrench space. Then, the allocation matrix maps the former one to the latter. Analysis of the allocation matrix can help us to extract the properties of the propeller allocation design. Particularly, the rank of the allocation matrix reflects the number of DOF that we can generate independently. Thus, for the considered above underactuated quadcopter with $t \in \mathbb{R}^4$ (4 propellers), we can state that the corresponding $\text{rank}(A_u) = 4$. On another hand, for the multirotor system with $t \in \mathbb{R}^{7+}$, we can write down that the $\text{rank}(A_m) = 6$.

2.1.3 Cable-driven robots

The typical cable-driven robot contains two main functional nodes, the base at which the world frame might be placed and the end effector platform the pose of which is defined by the body frame. The end effector is attached to the base through multiple length-controlled cables. The actuators which regulate the cable length are usually located in the base. Thus, controlling the cable length of multiple actuators, we can change the end effector pose. The forward kinematics for cable-driven robots is known to be challenging to resolve [Merlet and Alexandre-dit Sandretto, 2015], while the inverse kinematics problem might be addressed in straightforward manner [Bosscher et al., 2006, Pott and Kraus, 2016]. Indeed, let us have a look in Figure 2-6. By point $A_i$ we denote the location of the $i$-th actuator that controls the length of the cable, while point $B_i$ would represent
the connection of the cable to the end effector platform. Vector \( \mathbf{a}_i \) defines the position of the point \( A_i \) in the world frame, vector \( \mathbf{b}_i \) represents the location of point \( B_i \) in the body frame. The vector \( \mathbf{u}_i \) is a unit vector directed toward the cable length vector \( \mathbf{l}_i \). Thus, vector \( \mathbf{u}_i \) is aligned with the cable tension force.

Based on the Figure 2-6, the vector closure loop can be written as follows:

\[
\mathbf{l}_i = \mathbf{a}_i - \mathbf{p}_b^w - \mathbf{R}_b^w \mathbf{b}_i.
\]

Figure 2-6: Kinematics of the cable-driven robot.

### 2.1.4 Luca Dynamics library

The Luca Dynamics is a powerful kinematics and dynamics library that has been created at RMC [Garofalo et al., 2013]. It allows calculating the quantities used in rigid body robotics based on the Universal Robotic Description Format (URDF). Namely, based on the model and updated full state of a rigid body system, it provides the following outputs: inertia tensor, gravity vector, Coriolis/centrifugal terms, the position of COM, homogeneous transformation, centroidal momentum matrix, and Jacobian transformation for the point of interest.

### 2.2 Control of physical systems

A control of the dynamic system entails finding actuator efforts in order to provide a desired motion. In case when we command the desired joint motion, we deal with joint space control. The system dynamics for the serial chain and control input are related as follows:

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau + \tau_{\text{ext}}.
\]
In contrast, if we formulate the control law in the task space, then we deal with a task space control. By utilizing (2.3), we can write down the following:

\[
\dot{q} = J(q)^{-1} \dot{x}(q), \\
\ddot{q} = J(q)^{-1} \ddot{x}(q) + \dot{J}(q)^{-1} \dot{x}(q).
\]

Then, system dynamics and control law in the task space can be rewritten as:

\[
\begin{align*}
\bar{M}(q) & \ddot{x} + \bar{C}(q, \dot{q}) \dot{x} + \bar{g}(q) = F + \bar{\tau}_{ext} \\
J^{-T}M J^{-1} & J^{-T}MJ^{-1} \dot{\dot{x}} + J^{-T}MJ^{-1}J^{-T}g = J^{-T}\tau + J^{-T}\tau_{ext}.
\end{align*}
\]

Here, \(\bar{M}(q), \bar{C}(q, \dot{q}),\) and \(\bar{g}(q)\) are the effective parameters of the system dynamics as viewed from the task space.

Besides, it is worth mentioning that when we control each robotic subsystem (e.g., joint) independently, we utilize the so-called decentralized control approach. On the other hand, if we take into account the interaction between robotic subsystems, we work with centralized control strategy.

### 2.2.1 Compliant motion control

In the scope of this thesis, the compliant motion control is widely utilized. It involves methods for the interaction of the robot with the environment. This type of control includes two basic strategies: impedance control and admittance control [Hogan, 1985, Ott et al., 2010].

The impedance control treats the robotic system as admittance while the controller itself is considered as mechanical impedance, see Figure 2-7a. It can be defined as follows:

\[
F_{des} = C_{des}(\ddot{x}_{des} - \ddot{x}) + K_{des}(\dot{x}_{des} - \dot{x})
\]

Here, \(C_{des}\) and \(K_{des}\) describe desired contact damping and stiffness matrices, respectively, \(\dot{x}_{des}\) and \(\ddot{x}_{des}\) are the desired location and velocity of the system. As we can see, the presented controller for regulation task has a proportional-derivative structure. This is a special case of the impedance control called compliance control. In case of tracking problem, such a controller might be extended with feedforward term proportional to the desired acceleration \(\dddot{x}_{des}\) through reflected inertia. The controller outputs the desired wrench, which is kinematically mapped through Ja-
2.2. Control of physical systems

Impedance control

Torque control

Robotic system

Mechanical admittance

Mechanical impedance

+ -

(a) Cartesian impedance control

Inverse kinematics

Admittance control

Position control

Robotic system

F_{FS}

(b) Cartesian admittance control

Figure 2-7: Cartesian impedance and admittance control loops for the task space.

cobian $J(q)^T$ to the joint torques, which are directly applied to the robotic system torque controller.

In case, if we deal with position-controlled system, the admittance control technique can be utilized. In opposite to the impedance control, the robotic system is considered as mechanical impedance while admittance controller represents the mechanical admittance. It can be described as:

$$M_{adm} \ddot{x}_{s,p}^{des} + D_{adm} \dot{x}_{s,p}^{des} = F_{des} - F_{FTS}.$$ 

As shown in Figure 2-7b, the admittance controller takes as input the difference between desired force we want to generate and external force, e.g., measured by Force Torque Sensor (FTS), and outputs the desired pose in the task space. Further, the pose is transformed to the joint angles by IK to receive joint corrections $q_{cmd}$. These are fed to the position controller of the robotic system, which generates the torques $\tau_m$ applied to the robot joints. It is worth highlighting that the admittance controller allows tuning dynamics with desired inertia $M_{adm}$ and damping $D_{adm}$ matrices.
"Above all, don’t fear difficult moments. The best comes from them."

Rita Levi-Montalcini

Chapter 3

Physical system description

This chapter elaborates the cable-suspended aerial manipulator concept into more details and presents the particular demonstrator, cable-Suspended Aerial Manipulator (SAM), which is utilized for the experimental investigation in the scope of this thesis.

The chapter is organized as follows. First, an overview of the SAM is given. Secondly, the operational scenario for the cable-suspended aerial manipulator is discussed. Lastly, the main design features and frameworks are described for the SAM platform.

3.1 The cable-Suspended Aerial Manipulator (SAM)

In the chapters devoted to the experimental validation and investigation (chapter 6 and chapter 7), the cable-Suspended Aerial Manipulator (SAM) system is utilized. The SAM belongs to the cable-decoupled long-reach aerial manipulation type and serves for the conduction of safe aerial manipulation in a complex industrial environment, see detailed concept description and advantages in subsection 1.2.3.

The SAM is equipped with rate-controlled propeller-based actuation and position-controlled winch-based actuation in order to cope with any type of disturbances imposed on the aerial manipulator. During operation, the system is suspended by a long cable to the *external carrier* which brings the SAM close to the target point. Attached to the bottom of the platform, the redundant 7-DOF robotic arm KUKA-LWR4 performs the manipulation tasks while both actuation systems keep the system close to the motionless state by compensating for the undesired platform motion in a coordinated way. The overall integration concept of the manipulation
3.1. The cable-Suspended Aerial Manipulator (SAM)

(a) The SAM hanged at the crane’s hook

(b) The SAM composition

Figure 3-1: The SAM platform.

platform with external carrier and detailed design of the SAM system is shown in Figure 3-1. The presented system design is envisioned for the outdoor experimental scenarios in rough conditions in order to conduct industrial applications, e.g., the industrial inspection and maintenance of the cell towers, transmission towers, refinery pipelines, bridge supports, offshore drilling rigs.

The propeller-based actuation contains eight propellers, each of which is driven by BrushLess Direct Current motor (BLDC)\(^1\), and serves for the compensation of the dynamic disturbances by exerting agile omnidirectional wrench at the platform’s geometric center [Franchi et al., 2018]. Such a wrench can be realized by installing eight propellers in the special optimized configuration in order to generate a set of independent forces and torques. Obtained well-balanced force and torques allow to decouple the control of the position and the orientation.

The winch-based actuation consists of three servos, each of which can change the rigging cable length that connects the platform to the suspension cable point. Intuitively, when the length of the cables is equal, the SAM can be seen as equilateral triangular pyramid edges of which represent the cables. This actuation allows maintaining the SAM COM under the hanging point via control of horizontal displacements and the height of the platform. Due to integrated breaks, such an actuation helps to withstand quasi-static disturbances, e.g., the slow COM shift due to the motion of the robotic arm.

\(^1\)Through the thesis we also call this pair as a rotor group or propulsion unit.
Chapter 3. Physical system description

The exploitation of two actuation systems in parallel, i.e., propellers and winches, to stabilize the manipulator base allows reducing the amount of consumed energy on propeller motors since long-term continuous loads might be compensated by winches.

The SAM is suspended by means of the cable to the external carrier. The leading feature of such a configuration is that the weight of the SAM is supported by the external carrier. Since there is no need to compensate for the gravity, the required amount of thrust generated by the propeller propulsion in the manipulation platform is relatively low. As a result, the diameter of the propellers can be significantly scaled down. Having a compact size, the SAM can independently counter any disturbances and freely perform manipulation tasks in close proximity to obstacles in complex environments. Therefore, the safety of this system is higher.

It is worth mentioning that as external carrier the different types of aerial vehicles as well as a crane can serve. The latter is important for places and applications where the operation of UAVs is restricted or the permission to fly is difficult to obtain. The former can have any size rotor blades since it is distanced by cable from the operational environment.

The SAM is developed in the scope of the H2020 AEROARMS project by joint efforts of the several partners: DLR, Elektra Solar GmbH², LAAS-CNRS³, GRVC Robotics Labs⁴ of the University of Seville with contribution provided by Robo-Technology GmbH⁵ in design and manufacturing. The development process took less than 2 years, it is a very short time period for turning such a complex system from idea to working experimental setup.

3.2 Operational scenario

As shown in the schematic diagram in Figure 3-2, the aerial manipulator is hanged to the external carrier utilizing a long cable. At the beginning of the operation, the external carrier should deliver the cable-based platform to the target location. After that, the carrier central role is to maintain its own location while the system performs the manipulation task independently. Thus, the typical mission for the cable-suspended aerial manipulator is composed of two steps: transportation and manipulation.

²https://www.elektra-solar.com
³https://www.laas.fr
⁴https://grvc.us.es
⁵https://www.robo-technology.de
3.2. Operational scenario

During the first step, the SAM should follow the external carrier with minimum pendulum behavior. It achieves by exploiting the agile oscillation damping controller with the propeller-based actuation. Within the transportation, the robotic arm configuration is fixed, so winches are not utilized. The suspension cable length can be regulated depending on the operation height and type of the external carrier.

During the manipulation, the external carrier is motionless while the SAM conducts the task and should compensate for all disturbances depending on the certain task. At this step, both actuations should be activated, the propeller-based propulsion for reducing oscillation effect and winch-based actuation for compensating for the COM displacement due to robotic arm weight and external wrench due to contact with the environment.

It is worth noting that several critical operational aspects should be taken into account at the mission planning stage:

- environmental conditions might impose restrictions on the particular mission, i.e., weather (wind, rain), the proximity of obstacles, type of contact, type of disturbances the SAM should counter for safe operation,

- local aviation regulations dictate operational requirements, e.g., a type of carrier, the required length of the suspension cable, time of operation,
• emergency policy of the specific company might require additional measures such as the presence of firefighting apparatus on the site,

• level of autonomy is characterized by a certain task and defines the proportion of efforts between the manual control by the operator and autonomous control by the system.

We can distinguish two main external carrier types, see Figure 3-3. The first one is a heavy payload capable aerial vehicle, e.g., a helicopter with long rotor blades. In this case, it can operate far away from the environment obstacles while compact stabilizing platform SAM directly manipulates in close proximity to the object of manipulation. The second type is a crane. In this case, the SAM is suspended directly to the hook of the crane. The latter option is especially beneficial in terms of reliability, safety, and cost. Reliability and safety are due to the well-developed crane technological level, ease of its exploitation, and advanced operational control tools, e.g., remote joystick. Reduced cost is defined by low maintenance, deployment, exploitation expenses of the crane compared to the helicopter, no need in flight permission and piloting license since the system is considered just as a payload of the crane.

![Figure 3-3: Carrier variety for the cable-suspended aerial manipulators.](image-url)
3.2. Operational scenario

Moreover, since in the scope of this thesis we aim at the industrial applications, we consider the crane as a major external carrier leading by the following ideas:

• most of the objects for industrial inspection are built by the crane, so cranes can reach them to establish contact,

• cranes are widely accessible at the industrial spots,

• cranes have a high payload capability,

• the exploitation of the aerial manipulator should be affordable for the industry.

Let us briefly consider a crane taxonomy. All cranes might be categorized by two key principles [Ali et al., 2021, Ramli et al., 2017]. The first one is mobility, there are fixed and mobile cranes. The former is rigidly fixed to the ground and has a high payload capability, while the latter represents the crane attached to the crawler or truck vehicles. The second principle is kinematic structure. Three groups might be distinguished according to their type of motion, see Figure 3-4. Namely, a boom crane provides rotational motion of the base as well as the rotational motion of the boom, at the tip of which the payload might be hoisted up and down. It is typically utilized at the ships. A tower crane has a rotational DOF of the jib around the vertical axis at the base and translational DOF of the trolley along the jib. The payload can be moved up and down as well. Typically, the tower cranes might be found at the constructions sites. Lastly, an overhead crane (also known as a bridge crane or similar gantry crane) contains an elevated runway rails along which the bridge is traveling. The trolley with a hoist can move along the bridge. The overhead cranes are common for warehouses and factories.

It is worth noting that depending on the crane type, different requirements and a set of onboard sensors might be used for the task implementation. For example, the
overhead crane has a constant height of the trolley, so an onboard barometer at the SAM might be used in order to estimate the length of the crane’s chain for control purpose. At the same time, at the boom crane, the height of the boom’s tip varies, so the barometer would not help in estimating the crane’s chain length. Another example is conditioned by the dynamic coupling between the suspended platform and the trolley through the chain. The overhead crane has a rigid structure, so whenever the SAM oscillates, the trolley is fixed. On the other hand, at the boom crane, any oscillations of the system would result in the oscillations of the crane’s jib as well.

3.3 System design

In this subsection, we will describe the central functional nodes and present the key features of the SAM in detail.

3.3.1 Propeller-based actuation

The cable-suspended platform is equipped with 8 propeller-based rotor groups to resist disturbances induced by the pendulum nature and to stabilize its own dynamics. Below we consider the complete cycle of the propeller-based actuation development, including modeling, optimized allocation, hardware selection for specific requirements, design, and identification of the main parameters.

3.3.1.1 Propeller allocation

Let us introduce the following coordinate frames:

- world frame $\mathcal{F}_w$: $\{O_w, x_w, y_w, z_w\}$ fixed in the inertial space,
- body (platform) frame $^{b}\mathcal{F}_b$: $\{O_b, x_b, y_b, z_b\}$ attached to the Geometric Center (GC) of the SAM base with origin $O_b$,
- $i$–th actuator frame $^{a_i}\mathcal{F}_a$: $\{O_{a_i}, x_{a_i}, y_{a_i}, z_{a_i}\}$ defined in subsubsection 2.1.2.2.

The $^{a_i}\mathcal{F}_a$ can be described by three angles through three sequenced successive rotations as $^{b}_{a_i} = \mathbf{R}(\gamma_i)z\mathbf{R}(\alpha_i)x\mathbf{R}(\beta_i)y$ with respect to $^{b}\mathcal{F}_b$. Here, $\gamma_i$ defines a rotational

---

6Throughout the text we would use notation body frame whenever we deal with model containing single platform, and platform frame in cases when we deal with multi-body dynamics, i.e., platform with integrated robotic arm.
3.3. System design

(a) The SAM kinematics

(b) Orientation of the $F_b$ and $F_a$ with respect to the SAM structure

Figure 3-5: Reference frames for the SAM platform.

shift of the $i$–th frame arm in the $x_b y_b$ plane of $F_b$. The $\alpha_i$ describes the tilting angle of the $i$–th actuator spinning axis around $x_a$, and $\beta_i$ - around $y_a$. Relation between the aforementioned frames is illustrated in Figure 3-5, and convention of rotor-group installation angles $\alpha$ and $\beta$ is illustrated in Figure 2-3b. It is worth mention that all actuators lay on the same plane and equally spaced, i.e., they lay at the same circle centered in $O_b$. Such a design allows to minimize a collision risk with rigging cables and robotic arm.

Recent research results [Allenspach et al., 2020, Park et al., 2018, Ryll et al., 2016] have been shown that by installing propellers in the particular arrangement
Chapter 3. Physical system description

(non-collinear), an omnidirectional 6 DOF wrench can be realized.

**Remark.** *It would be worthwhile to mention that the minimum required number of rotor groups to realize 6 DOF wrench equals 7 [Tognon, 2018]. In the SAM, it was decided to use 8 units to lower the effort of each actuator, provide a rotor failure robustness, and address the exploitation of redundancy in the future studies.*

As was discussed in subsubsection 2.1.2.2, the body wrench \( w_b \) and the thrust input vector \( t \) can be related by the allocation matrix \( A \):

\[
\mathbf{w}_b = A t. \tag{3.1}
\]

Following the line of [Tognon and Franchi, 2018], to design the desirable arrangement for the 8 propellers at the SAM, an optimization was carried out by LAAS-CNRS researchers. In particular, given the number of propellers, their aerodynamic parameters, and positions, the cost function to minimize was chosen as the condition number of the allocation matrix in order to guarantee the equal distribution between the rotor group efforts required to generate an omnidirectional wrench. Additionally, the following constraints were imposed:

**C.1** the imposition of a particular (the unitary) eigenvector \( \mathbf{1}_{8 \times 1} \) for the allocation matrix such that \( A \mathbf{1}_{8 \times 1} = 0 \), in order to obtain a symmetrical design which maintains balanced effort distribution between all the actuators

**C.2** the normalization constraint for the unit vector defining the directions of the unidirectional thrusters

**C.3** an imposition of minimal installation angle of the rotor group \( \forall j \in \{2, 5, 7\} \) around the frame arm, \( \alpha_j \), which guarantees the lifting of the landing legs with attainable motor thrust\(^7\)

**C.4** the perpendicularity between the thrust directions and the frame arm axes for an ease of mechanical implementation, i.e., angle around the axis perpendicular to the frame arm, \( \beta_i \) is equal to 0 for \( \forall i \in \{1, \ldots, 8\} \).

The mathematical treatment of **C.1** and **C.2** can be found in [Tognon and Franchi, 2018]. The constraint **C.3** is added to provide a minimum projection of

\(^7\)Three out of eight frame arms serve as the landing gear for the SAM. More elaborated explanation of this constraint will be given in subsection 3.3.3.
3.3. System design

The motor thrust vector along the z-axis of the body frame. It guarantees the lifting of the landing leg by utilized rotor group for selected installation configuration. This constraint can be mathematically expressed as $|\mathbf{z}^T_{b} \mathbf{t}_j| \geq \cos(\alpha^\text{lim}_j)$, where $\mathbf{t}_j$ is the unit vector indicating the direction of the rotor group thrust expressed in the body frame and related to the landing $j$–th leg. $\mathbf{z}_B$ is the z-axis of the body frame, and $\alpha^\text{lim}_j = \frac{\pi}{3}$ is the limitation angle selected to allow the lifting of a leg given the maximum thrust attainable by the selected actuator in the worst configuration, the more details on that can be found in subsection 3.3.3. Such a constraint prevents allocation of the thrust vectors in the red area for the legs, see Figure 3-6a, and is needed to ensure the lifting of the landing gear legs after the take-off.

The constraint C.4, see Figure 3-6b, ensures $\beta_i = 0$ and can be written as $|\mathbf{p}_{ai}^b \mathbf{t}_i| = 0$, where $\mathbf{p}_{ai}^b$ is the vector of the $i$–th frame arm expressed in the body frame. 

![Figure 3-6: Constrains in the propeller allocation.](image)

![Figure 3-7: Optimized omnidirectional design for the propeller-based actuation of the SAM. The blue spheres represent the placement of the BLDC motors. The colored lines point to the thrust direction of each motor. The star symbol denotes the counterclockwise propeller, and the square indicates the clockwise propeller.](image)
Table 3.1: Configuration parameters of the propeller-based actuation frame. Such a constraint limits the mechanical complexity of the structure and also helps to avoid possible collision of the propellers with the frame arms.

The obtained design for the rotor group placement is depicted in Figure 3-7. The result of optimization, with respect to the angle definition is presented in Table 3.1.

With the obtained propulsion configuration, the octorotor platform is capable of generating a set of independent forces and torques, thus allowing to decouple the control of the position and the orientation. The allocation matrix\(^8\) can be defined as follows:

\[
A = \begin{bmatrix}
J_1 & J_2 & \ldots & J_8
\end{bmatrix} = \begin{bmatrix}
A_{1}^{3\times8} \\
A_{2}^{3\times8}
\end{bmatrix} =
\begin{bmatrix}
t_1 \\
p_{a1}^b \times t_1 + k_{ad} t_1 \\
p_{a2}^b \times t_2 + k_{ad} t_2 \\
p_{a3}^b \times t_3 + k_{ad} t_3 \\
\ldots \\
p_{a8}^b \times t_8 + k_{ad} t_8
\end{bmatrix},
\]

where \(J_i = \begin{bmatrix} t_i \\ p_{a_i}^b \times t_i + k t_i \end{bmatrix}\) and the aerodynamic gain \(k_{ad} = (-1)^{t+1} q^{t}\) is the constant ratio introduced in (2.7). The sign of the \(k_{ad}\) is positive (negative) when \(i\)-th propeller angular velocity has the same direction of \(-t_i (t_i)\), i.e., propeller spins CW.

---

\(^8\)It can be seen from the structure that the allocation matrix represents a geometric Jacobian.
3.3. System design

(CCW) seen from its tip. It results in the various drag torque directions as well.

The vector distance to the propeller hub from the $O_b^i$ is defined as follows:

$$\mathbf{p}_{a_i} = \mathbf{R} (\gamma_i) \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix},$$

where $L$ is the length of each frame arm and $\gamma_i = \frac{2\pi i}{8}$ with respect to the $x_b$ in CCW direction.

The unit thrust for $i$–th propeller in $\mathcal{F}_b$ can be defined as:

$$\mathbf{t}_i = \mathbf{R}_a \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Thus, each unit thrust generates three entities in $\mathcal{F}_b$. The first one is the force applied to the $O_b$, the second one is the torque applied around $O_b$ at the leverage $\mathbf{p}_{a_i}$, the last one is the aerodynamic drag produced by the propeller and proportional to the thrust force.

With resulted optimization angles and for $k_{ad} = (-1)^{i+1} \frac{0.2 N \cdot m}{10 N} = (-1)^i 0.02^9$, the final allocation matrix is defined as:

$$\mathbf{A} = \begin{bmatrix}
0.57 & -0.81 & -0.57 & 0.00 & -0.57 & 0.81 & 0.57 & 0.00 \\
-0.57 & 0.00 & -0.57 & 0.81 & 0.57 & 0.00 & 0.57 & -0.81 \\
0.60 & 0.50 & -0.60 & -0.59 & 0.60 & 0.59 & -0.60 & -0.59 \\
0.33 & 0.46 & -0.33 & 0.00 & -0.33 & -0.46 & 0.33 & 0.00 \\
-0.33 & 0.00 & -0.33 & -0.46 & 0.33 & 0.00 & 0.33 & 0.46 \\
-0.59 & 0.60 & 0.59 & -0.50 & -0.59 & 0.60 & 0.59 & -0.60
\end{bmatrix},$$

it corresponds to the optimized condition number of 1.8230.

3.3.1.2 Quantitative requirements

The remaining design parameter, which is not considered in the optimization problem, is the required thrust value per motor. Dual actuation provides a possibility to share stabilization efforts between winch-based and propeller-based actuations. We

\[9\text{The values are taken from the rotor group identification results for the input width of the ESC pulse of 1563 } \mu s \text{ as shown in Figure 3-13a.}\]
assume that slow disturbances should be suppressed by the winches while disturbing
dynamic transients should be counterbalanced by agile propellers. Thus, we need to
obtain a rough estimation of typical disturbances during manipulation in order to
understand the required magnitude for the actuator thrusts. To this end, we con-
ducted a simulational study at which the SAM was modeled as a double pendulum\textsuperscript{10}
without winches with zero-length of the second pendulum link. Initial conditions
of the pendulum joints had zero values, while the robotic arm started the motion
from the parking pose. The simulation was performed in the MathWorks MATLAB
Simulink (MATLAB Simulink) utilizing Luca Dynamics library (subsection 2.1.4).

During the simulation, the robotic arm moved through several configurations
typical for the manipulation tasks, see Figure 3-8a. Namely, the following sequence
of the robotic arm configurations $q_m$ was performed with 10 second step:

- parking: $q_m = [-\frac{\pi}{6.2}, -\frac{\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{2} + 0.343, 0, \frac{\pi}{2}, 0]^T$,
3.3. System design

- operational: \( q_m = [0, -\frac{\pi}{2.8}, 0, -\frac{\pi}{2.8}, 0, 0, 0]^T \),
- pick and place: \( q_m = [0, -\frac{\pi}{4}, 0, -\frac{\pi}{4}, 0, 0, 0]^T \),
- pick and place V2: \( q_m = [0, -\frac{\pi}{4}, 0, -\frac{\pi}{2}, 0, -\frac{\pi}{4}, 0]^T \),
- stretched: \( q_m = [0, -\frac{\pi}{4.5}, 0, \frac{\pi}{4}, 0, 0, 0]^T \),
- stretched V2: \( q_m = [\pi, -\frac{\pi}{4.5}, 0, \frac{\pi}{4}, 0, 0, 0]^T \),
- parking: \( q_m = [-\frac{\pi}{6.2}, -\frac{\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{2} + 0.343, 0, \frac{\pi}{2}, 0]^T \).

The wrench \( w_b \), required for the stabilization of the platform during arm motion, see Figure 3-9, was mapped to the thrust \( t \) space, see Figure 3-8b. As it can be seen, the thrust values reach pretty high values, e.g., 150 N. It is evident that the winch compensation at the static levels is a significant simplification for the real system, and the required thrust per actuator can be defined by transient magnitude between configurations, i.e., typically is less than 30 N per motor. For the stretched v2 pose, when manipulation is performed on the side of the SAM, the most significant jump for motor 4 reaches 96 N.

It is worth noting that the direct solution of (3.1) using Moore-Penrose pseudo inverse \( t^* = A^\dagger w_b \) would always contain at least one negative term, which cannot be exerted by unidirectional thruster [Hamandi et al., 2020]. To this end, the final thrust values contain additional term needed to meet the unidirectionality requirement and is defined as follows:

**Mapper 1:**

\[
t = A^\dagger w_b + \lambda^b b
\]

Here, \( ||b|| = 1 \) is the unit eigenvector of matrix \( A \) in its null-space, i.e., \( b \in N(A) \), \( \lambda^b = min(t^*) \in \mathbb{R}^+ \) is the smallest scalar to meet the constraint inequality \( t \geq t_{min} \mathbb{1}_{8x1} \), \( t_{min} \) is the minimum possible thrust, and \( \mathbb{1}_{8x1} \) is the vector of once with dimension 8.

As a result, the continuous thrust value per motor required for most operations is around 30 N, while the maximum value might reach 100 N. Selecting a maximum thrust value of 150 N for each motor guarantees some margin toward additional payload, disturbances, and insufficient winch performance.
Figure 3-9: Simulation study conditions.

3.3.1.3 Hardware

To provide the estimated thrust value, the BLDC Kontronik Pyro 650-65 [SOB, 2017] in pair with Aeronaut CAM carbon light-prop 16x6” (40.5 x15) propeller and Electronic Speed Controller (ESC) JIVE PRO 120+ HV [SOB, 2017] is chosen for the
3.3. System design

3.3.1.4 Identification

The rotor group parameter identification is performed at the specially designed testbench. The testbench contains the measurement unit installed at the long isolation pillar placed at the stand, see Figure 3-11a. The length of the isolation pillar is about 1.5 m, which is higher than 3 propeller diameters. Such a condition is required to avoid a ground effect [Zerihan and Zhang, 2000]. As shown in Figure 3-11b, the measurement unit includes the non-contact digital tachometer to measure angular velocity by registering white tape mark at the BLDC, 6-DOF FTS at which

---

\[11\] It is worth noting that common range of the pulse width pulse is of \([1000, 2000]\) \(\mu s\). However, due to additional use of the propeller-based actuation for the landing gear, the ESC are programmed to operate in the bidirectional mode, so the range of \([1500, 2000]\) corresponds to the direct rotation and the range of \([1000, 1500]\) to the reverse rotation.
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated maximum thrust</td>
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<td>N</td>
</tr>
<tr>
<td>Input voltage</td>
<td>16-50</td>
<td>V</td>
</tr>
<tr>
<td>Motor velocity constant</td>
<td>650</td>
<td>RPM/V</td>
</tr>
<tr>
<td>Number of poles</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>Maximum motor power</td>
<td>3000</td>
<td>W</td>
</tr>
<tr>
<td>ESC continuous current</td>
<td>120</td>
<td>A</td>
</tr>
<tr>
<td>Motor weight</td>
<td>295</td>
<td>g</td>
</tr>
<tr>
<td>ESC weight</td>
<td>140</td>
<td>g</td>
</tr>
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<td>$k_t$</td>
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<td>-</td>
</tr>
<tr>
<td>$b_t$</td>
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<td>-</td>
</tr>
<tr>
<td>$k_q$</td>
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<td>-</td>
</tr>
<tr>
<td>$b_q$</td>
<td>-12.4640</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.2: Technical characteristics of the propeller-based actuation.

(a) The composition of the test-bench

(b) The scaled view of the measurement unit of the test-bench

Figure 3-11: Testbench for the rotor group parameter identification.
the BLDC is installed to measure thrust force and drag torque, and additionally a current is roughly estimated by virtue of current clamp (not shown in the figure).

As a result, the thrust force, drag torque, angular velocity, and current data was collected for the operational range $[1518 - 1635] \mu s$ of PWM signal, see Figure 3-12. In this experiment, selected hardware could exert continuous 41 N thrust at 27% throttle level. At the same time, the drag torque did not exceed even 1 Nm. Angular

![Figure 3-12: Collected data at the testbench for the rotor group identification.](image)
velocity and current demonstrated quadratic behavior with respect to a width pulse.

After processing the raw data, see Figure 3-13, we could define the mapping between the thrust value as well as drag torque of the $i-$th rotor group and PWM control signal as follows:

\begin{align*}
t_i &= k_t \cdot \text{PWM} + b_t, \\
q_i &= k_q \cdot \text{PWM} + b_q,
\end{align*}

with the linear gains $k_t, b_t, k_q, b_q$, which can be found in Table 3.2. Thus, PWM command is proportional to exerted thrust and drag torque as well as to the square of the angular velocity of the BLDC.

As it can be seen, at the low PWM commands, the behavior is different from linear (outliers in Figure 3-13a), so we defined two complementary regions in addition to the operational one. The first one is the dead zone (blue piece) at which the motor does not spin at all (blue line) due to zero back electromotive force (BEMF), and the second one is the nonlinear piece (yellow line) at which the motor starts to spin with low BEMF, i.e., a transition between dead zone and operational range (green piece), see Figure 3-13b. During this transition, the minimum angular speed is generated in order to measure BEMF required for the rotor position estimation [Damodharan and Vasudevan, 2010]. As soon as the measurement of BEMF is reliable (when angular velocity exceeds a certain low threshold), the characteristic behaves linearly. Due to strong linear dependency, we plotted estimated thrust-PWM mapping for the whole PWM range as well.

Using the received characteristics, we can formulate the Mapper 2, which relates thrust vector and PWM control signal of the rotor group as follows:

\begin{align*}
\text{Mapper 2:} \\
\text{PWM} &= \frac{t - b_t}{k_t}
\end{align*}

It is worth mentioning that there is no built-in voltage regulator in ESC, so changing battery voltage level over time produces varying thrust output for the similar PWM command. Moreover, motor characteristics including BEMF coefficient might change. We will neglect this issues since all motors are powered by the same battery, so we assume that reducing in the thrust will be the same for all motors simultaneously. In general, such problems may be overcome by a feedback voltage regulator at the ESC [Franchi and Mallet, 2017].
3.3. System design

(a) Linear regression model of the thrust (left) and drag (right) with respect to the PWM signal for the operational range.

(b) Regression model of the thrust with respect to the PWM signal based on the data collected for the operational (left) and the whole (right) ranges.

Figure 3-13: Processed data for the rotor group identification.
3.3.1.5 Force and torque envelopes

To evaluate the reachable dynamic capabilities of the designed actuation system, it is worth analyzing the allocation matrix and corresponding wrench in $F_b$. Since the wrench is a six-dimensional vector, we utilize two projections of the wrench on three-dimensional spaces [Hamandi et al., 2020]. Namely, we analyze the set of body-frame admissible forces with zero torque and the set of admissible torques with zero force. In this analysis, we impose the saturation limit of 150 N on the identified propeller thrust.

Let us introduce the following space of positive thrust vectors $\lambda$:

$$\Lambda = \{ \lambda \in \mathbb{R}^8 \mid \forall \lambda_i \in \{0, 150\}, \ i = 1, \ldots, 8 \} .$$

By utilizing the definition of allocation matrix (3.1), we can calculate the envelopes of total force and total torque volumes corresponding to the whole space of $\lambda$ as follows:

$$w_b^{\text{total}} = \begin{bmatrix} f_b^{\text{total}} \\ \tau_b^{\text{total}} \end{bmatrix} = A\lambda = a_1J_1\lambda_1^{\max} + \cdots + a_8J_8\lambda_8^{\max} , \quad (3.5)$$

where $\lambda_i^{\max} = 150$, gain $a_i$ is defined as $\forall a_i \in \{0, 1\}$ to realize all extreme combinations of the thrust vector, $w_b^{\text{total}}$ is the six-dimensional shell of possible body-frame forces and torques. It is worth noting here that in contrast to cable-suspended systems, for common UAV the (3.5) should also contain the gravity force.

The total force and total torque represents the coupled dynamic capabilities. Due to omnidirectionallity, the platform is able to operate in decoupled manner. To this end, we aim to evaluate the set of admissible forces with zero torque and admissible torques with zero forces. First, let us introduce the following two spaces:

$$\mathcal{X} = \{ \ x \in \mathbb{R}^8 \mid x \in \mathcal{N}(A_2^{3\times 8}) \land 0 \leq x_i \leq 150, \ i = 1, \ldots, 8 \} ,$$

$$\mathcal{Y} = \{ \ y \in \mathbb{R}^8 \mid y \in \mathcal{N}(A_1^{3\times 8}) \land 0 \leq y_i \leq 150, \ i = 1, \ldots, 8 \} ,$$

where $\mathcal{X} \subseteq \Lambda$, $\mathcal{Y} \subseteq \Lambda$.

The space $\mathcal{X}$ (or $\mathcal{Y}$) contains all thrust vectors $x$ (or $y$), which result in the zero torque (or force) of the generated wrench, i.e., it is placed in the null-space of the $A_2$ (or $A_1$). In order to select only $\mathcal{X}$ space among the whole $\Lambda$ space, the following
3.3. System design

(a) Set of admissible forces with zero torque

(b) Set of admissible torques with zero force

Figure 3-14: Admissible forces and torques.

minimization problem should be resolved:

\[
\begin{align*}
\minimize_x & \quad \|x - \lambda\|^2 \\
\text{subject to} & \quad A_2 x = 0, \\
& \quad 0 \leq x_i \leq 150, \ i = 1, \ldots, 8.
\end{align*}
\]

Thus, \( x \in \mathbb{R}^8 \) is the vector of propeller thrusts which generate zero torque in the \( \mathcal{F}_b \). Thus, the vector of resulted thrusts is located in the null space of the matrix \( A_2 \), i.e. \( x \in \mathcal{N}(A_2) \), which corresponds to the first constraint. The second constraint imposes the lower and upper bounds on the thrust vector elements related to the physical limits of the rotor group. The main optimization condition implies minimization of a difference between total and admissible shell. The same approach might be applied to calculate the set of \( y \) thrust vectors. Figure 3-14 displays the resulted total envelopes and admissible sets.
Then, the resulted sets of admissible forces and admissible torques can be calculated using any solver that can find a minimum of constrained multivariable function\(^\text{12}\) as:

\[
\begin{align*}
w_{b}^{adm,f} &= \begin{bmatrix} f_{b}^{adm} \\ 0 \end{bmatrix} = Ax, \\
w_{b}^{adm,\tau} &= \begin{bmatrix} 0 \\ \tau_{b}^{adm} \end{bmatrix} = Ay.
\end{align*}
\]

It is interesting to highlight the high control authority around the yaw axis, which is useful since this DOF cannot be controlled by winch actuation and critical to maintain during the manipulation. Moreover, the force along the \(z_{b}\) axis is not required at first look since an external carrier performs the gravity compensation. However, any motion of the arm will result in the displacement of the system COM with respect to the platform GC. In this case, in order to generate the torque around horizontal axes through the new COM location, the force along \(z_{b}\) is required.

### 3.3.1.6 Frequency analysis

When the propellers start to spin, the IMU measurements become noisy. The frequency of the IMU noise is related to the propeller rotational speed. Thus, the noise is induced by propeller vibrations. They might have different sources, e.g., rotor and propeller imbalance or slight speed difference of several rotor groups. For a deeper investigation, 8 rotor groups were activated with 10 \% of the maximum achievable thrust, while the SAM was placed on the table and pressed with additional weights to prevent movements. The IMU-provided angular velocity in the body frame, i.e., \(\omega_{b} = [p, q, r]^T\), was collected and processed through fast Fourier transform (FFT), see Figure 3-15. It can be seen that when the motors are activated, the high-frequency noise with a frequency of 39.2 Hz appears. Since a periodic sources (8 rotor groups) produce the noise containing dominant harmonic component, the higher rotor group’s rpm the higher noise frequency. Thus, vibrations induced by rotor groups in a range higher than in the considered experiment would have even higher frequency (it is worth mentioning that intensity might vary).

In general, adaptive notch filtering could be applied to reject noise spikes in the IMU measurement for the commanded rpm.

\(^\text{12}\)In particular, the function \textit{fmincon} in MATLAB environment was utilized.
3.3. System design

3.3.2 Redundant robotic arm

In order to perform various manipulation tasks, a 7-DoF KUKA LWR IV\textsuperscript{13} [Albu-Schäffer et al., 2007] is mounted on the bottom side of the platform, see Figure 3-16.

Figure 3-16: The KUKA LWR IV.

\textsuperscript{13}https://www.kuka.com

---

\[\text{(a) The SAM pressed at the table (b) Single-sided amplitude spectrum of the IMU angular velocities. The } A(.) \text{ stands for Amplitude}\
\]

\[\text{Figure 3-15: Frequency analysis of the rotor group.}\]
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The arm contains $7R$ joints and has the total length of 1 meter. Two main postures of the robotic arm should be defined (Figure 3-17):

- operational with the following joint angle values:
  \[ \mathbf{q}_m = \begin{bmatrix} 0, -\frac{\pi}{2.8}, 0, -\frac{\pi}{2.8}, 0, 0, 0 \end{bmatrix}^T, \]

- parking with the following joint angle values:
  \[ \mathbf{q}_m = \begin{bmatrix} -\frac{\pi}{6.2}, -\frac{\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{2} + 0.343, 0, \frac{\pi}{2}, 0 \end{bmatrix}^T. \]

During the transportation and landing, the manipulator should be in the parking position in order to avoid collision between the robotic manipulator and the ground. The main advantage of this position is its compactness. Before initiating any manipulation task such as pick and place or peg-in-hole, the manipulator should be placed in the operational configuration as fewer movements are required to perform tasks from this position.

Additionally, it is worth mentioning that the manipulator mounting point is shifted from the center of the platform on 0.24 m. It allows to define the parking and operational configurations so that the resulting COM of the whole system is the same for both cases (in the horizontal plane) and is located under the suspension point, see the COM position in Figure 3-17. Therefore, the COM of the whole system for both configurations evolves close to the same point, minimizing required actuation during the transition.
3.3. System design

3.3.3 Foldable landing gear

In order to reduce the total weight and accommodate the large manipulation workspace, the traditional landing gear is not installed in the cable-suspended platform. To land, three out of eight frame propeller arms can be folded and converted to the frame legs of the landing gear, see Figure 3-17. Thus, the legs have a dual-use: landing legs and propeller arms. In order to switch from the landing leg to the propeller arm, each leg has to be risen from the lower to the upper position. In this transition, the leg rotates in the bearing through about 60 degrees. To lift up the leg, the thrust force of the rotor group is used. In order to compensate for a non-zero wrench caused by mentioned thrust during this procedure, the remaining five propellers are utilized. The movement downwards (transition from the frame arm to the landing leg) is done by the gravity force, damped by the thrust force.

3.3.3.1 Hardware

To fix the position of the leg/arm in the bottom and top points, the locker mechanism is used, see Figure 3-18. As it can be seen, the locker mechanism contains two fixed plastic grooves and a servo motor (Futaba S3152). The grooves are manufactured from "S" green plastic material with low friction, long lifetime, and high wear resistance. In order to lock the position, the servo motor should turn a bar installed at the shaft. A ground reaction force during landing is pointed toward the rotational bearing to avoid load on servo motor shafts. At the tip of each leg, the landing pad is installed to dampen the structure during the hard landing.
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3.3.3.2 Quantitative requirements

The main requirement for the landing gear is defined by the lift force generated by propeller vertical thrust projection. The length of the arm till the motor hub, i.e., the leverage at which thrust is applied is 0.75 m, while the estimated weight of the leg (aluminum tube, rotor group, holder, landing pad, and servomotor) which has to be lifted is 1.55 kg.

3.3.3.3 Validation

Based on the allocation matrix optimization results, the rotor group at one out of three frame legs is supposed to spin during the lifting in the opposite direction with respect to the designed one in regular operation. In such a scenario, the thrust coefficient is much lower than the identified one for the proper spinning direction in (3.3). To this end, the constraint angle of $\alpha^\text{lim} = 60^\circ$ w. r. t. the $z_b$ axis was found for the particular hardware selection and added as a constraint C.3 to the optimization problem for allocation matrix calculation.

Validation of the leg lifting was performed experimentally for the frame arm with the worst configuration resulted from the optimization in terms of the thrust generation. Figure 3-19 demonstrates the validation of the final configured propulsion unit that is tilted around alpha for 120 degrees, i.e., 30 degrees with respect to the horizontal plane, and spins in the opposite direction. As a result, the selected hardware completely satisfies the task requirements.

3.3.4 Winch-based actuation

The winch-based actuation, see Figure 3-20, allows controlling the length of three rigging cables that suspend the SAM platform to the crane’s hook in order to maintain the SAM COM location below the suspension point. With such an actuation,
3.3. System design

Figure 3-20: Winch-based actuation integrated into the SAM platform.

we can affect 3 out of 5 DOFs: platform roll, pitch, and COM displacement. Control of the height using winches allows us to completely exclude external carrier from the aerial manipulation.

3.3.4.1 Kinematics

Three rigging cables cross the platform plane \( x_b - y_b \) at points denoted by \( W_i \) for \( i \)-th winch. All three points are located at the circle with radius \( \| a_i \| = 415.7 \text{ mm} \) and center at the \( O_b \). Here, \( a_i \) is the vector pointing to the \( W_i \) in the body frame. All vectors are displaced with 120° shift around \( z_b \) axis, see Figure 3-21. The \( a_3 \) is turned with respect to the axis \( x_b \) at angle \( \gamma' = 52.49^\circ \).

3.3.4.2 Hardware

Each winch, see Figure 3-22a, is based on a motor assembly that contains the Maxon RE50 DC motor [Max, 2021], a planetary gearhead GP 52 C with reduction ratio 343/8 [Max, 2021], a 3 channel high precision encoder HEDL 9140 with 500 counts per turn [Max, 2021], and a static brake AB 44 [Max, 2020]. The regulation of each assembly is performed by controller EPOS4 compact 50/5 CAN with integrated motion controller based on the encoder feedback [Max, 2021].

The motor assemblies are installed inside of the platform, see Figure 3-22b. The controlled steel cable is wounded on the guided spool, which is attached to the motor gearhead output shaft, see Figure 3-23. The guided spool has special holes for weight reduction and is covered with a protection cover. The length of the cable
Figure 3-21: Allocation of all actuators with respect to the body frame. The point $W_i$ denotes $i$-th winch suspension point, and Arabic numbers indicate the corresponding rotor group.

corresponding to the one rotation of the guided spool is calculated as: $\pi \cdot d_{gs} = 3.14 \cdot 60.2 = 189$ mm. It can vary from 0.5 m to 2 m starting at $W_i$ point, marked by yellow circle in Figure 3-24a. For safety reasons, we limited an operational length range in bounds of 0.5-1.3 meters. In order to keep cable length within the operational range, an optical fiber sensing system from Keyence is used. If the cable length exceeds the precalibrated limit range, the light beam of the fiber unit will be interrupted as shown in Figure 3-24b, and the motor motion will be paused.

Figure 3-22: Winch-based actuation composition (CAD model).
3.3. System design

(a) The cable is wound at the guided spool

(b) Cabling of the winch suspension inside the platform

Figure 3-23: Winch suspension cabling inside the platform.

(a) Winch suspension points $W_i$ at the SAM

(b) Interruption of the fiber unit light beam by cable

Figure 3-24: Winch-based actuation (Real system).

The list of all elements of the winch-based actuation is presented in Table 3.3.

3.3.4.3 Communication

The overall architecture diagram of the winch-based actuation is presented in Figure 3-25. Three motor assemblies are integrated into a single communication Controller Area Network bus (CANbus) composed of the can-high and can-low signals and terminated by a 120 $\Omega$ resistance on both sides. The communication is established via the higher layer CANopen protocol [CAN, 2011] with a transmission...
### Table 3.3: Technical characteristics of the winch-based actuation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor power</td>
<td>200</td>
<td>W</td>
</tr>
<tr>
<td>Nominal voltage</td>
<td>48</td>
<td>V</td>
</tr>
<tr>
<td>Nominal speed</td>
<td>4620</td>
<td>rpm</td>
</tr>
<tr>
<td>Nominal torque</td>
<td>0.420</td>
<td>Nm</td>
</tr>
<tr>
<td>Nominal current</td>
<td>4.58</td>
<td>A</td>
</tr>
<tr>
<td>Torque constant</td>
<td>93.4</td>
<td>(\text{mNm/A})</td>
</tr>
<tr>
<td>Speed constant</td>
<td>102</td>
<td>(\text{rpm/V})</td>
</tr>
<tr>
<td>Motor profile velocity</td>
<td>1500</td>
<td>rpm</td>
</tr>
<tr>
<td>Gearbox reduction ratio</td>
<td>343/8</td>
<td>-</td>
</tr>
<tr>
<td>Gearbox efficiency (\nu)</td>
<td>75</td>
<td>%</td>
</tr>
<tr>
<td>Encoder counts per turn</td>
<td>500</td>
<td>-</td>
</tr>
<tr>
<td>Encoder max speed</td>
<td>12000</td>
<td>rpm</td>
</tr>
<tr>
<td>Break torque</td>
<td>2.5</td>
<td>Nm</td>
</tr>
<tr>
<td>Spool nominal diameter</td>
<td>60.2</td>
<td>mm</td>
</tr>
<tr>
<td>Baudrate</td>
<td>1</td>
<td>(\text{Mbits/s})</td>
</tr>
<tr>
<td>Cable length range</td>
<td>[0.5-2]</td>
<td>m</td>
</tr>
</tbody>
</table>

rate of 1 Mbits/s. Based on the CANopen addressing and communication scheme, each controller in the assembly, i.e., EPOS4, has an object dictionary (OD), which contains parameters that describe the behavior of the device, e.g., motor constants, controller gains. This standard also defines two communication objects. The first one is the Service Data Object (SDO) used to configure the device. The second is the Process Data Object (PDO), which is exploited to control the motor and to read measurement data in real-time.

Onboard flight control computer (FCC) with deployed QNX Neutrino Real-time Operating System (RtOS) is utilized as a leader device. In order to establish the Leader/Followers communication between FCC and EPOS4 controllers, a unique library to manage the CANopen stack for QNX Neutrino RtOS is developed. It allows the operation of the motors in the so-called Profile Position Mode [Max, 2019] at which the desired rotational angle in increments (encoder measurement...
3.3. System design

Winch-based actuation system

Leader

Flight control computer

 Followers

Winch I

Epos4

DC-motor

Encoder

Gearhead

Brake

Spool

CAN-BUS

120 Ω

Winch II

Epos4

DC-motor

Encoder

Gearhead

Brake

Spool

CAN-HIGH

120 Ω

Winch III

Epos4

DC-motor

Encoder

Gearhead

Brake

Spool

CAN-LOW

Figure 3-25: The architecture of the winch-based actuation.

In order to provide a reliable performance, the winch-based actuation should satisfy two main requirements. The first one ensures that the minimum value of the motor output torque and speed mapped via the gearhead ratio is higher than a load torque and velocity during the operation. Since we imply slow motion for the winch actuation, we consider the quasi-static process at which the system moves slowly enough to consider it static. The second demand is to provide brake torque high enough to hold the system arbitrary configuration. It is worth mentioning that the static breaks are integrated into the winch assemblies. It means that they should be turned on only after the full stop of the winch servos.

In the static or quasi-static regime, the tension forces $N$ along the winch cables
balance the gravitational force applied to the system COM. Depending on the angle \( \kappa \) between the \( i \)-th rigging cable and the vector \( \mathbf{a}_i \) located in the platform plane \((O_{bx,by})\), the gravity compensation term of the tension force is varied. We consider the critical case defined by the system structure with \( \kappa = 25^\circ \). In other words, \( \kappa \) cannot be less than \( 25^\circ \) due to the risk of touching the equipment located at the top of the platform by rigging cable. The weight of the platform is around 55 kg, then the force balance along the vertical axis might be formulated as follows:

\[
mg = 3N \sin(\kappa).
\]

Then the tension force for the critical angle is defined as \( N(\kappa = 25^\circ) = 425.56 \text{ N} \).

The tension force is applied to the gearhead via guided spool and generate the torque \( M_{\text{load}} \):

\[
M_{\text{load}} = N \frac{d_{gs}}{2} = 12.80 \text{ Nm},
\]

where \( d_{gs} = 0.0602 \text{ m} \) is the diameter of the guided spool.

Since the torque ratio between guided spool and gearhead is one and between the gearhead and the motor is \( i_{gh} = 43 \), the torque at the motor shaft can be calculated as:

\[
M_{\text{motor}} = \frac{M_{\text{load}}}{i_{gh} \nu} = \frac{M_{\text{load}}}{43 \cdot 0.75} = 0.3983 \text{ Nm}.
\]

Let us compare this value with torque parameters of the selected hardware:

\[
M_{re50} = 0.42 > M_{\text{motor}} = 0.3983,
\]

\[
M_{ab44} = 2.5 > M_{\text{motor}} = 0.3983.
\]

In order to analyze the motor speed parameter, we assume that the cable slowly moves with linear velocity of \( v_{\text{cable}} = 10 \text{ cm sec}^{-1} \), then:

\[
n_m = i_{gh} n_{gh} = \frac{60 i_{gh} v_{\text{cable}}}{\pi d_{gh}} = 1364.8 \text{ rpm}.
\]

Here, \( g_{gh} \) is the nominal diameter of the gearhead. Estimated torque-speed motor parameters fully comply with the continuous operating range for the selected Maxon RE50 motor. Moreover, due to system symmetry, defined parameters are the same for all three winch motors.
3.3. System design

Additionally, it is worth noting that each winch actuator is also unidirectional (in terms of tension): cables can be pulled but not pushed. Since we have an under constrained system, i.e., the SAM platform has 6 DOF and only 3 cables, there is no null-space at which we can set additional control effort as we did in (3.2). So, the corresponding allocation matrix relating the positive tension forces and corresponding generated wrench will not provide arbitrary solution [Borgstrom et al., 2009]. This restriction leads to the limitation in the workspace that has to be kept in mind during the control strategy selection.

3.3.5 Mechanical structure

As seen in Figure 3-26, the SAM consists of the two plates. The winch motors are placed inside of the platform, while other electronic blocks are either attached to the top plate or installed on the special fiberglass plates connected to the installation
ears between two plates, see Figure 3-26c, the redundant robotic arm is mounted on the bottom plate. The possibility to remove the top plate provides easy and fast access to all electronic components. The rotor groups are installed on the frame arms. The orientation of each propulsion unit can be adjusted in the range of 0 to 360 degrees around $\alpha$ and $\pm 12$ degrees around $\beta$, using a clamping mechanism.

A weight reduction is one of the main challenges in the design and production of common aerial manipulators. Thus, the general mechanical frame, including arms, plates, ribs (stiffeners), and motor holders, is manufactured from insulated

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of propellers</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>Number of winches</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Diameter of placement circle for propeller hubs</td>
<td>1.5</td>
<td>m</td>
</tr>
<tr>
<td>Diameter of circle where winch cable crosses the platform plane</td>
<td>0.831</td>
<td>m</td>
</tr>
<tr>
<td>Frame arm length</td>
<td>0.765</td>
<td>m</td>
</tr>
<tr>
<td>Frame leg length</td>
<td>0.905</td>
<td>m</td>
</tr>
<tr>
<td>Robotic arm length</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td>Weight of the frame with integrated electronic components</td>
<td>35.7</td>
<td>kg</td>
</tr>
<tr>
<td>Battery pack weight</td>
<td>3.7</td>
<td>kg</td>
</tr>
<tr>
<td>Weight of the robotic arm with FTS and tool</td>
<td>16.7</td>
<td>kg</td>
</tr>
<tr>
<td>Principal components of the inertia tensor obtained from the CAD detailed model</td>
<td>diag(4.275, 4.275, 8.438)</td>
<td>kgm$^2$</td>
</tr>
<tr>
<td>Displacement of the arm installation point with respect to GC</td>
<td>0.24</td>
<td>m</td>
</tr>
<tr>
<td>Cable length range</td>
<td>[0.5-2]</td>
<td>m</td>
</tr>
</tbody>
</table>

Table 3.4: Key parameters of the SAM system.
lightweight aluminum. Applying a set of methods for the weight optimization in the mechanical structure after several iterations, we managed to achieve the total weight of the SAM with the installed manipulator, electronics, and batteries roughly 55 kg. The stiffness of the platform with conducted design optimization is high enough and is validated by finite element analysis. Namely, the stress-strain analysis was performed by engineers from the Robo-Technology GmbH for the flight mode when all actuators exert non-zero forces and for the hard landing when the system is dropped at the ground from 10 cm height.

It is worth noting that with given weight-geometric characteristics even for fully horizontally stretched robotic arm with maximum payload at the end effector, in the quasi-static condition, the COM of the SAM will be located below the mechanical frame border. Moreover, it will be mostly inside of triangle $W_1W_2W_3$, see Figure 3-21. Thus, the slacking of the rigging cables in normal and close to the extreme (but possible) conditions is unattainable for the considered system.

The list of all geometric-weigh parameters of the SAM is presented in Table 3.4

### 3.3.6 Architecture

The architecture of the SAM is presented in Figure 3-27. As it can be seen, three different computers are installed onboard: Robot Control Computer (RCC), Vision Processing Computer (VPC), and Flight Control Computer (FCC). The FCC is based on a field-programmable gate array and responsible for the control of the winch-based and propeller-based actuation systems. Also, it is directly connected to the set of sensors for estimation of the platform state during the operation: Inertial Measurement Unit (IMU) and Global Positioning System (GPS) with real-time kinematics (RTK). The VPC is connected to the two cameras and processes vision data for robot localization. It is worth noting that cameras have a dual-use. Their second purpose is to provide visual feedback to the human operator during teleoperation tasks. The main task of the RCC is to control the robotic arm. The connection between computers is established via a network switch, which is connected to the access point. Through the access point, the point-to-point communication channel between the SAM and the ground station is established. Additionally, there is a communication channel between the FCC and manual command transmitter via a radio link.

A data management between computers is organized through the Links and Nodes (LN) package. The LN is the software framework developed at DLR that
provides middleware, process management, and monitoring functionalities.

The power distribution system is mounted at the top-plate of the platform and contains three main blocks, see Figure 3-28:

- Two power bars to distribute power from battery pack to all actuators and robotic arm.
- Specially designed at DLR power distribution system to power all components with reduced input voltage.
- 12S Battery pack installed in the case that can be easily mounted and dismounted.

The battery pack can provide more than 250 A output current with nominal voltage of 48 V. Different electronic components onboard require 5 V, 12 V, and 24
3.3. System design

V as an input, and the total current consumption of the cable-suspended platform is around 120 A during the common manipulation task. Thus, to power the whole system, 48 V with 120 A input is enough in terms of power consumption. As a battery pack, SLS 12S 21000 mAh is used. Then, selected battery pack can maintain the SAM during the time period of:

\[ \frac{V \cdot \text{Capacity}}{P_{\text{consumption}}} = \frac{12 \cdot 3.7 \cdot 21 \cdot 60}{48 \cdot 120} = 9.71 \text{ min.} \]

The complete list of the SAM components is presented in Table 3.5.

3.3.7 Control framework

Taking into account aforedescribed design process, a special control framework is deployed onboard, see Figure 3-29. It includes two main blocks. The first one is a control block which represents the black box for now. It contains the control algorithms that should generate control signals for system actuation to fulfill specific tasks. This block will be developed and investigated in the following chapters. The second block is the actuation-perception block. It includes actuators and sensors that will be utilized in the scope of this thesis for proposed control algorithms: propeller-based actuation, winch-based actuation, robotic arm, and IMU.

Since actuators exploit different control interfaces, the control signals should be appropriately transformed before being applied. Thus, two above developed mappers, (3.2) and (3.4), are utilized for propeller-based actuation in order to calculate control PWM signal from the desired body wrench. The robotic arm and winch servos are fed with inputs corresponding to their control interfaces, i.e, required
### Table 3.5: List of the SAM components and their functional roles.

<table>
<thead>
<tr>
<th>Functional role</th>
<th>Item</th>
<th>Qty</th>
<th>Current [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>48 V consumers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Robotic arm</td>
<td>KUKA LWR IV</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Rotor group</td>
<td>Kontronik-based</td>
<td>8</td>
<td>up to 100 per group</td>
</tr>
<tr>
<td>Winch assemblies</td>
<td>Maxon-based</td>
<td>3</td>
<td>up to 5 per group</td>
</tr>
<tr>
<td><strong>24 V consumers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Winch break</td>
<td>Maxon-based</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>Stereo camera</td>
<td>Roboception RC Visard 65</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Access point</td>
<td>Ubiquiti Bullet M5</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>Red button</td>
<td>custom-built</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>12 V consumers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Robot Control Computer (RCC)</td>
<td>Kontron KTH81 Flex</td>
<td>1</td>
<td>1.5-3.3</td>
</tr>
<tr>
<td>Flight Control Computer (FCC)</td>
<td>Elektra Solar-based</td>
<td>1</td>
<td>0.6</td>
</tr>
<tr>
<td>Vision Processing Computer (VPC)</td>
<td>NVIDIA Jetson series</td>
<td>1</td>
<td>4-8.5</td>
</tr>
<tr>
<td>Network switch</td>
<td>Netgear GS105</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>Lidar</td>
<td>Velodyne series</td>
<td>1</td>
<td>up to 1</td>
</tr>
<tr>
<td><strong>5 V consumers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force Torque Sensor (FTS)</td>
<td>DLR custom-built</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Leg locker servo</td>
<td>Futaba S3152</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td><strong>Others</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8-channel S.BUS receiver 2.4 Ghz</td>
<td>Futaba R6308SBT</td>
<td>1</td>
<td>FCC-powered</td>
</tr>
<tr>
<td>IMU</td>
<td>Xsens MTi 100-Series</td>
<td>1</td>
<td>FCC-powered</td>
</tr>
<tr>
<td>GPS module</td>
<td>-</td>
<td>1</td>
<td>FCC-powered</td>
</tr>
<tr>
<td>Monocular camera</td>
<td>Allied Prosilica GC1600H</td>
<td>1</td>
<td>VPC-powered</td>
</tr>
<tr>
<td>Power distribution system</td>
<td>DLR custom-built</td>
<td>1</td>
<td>passive</td>
</tr>
<tr>
<td>Remote transmitter</td>
<td>Futaba FX-30 2.4GHz</td>
<td>1</td>
<td>External power</td>
</tr>
<tr>
<td>12S battery pack</td>
<td>SLS Magnum APL V2 6S1P 22.2v 21000 mAh</td>
<td>2</td>
<td>Power source</td>
</tr>
</tbody>
</table>
3.3. System design

![Control framework diagram](image_url)

Figure 3-29: The control framework.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>DOF</th>
<th>Control interface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller-based actuation</td>
<td>$3R3T$</td>
<td>PWM</td>
</tr>
<tr>
<td>Winch-based actuation</td>
<td>$3T$</td>
<td>Position</td>
</tr>
<tr>
<td>Robotic manipulator</td>
<td>$7R$</td>
<td>Torque</td>
</tr>
</tbody>
</table>

Table 3.6: Actuated DOF and corresponding control interfaces.

mapping should be performed within the controllers. Moreover, all control signals are transmitted through saturation blocks for safety reasons. All actuated DOF of the SAM and corresponding control interfaces are listed in the Table 3.6.

The main control loop at the FCC, which is responsible for the control of winches and rotor groups, runs with a frequency of 200 Hz, while the manipulator joint torque controller operates at a sampling rate of 3 kHz.
"Research is to see what everybody else has seen, and to think what nobody else has thought."

Albert Szent-Györgyi

Chapter 4

Oscillation damping control with the propeller-based actuation

4.1 Problem statement

The cable-suspended aerial manipulation system is designed to perform various manipulation tasks in complex industrial sites. The manipulation platform is suspended on the crane’s hook by means of cable and equipped with a robotic arm. Thus, the system can approach most of the task locations. Once it is close to the target, the robotic arm performs a manipulation task. As an example, Figure 4-1 shows how the SAM performs a mobile crawler deployment on the pipe. In this task, the SAM should press the cage (with a crawler) to the pipe with a certain force to maintain

Figure 4-1: The SAM platform deploys the mobile inspection robot.
enough contact force required for secure extraction.

The main challenge to utilize the cable-suspended aerial manipulator in such a scenario is the pendulum motion caused by the suspension chain, so a self-stabilization of the system is essential. In particular, this chapter is interested in an oscillation damping of the suspended aerial manipulator, because it is crucial to dissipate the pendulum motion as quickly as possible in order to perform manipulation precisely. To this end, the SAM is equipped with an agile propeller-based actuation system that can produce the omnidirectional wrench at the GC of the platform. Here, we rely only on the IMU sensor for designing the controller because it is the most robust one among all onboard options. Indeed, GPS may not provide an accurate position in a complex industrial or indoor environment, and the lighting conditions limit the reliability of the vision sensor. In contrast, modern IMU with AHRS functionalities exploits real-time gyroscope drift correction and yaw adaptation to the disturbed magnetic environment.

However, as will be discussed in detail later, the cable-suspended aerial manipulator behaves like a double pendulum, not a single pendulum. Indeed, suspended aerial manipulator is hanged to the crane’s hook, which has a significant own mass. Thus, resulted oscillation (vibration) of the platform consists of sum of two vibration modes. Since the resulted motion of a double pendulum is more complex, the onboard IMU sensor does provide neither all vibration modes nor all the states needed for the damping control. In this chapter, to overcome this issue, we design a controller motivated by a simplified model of the SAM. Moreover, we consider two criteria in the control design. First, as addressed earlier, oscillation damping should be accomplished as quickly as possible to perform manipulation tasks. Second, since the aerial system is operated by a battery, we should take the power consumption into account. To this end, we seek an optimal controller that minimizes linear-quadratic function that balances two criteria. Additionally, it is worth noting that various crane might be used as the carrier for the cable-suspended aerial manipulator. Therefore, we design the model-free controller that should be robust to uncertainties and disturbances.

Thus, the main objective of the chapter is to design an oscillation damping controller for the cable-suspended aerial manipulator to achieve fast damping behavior while minimizing power consumption using data from a single onboard IMU sensor. The rest of the chapter is organized as follows. Section 4.2 presents the mathematical modeling and behavior analysis of the system. Section 4.3 describes an entire
cycle of the controller design, including control goal formulation, control derivation, stability analysis, and performance assessment in simulation studies.

4.2 Cable-suspended aerial manipulator modeling

4.2.1 Modeling assumptions

To model the cable-suspended aerial manipulator and to derive equations of motion that highlight the main system features, several assumptions were taken into account:

- The aerodynamic effects, e.g., blade flapping, platform drag and lift, airflow disruption, are negligible.

- The influence of the crane motion on the platform dynamics is negligible. Thus, we consider the crane as a fixed base. This assumption is reasonable since the crane is stopped as soon as the platform reaches the target location.

- The crane’s chain is a massless link with negligible inertia. Although this assumption reduces the model accuracy for a particular setup, the future controller should be robust enough to adapt to different crane chains. As an additional consequence, the wave effect along the chain might be neglected.

- The crane’s chain might be modeled as the rigid link with passive spherical joints at the tips. Such an assumption is valid since the platform suspension is always under tension because of the hook and the platform weight.

- The chain’s hook and the platform might be considered as rigid bodies.

- The motion of the platform suspended at the chain’s hook can be described by the double pendulum model restricted to small angles.

- The chain length, i.e., the length of the pendulum’s first link, changes with a slow constant speed which is reasonable for most cranes.

- The rotor group dynamics is negligible, i.e., we assume an immediate propeller response. Namely, actuator dynamics is ignored because the bandwidth of the BLDC is much larger (subsubsection 3.3.1.6) than the suppositional frequencies of the pendulum oscillation modes (section 6.2)).
4.2. Cable-suspended aerial manipulator modeling

![Diagram of SAM platform](image)

Figure 4-2: The model of the SAM platform (side view).

- The full system state is not available. It is possible to install the second IMU at the middle of the crane’s chain [Neupert et al., 2009] to track the hook motion, but the system applicability will be significantly reduced in such a case.

### 4.2.2 Dynamic formulation

In general, the cable-suspended aerial manipulator can be modeled as a spherical double pendulum, where the first link of length $l_1$ is the chain between the crane’s jib tip and crane’s hook, and the second link is the distance between the hook and the platform itself with length $l_2$, see the link depiction for the SAM system described in chapter 3 in Figure 4-2. Thus, the state of such a system can be characterized by 6 generalized coordinates: the Euler angles of the first $(q_1 \in \mathbb{R}^3)$ and the second $(q_2 \in \mathbb{R}^3)$ spherical passive joints of the double pendulum.

Let us consider three coordinate frames. The body frame $F_b$ is located at the GC of the platform, while its $y_b$ axis is aligned with the pendulum second link and directed upward. A hook frame $F_h : \{O_h, \ x_h, \ y_h, \ z_h\}$ is placed at the second spherical passive joint, and its $y_h$ axis is aligned with the first link and also directed upward. Finally, the world frame $F_w$ is placed at the suspension point. Its orientation coincides with the body frame at the moment when onboard IMU is initialized, i.e., in the beginning of the operation. Thus, onboard IMU provides

---

1: It is worth noting that in the scope of this chapter, we consider platform without robotic arm in our model. To this end, platform GC and COM coincide.
orientation $R_b^w$ and angular velocity $\omega_b$ of the platform. The weight of the links is neglected, and the two masses, $m_1$ and $m_2$, which correspond to the weight of the hook and cable-suspended aerial manipulator, are located at the origins of the $\mathcal{F}_h$ and $\mathcal{F}_b$ frames, respectively.

It is worth mentioning that joints of the double pendulum are not actuated, i.e., they are passive. To control the system, as described previously, we can generate a body wrench $w_b = [f_b, \tau_b]^T$ at the origin of $\mathcal{F}_b$ frame. Based on the preceding description, the equation of motion for the suspended aerial manipulator can be written as follows:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \begin{bmatrix} J^T w_b \\ \tau_m \end{bmatrix},$$  

(4.1)

where $M(q)$ is the inertia matrix, $C(q, \dot{q})$ is the centrifugal/Coriolis terms, and $g(q)$ is the gravity vector. The double pendulum configuration is defined as:

$$q = [q_1, q_2, q_m]^T \in \mathbb{R}^{13}. \quad (4.2)$$

Here, $q_m, \tau_m$ represent the robotic arm generalized coordinates and torque input. $J$ is the Jacobian matrix that maps pendulum joint velocities to a body twist coordinates $V_b$:

$$\begin{bmatrix} v_b \\ \omega_b \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}.$$  

In order to reinforce an intuition behind the complex system, the following analysis is performed for the simplified planar case, while the controller evaluation will be conducted for a generalized 3D case.

### 4.2.3 Double pendulum physics

Based on introduced assumptions, the cable-suspended aerial manipulator can be modeled as a double pendulum. Let us investigate the planar double pendulum dynamics in order to grasp some useful insights for later convenience. Hereafter, we will denote by a bar over a parameter $(\bar{.})$ the 2D version (planar case) of the corresponding parameter $(.)$.

Figure 4-3a illustrates the planar double pendulum configuration which is de-
4.2. Cable-suspended aerial manipulator modeling

(a) Planar double pendulum  (b) Planar single pendulum  (c) Wrench and twist

Figure 4-3: Planar pendulum modeling.

scribed by vector of generalized coordinates \( \bar{q} = [q_1, q_2]^T \) and angle \( \theta = q_1 + q_2 \) corresponding to the onboard IMU measurement.

Let us define the hook and platform COM locations with respect to \( F_w \) as:

\[
\begin{align*}
    x_1 &= l_1 \sin(q_1) \\
    y_1 &= -l_1 \cos(q_1) \\
    x_2 &= x_1 + l_2 \sin(\theta) \\
    y_2 &= y_1 + l_2 \cos(\theta)
\end{align*}
\]  

(4.3)

Taking the first and second derivatives over time in (4.3) and applying Lagrangian formulation (2.6), the equations of motion for such a system can be formulated as follows:

\[
\begin{align*}
    (m_1 + m_2)l_1 \ddot{q}_1 + m_2l_2 \dot{\theta} \cos(q_1 - \theta) + m_2l_2 \dot{\theta}^2 \sin(q_1 - \theta) + (m_1 + m_2)g \sin(q_1) &= 0, \\
    m_2l_2 \ddot{\theta} + m_2l_1 \dot{q}_1 \cos(q_1 - \theta) - m_2l_1 \dot{\theta}^2 \sin(q_1 - \theta) + m_2g \sin(\theta) &= 0.
\end{align*}
\]  

(4.4)

The solution of (4.4) linearized around equilibrium point \( q_{des} = [0, 0]^T \) can be found in the following form [Morin, 2002]:

\[
\bar{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} C_{11} \\ C_{21} \end{bmatrix} q^{slow} + \begin{bmatrix} C_{12} \\ C_{22} \end{bmatrix} q^{fast},
\]  

(4.5)

where \( q^i = A_i \cos(\nu_i t + \phi_i) \) is the oscillation term with amplitude \( A_i \), frequency \( \nu_i \), and phase shift \( \phi_i \) for \( i \in \{slow, fast\} \), \( C_{ij} \) with \( i, j \in \{1, 2\} \) are constant parameters determined by the weight-geometric parameters and initial conditions.
Chapter 4. Oscillation damping control with the propeller-based actuation

Thus, the double pendulum (coupled) system contains two oscillation motions [Magnus, 1965]: the first component with low frequency, \( \nu_{\text{slow}} \), is modulating the second one with a high frequency, \( \nu_{\text{fast}} \). Although the solution for joint angles \( q_1 \) and \( q_2 \) contains both high and low frequency motions, in the system with parameter relation such as \( l_1 > l_2 \) and \( m_1 < m_2 \), roughly speaking, the slow motion dominates in \( q_1 \), and fast motion - in \( q_2 \).

Fast and slow frequencies of the planar double pendulum system can be calculated using the following:

\[
\nu_{\text{fast,slow}}^2 = \frac{g(m_1 + m_2)}{8\pi^2 m_1 l_1 l_2} \left( (l_1 + l_2) \pm \sqrt{(l_1 + l_2)^2 - \frac{4m_1 l_1 l_2}{m_1 + m_2}} \right).
\]

Moreover, in [Singhose and Kim, 2007], it was shown that in case of \( l_1 = l_2 \) and \( m_1 < m_2 \), the \( \nu_{\text{slow}} \) is maximized and amplitude \( C_{i1} \) is much higher than \( C_{i2} \), so both links oscillate with the same frequency, and system behavior can be described by a single pendulum, see Figure 4-3b.

In the case of a single pendulum, the system can be described by the equation:

\[
(m_1 l_1^2 + m_2 l_2^2) \ddot{q}_1 = -g(m_1 l_1 + m_2 l_2) \sin(q_1).
\]

Let us denote the squared oscillation frequency by \( \nu_{\text{pend}}^2 = \frac{g m_1 l_1 + m_2 l_2}{m_1 l_1^2 + m_2 l_2^2} \), the amplitude by \( A_1 \), and the phase shift as \( \phi_1 \), then for small oscillations (\( \sin(q_1) \approx q_1 \)) the solution of (4.6) can be formulated as \( q_1 = A_1 \cos(\nu_{\text{slow}} t + \phi_1) \).

4.3 Optimal oscillation damping controller

4.3.1 Control goal

Let us consider unforced double pendulum dynamics (4.4) at a steady state. It can be seen that there are 4 equilibrium points:

\[
\dot{q}^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \pm \pi \\ \pm \pi \end{bmatrix} = \begin{bmatrix} 0 \\ \pm \pi \end{bmatrix} = \begin{bmatrix} \pm \pi \\ \pm \pi \end{bmatrix}.
\]

Among all of them, there is only one stable equilibrium which we consider as the operational point of interest. It corresponds to the bottom position with the stretched
4.3. Optimal oscillation damping controller

configuration, \( \mathbf{q}^* = [0, 0]^T \), i.e., where the potential energy is minimum. Nevertheless, any external perturbation in this position can cause an oscillation of the platform. In fact, due to the internal joint damping, the system itself is asymptotically stable with respect to the equilibrium point without any control. However, the damping value is very low, so natural stabilization takes a long time which is not acceptable for real industrial applications. Therefore, our goal is to design an oscillation damping controller to achieve faster damping behavior with minimal power consumption. One challenge is that, as will be shown shortly, the IMU sensor does not provide enough information for the control.

4.3.2 Reduced model for control design

In this chapter, we utilize a decentralized control strategy. Namely, the suspended platform control and manipulator control are decoupled. Regardless of chosen control strategy [Albu-Schäffer et al., 2007, Iskandar et al., 2020, Tsetserukou et al., 2008] and compensators [Iskandar and Wolf, 2019, Kim et al., 2019, Wolf and Iskandar, 2018] for the robotic arm, from the platform’s point of view, the dynamic behavior of the manipulator is treated as an external disturbance that causes oscillations.

In subsubsection 3.3.1.5, we found out experimentally that yaw control of the SAM can be independently performed because:

(i) we have a good control authority in yaw direction as shown in Figure 3-14

(ii) dynamics of yaw is rather decoupled from the others around the operational point. Indeed, we have achieved very strong yaw control by applying a common geometric control approach [Lee et al., 2010], see Appendix A.

Therefore, we can eliminate yaw- and manipulator-related variables in (4.1), which then reduces to:

\[
\ddot{\mathbf{M}}(\mathbf{q}) \ddot{\mathbf{q}} + \dot{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{J}^T \mathbf{w}_b + \mathbf{d}, \tag{4.7}
\]

where \( \mathbf{q} = [\mathbf{q}_1^T, \mathbf{q}_2^T]^T \) is the configuration (4.2) without variables related to yaw and manipulator; \( \ddot{\mathbf{M}}(\mathbf{q}), \dot{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}), \mathbf{g}(\mathbf{q}), \mathbf{w}_b, \) and \( \mathbf{J} \) represent components of remaining dynamics of (4.1). Motion of manipulator causes an uncertain disturbance \( \mathbf{d} \) that we further omit and treat as a source of oscillations that have to be dampened by controller.
If we restrict the pendulum motion in the plane (see Figure 4-3), we can further simplify the model as follows:

\[
\ddot{\bar{M}}(\bar{q})\ddot{\bar{q}} + \dot{C}(\bar{q}, \dot{\bar{q}})\dot{\bar{q}} + \ddot{\bar{g}}(\bar{q}) = \bar{J}^T\bar{w}_b, \tag{4.8}
\]

where, \(\bar{q} = [q_1, q_2]^T\) corresponds to angles of first and second links, and \(\bar{w}_b = [f_b, \tau_b]^T\) is the control wrench, see Figure 4-3c. The IMU measurement is then \(\theta = q_1 + q_2\) for the planar case. The body twist coordinates \(\bar{V}_b = [v_b, \omega_b]^T\) and configuration \(\dot{\bar{q}}\) are related by the Jacobian matrix \(\bar{J}\):

\[
\begin{bmatrix}
  v_b \\
  \omega_b \\
  \dot{v}_b
\end{bmatrix} =
\begin{bmatrix}
  l_1 \cos q_2 + l_2 \\
  1 \\
  0
\end{bmatrix}
\begin{bmatrix}
  \dot{q}_1 \\
  \dot{q}_2 \\
  \dot{q}
\end{bmatrix} =
\begin{bmatrix}
  l_1 \cos q_2 \\
  0 \\
  1
\end{bmatrix}
\begin{bmatrix}
  \dot{q}_1 \\
  \dot{\omega}_b
\end{bmatrix}. \tag{4.9}
\]

The Jacobian matrix \(\bar{J}\) can be calculated by utilizing (2.2). For later convenience, we also express a mapping \(\bar{J}_\theta\) of the body twist coordinates to the \([q_1, \omega_b]^T\). Let us notice that in a certain range of \(q_2 (q_2 < 90^\circ)\), the both mappings are invertible.

In the later section, we design a controller using the simplified planar model, while stability analysis and validation will be performed for the original system model (4.7).

### 4.3.3 Controller derivation

In the planar model (4.8), we first apply coordinate transformation from \(\bar{q}\) to \(\bar{V}_b\) using (4.9). Then, we obtain:

\[
\Lambda(\bar{q})\dot{\bar{V}}_b + \Gamma(\bar{q}, \dot{\bar{q}})\dot{\bar{V}}_b + \zeta(\bar{q}) = \bar{w}_b, \tag{4.10}
\]

where \(\Lambda(\bar{q}), \Gamma(\bar{q}, \dot{\bar{q}}), \zeta(\bar{q})\) represent inertia, Coriolis/centrifugal, and gravity terms in the new coordinate system. This coordinate transformation is valid since the Jacobian matrix \(\bar{J}\) is invertible for angle \(q_2\) which can be experienced in the real setup, e.g., the SAM. From (4.10), damping can be artificially injected by letting:

\[
\bar{w}_b = \begin{bmatrix}
  f_{b\text{des}} \\
  \tau_{b\text{des}}
\end{bmatrix} = \begin{bmatrix}
  -K_v v_b \\
  -K_\omega \omega_b
\end{bmatrix}, \tag{4.11}
\]

where \(K_v, K_\omega\) are positive control gains. Using (4.11), the control goal addressed earlier can be achieved.
4.3. Optimal oscillation damping controller

However, this control law cannot be directly applied because we have no measurements for $v_b$, whereas $\omega_b$ is directly obtained by IMU sensor. Assuming small angle for $q_2$ (hence, $\cos(q_2) \approx 1$) in (4.9), $f_{b\text{des}}$ can be approximated as:

$$f_{b\text{des}} = -K_v(l_1\dot{q}_1 + l_2\omega_b).$$

As addressed in subsection 4.2.3, $\dot{q}_1$ is dominated by slow oscillation motion with a low-frequency mode while $\omega_b = \dot{\theta}$ contains both modes. Therefore, using the frequency separation technique [Kuo and Puvvala, 2006, Sakata et al., 2013], we can extract $\dot{q}_1$ from $\omega_b = \dot{\theta}$ by applying a low-pass filter (LPF) to the signal, i.e., $\dot{\hat{q}}_1 \approx \omega_b^{lpf}$, where $(\cdot)$ is the estimation of parameter $(\cdot)$. Then, (4.11) can be rewritten as follows:

$$\dot{\tilde{\omega}}_b^{\text{des}} = \begin{bmatrix} f_{b\text{des}} \\ \omega_b^{lpf} \end{bmatrix} = \begin{bmatrix} -K_v(l_1\omega_b^{lpf} + l_2\omega_b) \\ -K_{\omega}\omega_b \end{bmatrix}, \quad \text{wi}$$

where

$$\omega_b^{lpf} = \frac{1}{\tau s + 1}\omega_b$$

is the $\omega_b$ processed through the first-order LPF with the time constant $\tau$. The time constant for the low-pass filter can be calculated as $\tau = \frac{1}{2\pi\nu_{\text{cutoff}}}$ based on the cut-off frequency $\nu_{\text{cutoff}}$[Hz]. In our control design, the cut-off frequency of the LPF is selected in the middle between slow and high frequency modes, i.e.,

$$\nu_{\text{cutoff}} = \frac{\nu_{\text{slow}} + \nu_{\text{fast}}}{2}. \quad (4.13)$$

Let us extend the presented control law to the original system (4.7) as follows:

$$\dot{\tilde{\omega}}_b = \begin{bmatrix} f_b \\ \tau_b \end{bmatrix} = -\begin{bmatrix} K_v & 0 \\ 0 & K_{\omega} \end{bmatrix} \tilde{V}_b = \begin{bmatrix} -K_v(l_1\tilde{\omega}_b^{lpf} + l_2\tilde{\omega}_b) \\ -K_{\omega}\tilde{\omega}_b \end{bmatrix}.$$ \quad (4.14)

Here, $K_v$ and $K_{\omega}$ are diagonal positive definite gain matrices.

The presented control approach is shown in Figure 4-4. It is interesting to notice that the controller does not contain a gravity compensation because the gravity works toward the control goal, i.e., stabilization around the operational point.
4.3.4 Stability analysis

Let us select the total energy of the system (4.7) as the Lyapunov function candidate:

\[ V = T(\dot{q}, \dot{q}) + U(q) = \frac{1}{2} \dot{q}^T \dot{q} + \frac{1}{2} \ddot{q}^T \ddot{q} + \sum_{i=1}^{2} \dot{q}^T \dot{q} + g^T r_i(q) m_i, \]

where \( g = [0, 0, g]^T \) is the vector giving the direction of the gravitational acceleration \( g = 9.81 \frac{m}{s^2} \) in the world frame and the vector \( r_i \) gives the coordinates of the COM of \( i \)-th pendulum weight. Derivation of the inertia tensor \( \dot{M}(\dot{q}) \) and \( r_i(\dot{q}) \) is shown in Appendix B.

It can be seen that \( V \) is a positive definite function, derivative of which with respect to time is defined as:

\[ \dot{V} = \frac{1}{2} \dot{q}^T \dot{q} + \frac{1}{2} \ddot{q}^T \ddot{q} + \dot{q}^T \dot{q} + g^T r_i(q) m_i. \]  

(4.15)

Extracting \( \ddot{q} \) from (4.7) and inserting it into (4.15), we receive the following:

\[ \dot{V} = \dot{q}^T \dot{q} + \frac{1}{2} \dot{q}^T \dot{q} + \dot{q}^T \dot{q} + \dot{q}^T \dot{q} + \dot{q}^T \dot{q} + \dot{q}^T \dot{q} = \dot{q}^T (\dot{q} + \dot{q} + \dot{q} + \dot{q} + \dot{q} + \dot{q}). \]  

(4.16)

Due to passivity property that implies a skew-symmetry nature of \( \dot{M}(\dot{q}) - 2\dot{C}(\dot{q}, \dot{q}) \) [Murray, 1994], (4.16) can be rewritten as follows:

\[ \dot{V} = \dot{q}^T \dot{q} = -\dot{q}^T \dot{q} + \dot{q}^T \dot{q} = -\dot{q}^T \dot{q}. \]
Since the control gain matrix $F_q$ scaled to the joint space is diagonal positive definite, the $\dot{V}$ is negative semi-definite. Therefore, the system is locally stable at the equilibrium point. Let us notice that $\dot{V} = 0$ only at the origin $\dot{q} = 0, \ddot{q} = 0$. Thus, by invoking the LaSalle’s invariance principle [Khalil, 2014], global asymptotic stability of the equilibrium point is followed.

As shown, the spherical double pendulum system with artificially injected damping in joints is asymptotically stable. This statement is true for the general case. However, the phase-lag introduced by LPF might compromise the passivity and induce the instability to the system with a feedback. In order to validate that introduced time delay does not penalize strongly the damping performance, additionally, we have numerically investigated the stability of our controller in a simulator. We report that for the $l_t = 4..10$[m] while changing the initial angles of the passive joints from 2 till 45 degrees in arbitrary configuration with a step of 7 degrees, the closed-loop system is always stable. Moreover, initial angular velocities at these joints were varied from 0 to 1 $\text{rad} \text{s}^{-1}$ with a step of 0.5. The performed analysis also confirms that the proposed controller is robust against model uncertainties.

4.3.5 Gain tuning rule

Since the system is stable for (almost) any choice of parameters and gains, it is important to seek the best gain in some sense. In particular, we seek for the control gains which minimize the following linear quadratic cost function:

$$J = \int_0^t \left( X(t)^T Q X(t) + U(t)^T R U(t) \right) dt,$$  \hspace{1cm} (4.17)

where weighting matrix $Q \geq 0$ penalizes the state $X$, and weighting matrix $R > 0$ penalizes the amount of control input $U$.

In this subsection, we again use the planar double pendulum for simplicity. Let us first linearize (4.8) around operational point as follows:

$$\dot{X} = AX + BU,$$  \hspace{1cm} (4.18)

where the state is $X = [q_1, \dot{q}_1, \theta, \omega_b, \omega_{b_{lpf}}]^T$ and state and input matrices\footnote{It is worth mentioning that in 3D case the linearized system can be modeled as two decoupled planar pendulums each of which can be described by similar matrices $A$ and $B$.} are:
The last row of matrix $A$ corresponds to the dynamics of the low-pass filter:

$$
\tau \ddot{\omega}_b^{lpf} + \omega_b^{lpf} = \omega_b.
$$

To express the control input (4.12) in an output feedback form, we define output as follows:

$$
Y = CX, \quad (4.19)
$$

where

$$
C = \begin{bmatrix}
0 & 0 & 0 & l_2 & l_1 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}.
$$

Consequently, the control input $U$ is presented as the output feedback form:

$$
U = -FY = - \begin{bmatrix}
K_V & 0 \\
0 & K_\omega
\end{bmatrix} Y. \quad (4.20)
$$

For our system described by (4.18)-(4.19), we apply the output feedback Linear Quadratic Regulator (oLQR) technique, which can be formulated using linear matrix inequalities (LMIs). Quadratic stability criteria of oLQR [Kučera and De Souza, 1995, Veselý, 2001, 2002] are defined directly in terms of LMIs.

**Theorem 1.** Let us consider the system (4.18) with the output (4.19). There exists an optimal controller in the form of (4.20) which minimizes the cost function (4.17), if the following problem has a solution for the given matrix $\Xi > 0$ and weighting matrices $Q \geq 0, R > 0$:

$$
\min_{F, P} \text{trace}(P)
$$
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subject to LMIs:

\[ M \leq 0, \quad P > 0, \]

where

\[ M = \begin{bmatrix} A^T P + PA + Q + H & G^T \\ G & -R^{-1} \end{bmatrix}, \]

with

\[ G = FC - R^{-1}B^T P, \]
\[ H = -(\Xi B)R^{-1}(B^T P) - (PB)R^{-1}(B^T \Xi) \]
\[ + (\Xi B)R^{-1}(B^T \Xi). \]

To solve the given LMI problem, we used offLQR library [Ilka, 2018] with the LMI solver in YALMIP [Löfberg, 2004]. Such a controller guarantees the robust closed-loop stability (quadratic stability) and minimizes the quadratic cost function [Amato, 2006; Lewis et al., 2012; Veselý, 2006]. Moreover, the stability is provided by checking the eigenvalues of the closed-loop system. In our control design, we selected \( Q \) and \( R \) matrices as follows:

\[ Q = \text{diag}\{0, 10, 0, 1, 0\} \quad \text{and} \quad R = \sigma \cdot \text{diag}\{1, 10\}. \quad (4.21) \]

Since our control goal is to dampen the oscillations, we penalized only \( \dot{q}_1 \) and \( \omega_b \). Moreover, we applied stronger control action on slow oscillation mode, which might be more critical when performing manipulation tasks in a real industrial scenario. For this reason, we penalized \( \dot{q}_1 \) more than \( \omega_b \) in \( Q \), and allowed more control input for \( f_b \) in \( R \) design.

In (4.21), \( \sigma \) is a new parameter that allows us to investigate the optimal control gains over admissible control inputs; note that smaller \( R \) implies more considerable control input. Therefore, we solved the optimization problem with varying \( \sigma \): from \( 1e^{-6} \) to \( 8e^{-5} \), as shown in Figure 4-5. Depending on the designer’s choice (balance between oscillation damping and power consumption), we can select one proper combination of gains. It might be useful to consider the rotor group saturation at the specific system as one of the criteria for the \( \sigma \) selection so we do not command too high wrench. Another beneficial criterion might be the proximity of the obstacles in
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Figure 4-5: Relation between optimal control gains $K_v$, $K_\omega$ and parameter $\sigma$.

The deep investigation on strategies for the $\sigma$ selection is beyond the scope of this thesis.

Optimization was conducted using parameters measured for the real system:

$$m_1 = 18.5 \text{ [kg]}, \quad m_2 = 55 \text{ [kg]}, \quad l_1 = 6 \text{ [m]}, \quad l_2 = 2.2 \text{ [m]}.$$  \hspace{1cm} (4.22)

The cut-off frequency $\nu_{\text{cutoff}} = 0.4499 \text{ Hz}$ was estimated by utilizing (4.13) for obtained $\nu_{\text{slow}} = 0.1665 \text{ Hz}$ and $\nu_{\text{fast}} = 0.7334 \text{ Hz}$ using FFT applied to the platform angular velocity $\omega_b$ for simulation data, see Figure 4-6.

Figure 4-6: Single-sided amplitude spectrum of the platform angular velocity. Amp stands for Amplitude.
4.3.5.1 Pendulum link length variation

In real applications, it is essential to change the length of the crane’s chain during the operation, e.g., to overcome the obstacles. That is why it is important to analyze the gain evolution over the different lengths of the first link. To this end, the optimization was conducted for the arbitrarily selected $\sigma = 1e^{-5}$, weight matrices as in (4.21), and $l_1 = \{1, 2, 4, 6, 8, 10\}$, see Figure 4-7. As we can see, the optimal gain $K_v$ changes within a reasonable range regardless of the length $l_1$, while the gain $K_\omega$ increases dramatically for the short length of the first link. It might be explained by decreased contribution of the $q^{fast}$ term in (4.5) when $l_1 \approx l_2$.

![Figure 4-7: Relation between optimal control gains $K_v$, $K_\omega$ and first link length $l_1$ for the $\sigma = 1e^{-5}$.](image)

As we discussed in subsection 4.2.3, when the length of the crane’s chain is equal to the distance from the hook to the platform, the system demonstrates a single pendulum behavior. Such behavior provides us with a significant advantage of being able to estimate the full system state by utilizing onboard IMU. Thus, it is essential to introduce a separated controller for such a case, see Appendix C.

4.3.6 Simulation studies

Extensive simulation studies were conducted to validate the proposed control approach performance. The suspended aerial manipulator was modeled as a double pendulum with parameters chosen to be as close as possible to the real setup given in (4.22). In addition, gyro noise density $0.009 [\text{deg}/\sqrt{\text{Hz}}]$ from the calibration certificate of IMU was introduced in simulation. The optimal control gains were selected
from Figure 4-5 for the $\sigma = 5e^{-6}$ as $K_v = 54.47$ and $K_\omega = 72.98$. The influence of unmodeled dynamics (e.g., the weight of the link), airflow, and actuator dynamics was not considered in the simulation. In the scope of the simulation, the $\sigma$ was selected to provide high control gains, and, consequently, to decrease convergence time.

Several different numerical simulations were studied and performed in the MathWorks MATLAB Simulink (MATLAB Simulink) utilizing Luca Dynamics environment (subsection 2.1.4).

### 4.3.6.1 Planar double pendulum

First, we tested controller at the planar double pendulum model with rigid links. Initial conditions for the system state were selected as $\bar{q} = [\frac{\pi}{25}, \frac{\pi}{25}]^T$ and $\dot{q} = [0, 0]^T$, and actuators were not saturated by any limits.

The behavior of the undamped (free)\(^3\) double pendulum is shown in Figure 4-8a. The corresponding joint parameters are presented in Figure 4-9a and Figure 4-9b. It can be seen that joint angles contain two modes, but as we mentioned before, the low frequency motion dominates in $q_1$, while the high frequency motion has a strong effect on $q_2$. The $\theta$ angle contains both motions with approximately equal influence.

The behavior of the planar double pendulum with non-zero initial conditions under the proposed controller with optimal gains is shown in Figure 4-8b. Simulational results are presented in Figure 4-9c and Figure 4-9d. The gravitational field applied to the platform (second pendulum weight) forces the $q_1$ angle of the hook deflection to slightly increase in the beginning for the given initial conditions. After that, the platform quickly converges to its equilibrium point, so the system behaves as a damped double pendulum. The corresponding control torques applied to the pendulum joints are presented in Figure 4-10. The generated control torque is based on the angular velocities with added high-frequency IMU noise. It can be seen that it does not affect the stability of the system.

Let us calculate the required planar wrench corresponding to the resulted maximum torques. As can be seen in Figure 4-9 and Figure 4-10, at $t = 0.285$ s the $\tau_1 = 1172.62$ Nm, $\tau_2 = 332.42$ Nm, and $q_2 = 4.62$ degrees. The planar wrench can be expressed from (2.4) after calculating the Jacobian matrix (4.9) as $\mathbf{w}_b = \begin{bmatrix} 140.48 & N \\ 23.35 & Nm \end{bmatrix}$. As we can see in Figure 3-14, the calculated wrench is reachable.

\(^3\)The model contained very small internal damping.
4.3. Optimal oscillation damping controller

Figure 4-8: The behavior of the planar double pendulum modeled in the Luca Dynamics environment.
Chapter 4. Oscillation damping control with the propeller-based actuation

Figure 4-9: Joint angles and velocities of the planar double pendulum.

by the SAM platform presented in chapter 3.
4.3.6.2 Cable-suspended aerial manipulator equipped with the robotic arm

Additionally, the behavior of the real system equipped with 7-DOF robotic arm in 3D space under the proposed controller with optimal gains has been studied through the validation simulation. The suspension was modeled as a spherical double pendulum. The first spherical joint is modeled by three rotational DOFs in \( RPY \) sequence, and the second one - by \( YPR \). Since we designed the oscillation damping controller with the decentralized control approach (subsection 4.3.2), we investigate its performance under various external disturbances such as oscillations caused by non-zero initial angles and robotic arm motion. To this end, the system starts with non-zero pendulum conditions, while the robotic arm is placed in the parking configuration. After 10 seconds (when stabilization is fully finished), the system experiences the jerky maneuver performed by the robotic arm. Namely, it moves from the parking configuration to the stretched one as defined in subsubsection 3.3.1.2. At the 12-th second after starting, the arm goes back to the parking position, see Figure 4-11. In addition, during the whole operation, the desired yaw angle is set to be 25 degrees. The initial configuration of the system is described by zero velocities and \( \begin{bmatrix} q_1, q_2 \end{bmatrix}^T = [0, \pi/12, 0, 0, -\pi/21, 0]^T \) for the pendulum joints and parking configuration for the robotic arm.

Simulation results are presented in Figure 4-12 and Figure 4-13. It can be seen that the proposed controller successfully dissipated the oscillations caused by releasing from non-zero initial conditions within 6.7 seconds and by the jerky motion of the robotic arm within 1.5 seconds. The desired yaw was reached gradually within
Figure 4-11: Illustration of the validation simulation. Here, at 1 double pendulum starts to move from non-zero initial conditions, at 2 it reaches the maximum negative oscillation displacement, at 3 – 4 it comes to the equilibrium point, at 5 is the 10-th second of simulation when arm performs the jerky motion by placing in the stretched configuration, at 6 is the 13.5-th second of simulation when the arm comes back to the parking configuration.

12 seconds with poorly tuned control gains. It is worth mentioning here that initial configuration angles of the pendulum were selected twice higher than expected in normal operational condition, and maximum robotic arm joint velocity in the
simulation equals to the $5 \text{ rad/s}$, while for the real robot it reaches a value of $3 \text{ rad/s}$ only in extreme conditions. So, simulation conditions can be considered as tough in comparison with a real mission.

At the second step, we compared the proposed control law (4.14), and the "ideal" controller defined as:

\[
\mathbf{w}_b^{des} = \begin{bmatrix} \mathbf{f}_b^{des} \\ \tau_b^{des} \end{bmatrix} = \begin{bmatrix} -K_v \mathbf{v}_b \\ -K_\omega \mathbf{w}_b \end{bmatrix},
\]

which is the 3D version of (4.11). By "ideal", we call the controller, which has access to the full state of the system. Recall that we proposed (4.14) because $\mathbf{v}_b$ is not measurable for the real system. As shown in Figure 4-14, overall shapes of resulting behavior were quite similar for both controllers. In fact, the proposed controller resulted in fewer oscillations than the ideal one due to the effect of filters. At $t = 10$, the robotic arm was commanded to cause jerky motions to generate some disturbances on the platform. The simulation results validate that the proposed control dissipates the oscillations caused by the dynamic disturbances.
Figure 4-14: Comparison of the proposed (blue) and "ideal" (red) controllers.
Chapter 5

Hierarchical whole-body control with the winch-based actuation and kinematically redundant manipulator

5.1 Problem statement

During the operation, the cable-suspended aerial manipulator encounters two types of disturbances: dynamic and static. As was shown in the previous chapter, the former might be effectively compensated by the short-term actions produced by the propeller-based actuation, while the latter requires a constant long-term impact that cannot be generated by the rotor groups because of the possible overheating. Most common examples of static disturbances in aerial manipulation include the displacement of the system COM because of the robotic arm motion and external wrench applied to the system during the interaction with the environment. In fact, the static disturbances negatively affect manipulation performance by perturbing the base dynamics and, consequently, the IMU measurements and onboard vision camera’s field of view.

To this end, to further enhance the manipulation and stabilization performance of the SAM, the second actuation system, winch-based actuation, is integrated into the considered system. Three rigging cables are connecting the SAM platform to the hook of the crane. Winch-based actuation inherited from the cable-driven robotics [Begey et al., 2018, Bosscher et al., 2006, Kraus et al., 2014] allows to slowly change the length of these cables affecting the tension force distribution and, consequently, the SAM pose relative to the hook. The exploitation of two actuation systems should
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Figure 5-1: The behavior of the SAM while manipulating below the platform (left) and on the side (right).

reduce the amount of the consuming energy.

In the scope of this chapter, we investigate a winch-based actuation utilized to compensate for the static gravitational torque caused by the weight of the robotic manipulator, see Figure 5-1. During the operation below the platform, the system COM is located close to the vertical axis, passing through the suspension point. However, whenever the robotic arm manipulates on the side of the platform, the shifted system COM generates the strong gravitational torque resulting in the platform tilt.

The main challenges for the winch-based actuation utilization include complex modeling of closed-chain dynamics [Freitas et al., 2011, McGrath et al., 2017, Murray and Lovell, 1989] and mixed actuation in terms of control input: winch servos are position controlled, while the robotic arm is torque controlled. To this end, modeling is performed in terms of the equal quasi-joint dynamics directly representing the translational motion of the platform and robotic arm joints. Also, an admittance interface is used for the winch servos to transform control torques to position commands. Moreover, as will be shown below, the system under investigation is redundant. Therefore, it is decided to adapt a hierarchical whole-body controller
aimed at two main control tasks: to cope with static disturbances and to perform manipulation.

Therefore, the main objective of the chapter is to design a whole-body controller that compensates for the system COM displacement without degradation of manipulation performance by utilizing kinematically redundant robotic arm and winch-based actuation. The rest of the chapter is organized as follows. The section 5.2 shows a modeling process of the winch suspension integrated into the double pendulum and formulates the dynamics. The section 5.3 provides the adaptation of a hierarchical whole-body controller with an integrated admittance interface.

5.2 Winch suspension modeling

5.2.1 Modeling assumptions

In this section, we aim to derive the winch suspension model. Due to high system complexity, in addition to the list of previously defined assumptions in subsection 4.2.1, we adopt several more points:

- The winch-based actuation switches on only after the platform reaches the target point and the propeller-based actuation dissipates all oscillations. This assumption fully meets our operational strategy at which the static disturbances appear mainly during the robotic arm motion, i.e., during the manipulation process.

- The rigging cables controlled by winches are under tension, so they can be represented as massless rigid links without sagging (mass of each rigging cable is around 40-50 grams). Each tip of the cable is modeled as a rotational joint.

- The actuator dynamics is negligible, i.e., we assume an immediate winch servo response due to a high gain controller.

To gain better intuition behind the complex system, first, we investigate a planar case that can be further generalized to the 3D model.
5.2.2 Dynamic formulation

5.2.2.1 Coordinate frames

Let us introduce the following coordinate frames\(^1\), see Figure 5-2:

- The **world frame** \(F_w\) centered in the suspension point \(O\) such that its vertical axis is opposite to downward gravity direction, and the horizontal axis is perpendicular to it.

- The **hook frame** \(F_h\) attached at point \(A\) represents a crane’s chain swinging, i.e., rotational angle of the pendulum first link relative to the world frame, \(q_1\). The vertical axis of this frame is aligned with the chain, and the horizontal axis complements it to the right-hand frame.

- The **platform frame** \(F_p\) is placed at the point \(C^2\), which is located in the platform center. It is aligned with the principal platform axes.

- The **COM frame** \(F_{\text{com}}\) is located at the total system (platform and robotic arm) COM, at point \(G\), i.e., the location of the frame origin changes over-time and depends on the configuration of the arm. Thus, for the double pendulum system without the integrated manipulator, the point \(G\) coincides with point \(C\). The vertical axis of the frame is pointing towards rigging cable suspension point \(A\), and the horizontal one is perpendicular to it. The **COM frame** represents the rotation of the pendulum’s second link relative to the first one, \(q_2\).

- The **tool frame** \(F_t\) : \(\{F; x_t, y_t, z_t\}\) (or end effector frame) is attached to the manipulator end effector at point \(F\). The point \(F\) is also named as tool center point (TCP). The orientation of the \(F_t\) is task dependent.

---

\(^1\)The color of the font corresponds to the color of the frame in figures.

\(^2\)It is worth noting that in chapter 4 we addressed this frame as the body frame since we dealt with a single body (platform) model. In this chapter, to avoid confusion and distinguish the "platform" and the "platform with the integrated robotic arm", we introduce the platform frame.
5.2. Winch suspension modeling

5.2.2.2 Planar model

As we mentioned in the previous chapter, with fixed-length rigging cables the system can be represented as a double pendulum suspended to the crane jib’s tip, O, see Figure 5-3a. The length of the first link, $OA$, is equal to the length of the chain, $l_1$. We assume that the weight of the hook, $m_1$, is concentrated at point A. The second link consists of two objects: the platform and the manipulator with masses $m_p$ and $m_m$, respectively. Rigging cables of the winches correspond to $AB$ and $AE$, see Figure 5-3b. The rigging cable lengths can be controlled and initially are equal to each other, $l_0$. The robotic arm is rigidly attached to the platform at point $D$.

Resulted system model contains a closed chain, see Figure 5-3c. The state of such a model can be described by the vector of redundant generalized coordinates:

$$
q = \begin{bmatrix}
q_1, q_3, q_4, q_5, q_6, q_7, q_{\text{winch}}
\end{bmatrix}^T \in \mathbb{R}^{6+m}.
$$

Here, $q_{\text{winch}}$ is the vector of redundant winch joints containing passive revolute joints, $q_3, q_5, q_6$, representing the connectors of the rigging cables relative to the platform\(^3\) and active prismatic joints, $q_4$ and $q_7$, corresponding to the rigging cable length. For the sake of readability, we imply that $q_4$ and $q_7$ include $l_0$ and represent the total current lengths of the rigging cable. Finally, $q_m \in \mathbb{R}^m$ is the vector of the robotic arm joint angles. It is worth noting that $q_2$ is not a part of the state since it is a

\[^3\text{There is one more revolute joint } q_8 \text{ that coincides with } q_3, \text{ it can be easily eliminated from the state } q \text{ by considering angle constraint of the closed-chain [Tang, 2010].}\]
virtual quantity that might be computed based on the system state $q$.

Since our model contains a closed chain $ABE$, the system dynamics is partially constrained. Applying Lagrangian formulation, equation of motion can be written as follows:

$$
\ddot{M}(q)\ddot{q} + \dot{C}(q, \dot{q}) + \dot{g}(q) = \ddot{\tau} + A^T \lambda + \ddot{\tau}_{ext},
A(q)\dot{q} = 0,
$$

where $\ddot{M}(q)$ is the symmetric and positive definite inertia matrix, $\dot{C}(q, \dot{q})$ contains the centrifugal/Coriolis terms, $\dot{g}(q)$ is the gravity vector, $\ddot{\tau}$ is the vector of the joint torques, $\ddot{\tau}_{ext} \in \mathbb{R}^{6+m}$ is the vector of external forces and torques. Let us denote by $n_w = 5$ a number of passive and active DOFs of the winch-related joints which cannot be controlled independently because of the $n_c = 2$ loop-closure constraints. Thus, there are only $n_p = n_w - n_c = 3$ DOFs along which platform can be moved by utilizing winch-based actuation. Then, $A \in \mathbb{R}^{n_c \times 6+m}$ is the Pfaffian matrix which will be defined later, and $\lambda \in \mathbb{R}^{n_c}$ are Lagrange multipliers that parametrize the interaction forces acting along the constraints.

In order to formulate holonomic constraints in the system dynamics, let us formulate a vector loop-closure equation which represents the forward kinematics of the point $C = [x_c, \ y_c]^T$ at which weight of the platform is concentrated as follows:

$$
\begin{bmatrix}
  x_c \\
  y_c
\end{bmatrix} =
OA + AB + BC = OA + AE + EC,
=f(q_1, \ q_3, \ q_4, \ q_5)
\Rightarrow
=f(q_1, \ q_5, \ q_6, \ q_7).

(5.2)
$$

Projecting (5.2) on $x_w$ and $y_w$ axes of the world frame and taking its time-derivative, we can derive two constraints in the Pfaffian form:

$$
A(q)\dot{q} = 0.
$$

To resolve constraints and reduce the dimension of the state (5.1), we might apply a coordinate transformation. To this end, let us define a vector of constraint-consistent independent generalized coordinates as follows:

$$
\delta = [q_1, \overbrace{q_4, q_5, q_7}^{q_{\text{winch}}}, q_m]^T \in \mathbb{R}^{4+m},
$$

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and a vector of dependent generalized coordinates as:

\[ z = [q_3, q_6]^T \in \mathbb{R}^2. \]

Then, the relation between \( \dot{\delta} \) and \( \dot{q} \) can be formulated as:

\[ \dot{q} = S(q)\dot{\delta}, \quad (5.3) \]

where \( S(q) \in \mathbb{R}^{(6+m) \times (4+m)} \) is chosen such that:

\[ S^T(q)A^T(q) = 0. \quad (5.4) \]

With (5.3), one can conduct a coordinate transformation. Then the unconstrained dynamics in terms of the independent generalized coordinates can be formulated as follows:

\[
\begin{align*}
\dot{\delta} = & \begin{bmatrix}
M(q) \delta + \dot{C}(q, \delta) \delta + \dot{g}(q) = \tau + \tau_{ext}
\end{bmatrix} \\
& \begin{bmatrix}
S^TMS \\
S^T(MS+CS)
\end{bmatrix}
\begin{bmatrix}
\dot{\delta}
\end{bmatrix} \\
& \begin{bmatrix}
S^T \dot{\theta}
\end{bmatrix} \\
& \begin{bmatrix}
S^T \dot{\tau}
\end{bmatrix} \\
& \begin{bmatrix}
S^T \dot{\tau}_{ext}
\end{bmatrix}
\end{align*}
\]

As we mentioned above, there are \( n_p = 3 \) DOFs along which platform can perform motion using winches, i.e., \( q_4, q_5, q_7 \). In terms of practical application, we are interested in more intuitive motion directions, e.g., platform horizontal \( x_c \), vertical \( y_c \), and one rotational \( q_5 \) motions with respect to the world frame. To this end, solving an IK for (5.2) it is possible to express winch parameters \( q_{winch} = [q_4, q_5, q_7]^T \) as vector of the platform state \( q_p = [x_c, q_5, y_c]^T \) and \( q_1 \).

Thus, we can define an equal quasi-state vector of feasible motions, see Figure 5-4, as follows:

\[ \eta = [q_1, x_c, q_5, y_c, q_m]^T \in \mathbb{R}^{4+m}, \]

such that:

\[ \dot{\delta} = B\dot{\eta}. \]

Figure 5-4: Serial chain (quasi-state) coordinates of the feasible motions.
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The vector $\eta$ corresponds to the equal serial-chain representation of the independent generalised coordinates describing closed-chain dynamics.

Then, the dynamics (5.5) with respect to $\eta$ can be rewritten as follows:

$$\ddot{M}(q) \ddot{\eta} + \ddot{C}(q, \dot{\eta}) \dot{\eta} + \ddot{g}(q) = \bar{\tau} + \bar{\tau}_{ext}. \quad (5.6)$$

A vector of generalized torques applied to the serial dynamics (5.6) can be defined as $\bar{\tau} = [\tau_{q_1}, \bar{\tau}_p, \tau_m]^T \in \mathbb{R}^{4+m}$, where $\tau_{q_1}$ is the torque applied to the pendulum first joint, $\bar{\tau}_p = [\tau_{x_c}, \tau_{q_1}^{y_c}, \tau_{y_c}]^T$ is the vector of control inputs applied to the quasi joints corresponding to the platform motion, and $\tau_m$ are torques applied to the joints of the robotic arm. Since in the winch-based actuation we have only two actuated joints, $q_4$ and $q_7$, we should select two out of three quasi joints that we aim to control. In our case, $x_c$ and $y_c$ are of main interest. Thus, the vector of actuated torques, $\bar{\tau}_a = [\tau_{q_1}, \tau_{x_c}, \tau_{y_c}, \tau_m]^T \in \mathbb{R}^{3+m}$ can be defined such that $\bar{\tau} = O^T \bar{\tau}_a$. Here, $O \in \mathbb{R}^{(3+m) \times (4+m)}$ maps actuated torques $\bar{\tau}_a$ to the generalized torques $\bar{\tau}$. As it will be shown later, in our control strategy we utilize IK to map the desired motion in quasi state joints to the actuated joints, so we do not need to calculate the $O$. For detailed derivation of the matrices $A$, $S$, $B$, and $O$, the reader is referred to Appendix D.

5.2.2.3 3D case

Since winches cannot affect the motion of the platform around the yaw, a spatial case might be represented as two decoupled planar dynamics similar to the considered above. Therefore, the first joint of the double pendulum can be modeled by spherical joint with corresponding coordinates $q_{s,pend1} = [q_1, q_2, q_3]^T$, each tip of the rigging cables can be modeled as a universal joint with 2 DOF at each fixation point, and three closed-chain loops in the space provide 9 constraint equations. The platform can be moved by winch-based actuation in the space along 3 translational DOFs, i.e., $x_{s,p} = [x_c, y_c, z_c]^T$ and 2 rotational DOFs corresponding to the roll and pitch, i.e., $q_{s,p} = [x_{s,p}, \varphi, \theta]^T$. Then, a quasi-state vector of feasible motions can be rewritten as: $\xi = [q_{s,pend1}, q_{s,p}, q_{m}]^T \in \mathbb{R}^{15}$, where $m = 7$ for the redundant robotic arm, and resulted dynamics in terms of the quasi-state vector in 3D space can be formulated
5.3. Hierarchical impedance-based whole-body controller

as follows:

\[
M_s(\xi)\ddot{\xi} + C_s(\xi, \dot{\xi})\dot{\xi} + g_s(\xi) = O_s^T \begin{bmatrix}
\tau_{s,\text{pend}_1} \\
\tau_{xc} \\
\tau_{yc} \\
\tau_{zc} \\
\tau_m \\
\tau_{s,a}
\end{bmatrix} + \tau_{s,\text{ext}}. \tag{5.7}
\]

All values with subscript "s" in the dynamics (5.7) correspond to the same values of dynamics (5.6) generalized for the 3D case. For further convenience, let us also define a vector of the spherical joint \(\mathbf{q}_{s,\text{pend}_2} \in \mathbb{R}^3\) corresponding to the motion of the second link of the double pendulum. Similar to the \(q_2\) for the planar case, \(\mathbf{q}_{s,\text{pend}_2}\) is an abstract quantity that can be calculated based on the \(\xi\) vector.

5.3 Hierarchical impedance-based whole-body controller

5.3.1 Reduced model for control design

The received equations of motion (5.7) for the cable-suspended aerial manipulator with integrated winch suspension are highly complex. Therefore, we want to simplify them by finding a reasonable trade-off between the system behavior description and equations that can be efficiently utilized in the controller.

In the previous chapter, we have introduced the oscillation damping controller that can eliminate oscillations in the pendulum joints while keeping the constant yaw of the platform using propeller-based actuation. Thus, in the case when the oscillation damping controller is applied to the system (5.7), the dynamics of the pendulum joints can be neglected, i.e., \(\dot{q}_{s,\text{pend}_1} = 0\) and \(\dot{q}_{s,\text{pend}_2} = 0\), and corresponding pendulum joint angles will be defined over time only by effect of the external and gravitational torques. The latter one appears because of the robotic arm weight, which shifts the total system COM. As a result, gravity force generates disturbing torque around the suspension point, which leads to the undesired non-zero configuration in pendulum joints of the system\(^4\), see Figure 5-5, and, consequently, to the platform tilt, shift in end effector pose, and the onboard sensor perturbations.

\(^4\)Here we neglected by the hook weight for clarity of explanation.
The winch-based actuation affects three controllable rigging cables that allow translational shifts of the platform with respect to the suspension point in order to counterbalance the COM displacement. Such action should result in zero pendulum joints. To this end, we define a reduced configuration $\gamma$ which contains only joints responsible for the system COM location and robotic arm end effector pose as follows:

$$\gamma = [x_c, y_c, z_c, q_m]^T \in \mathbb{R}^{10}.$$  

The $\gamma$ defines DOFs that will be accessed by the controller. In our case, based on the desired quasi-state of the platform, we have to calculate control inputs to the real system, i.e., cable lengths, by utilizing IK. This task is fully determined as it will be shown in subsection 5.3.3.5. Thus, we can always generate such cable lengths that nullify the tilting angles ($\varphi$, $\theta$) while keeping desired displacements ($x_c$, $y_c$, $z_c$). Taking into account the slow winch dynamics, it allows us to neglect dynamics along rotational DOFs as well. To this end, state $\gamma$ does not contain tilting roll $\varphi$ and pitch $\theta$.

When the double pendulum joint angles as well as the platform tilt $q_5$ are equal to zero, the COM is located under the suspension point, and the mapping matrix $O$ has the form of a unit matrix. Then, the dynamics (5.7) can be rewritten in terms of the $\gamma$ state as follows:

$$M_r(\gamma)\dot{\gamma} + C_r(\gamma, \dot{\gamma})\dot{\gamma} + g_r(\gamma) = \begin{bmatrix} \tau_{x_c} \\ \tau_{y_c} \\ \tau_{z_c} \\ \tau_m \end{bmatrix} + \tau_{ext,r}. \quad (5.8)$$

Here, $\tau_\gamma = [\tau_p, \tau_m]^T$, and all elements with subscript "r" correspond to the reduced dynamics.

Figure 5-5: Gravitational torque brings the system COM under the suspension point by changing the pendulum configuration.
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5.3.2 Control goals

Let us define the following control goals for our system:

1. The main task for aerial manipulator is to change the end effector Cartesian pose (three coordinates and three Euler angles), \( \mathbf{x}_e \in \mathbb{R}^6 \), expressed in the world frame toward desired value, \( \mathbf{x}_e^{\text{des}} \). Let us define configuration of the end effector in terms of the \( \gamma \) as \( \mathbf{x}_e = f_e(\gamma) \), then our control goal is to provide: 
\[
\mathbf{x}_e - \mathbf{x}_e^{\text{des}} = 0.
\]

2. The second task is to cope with the static disturbances caused by robotic arm weight, i.e., neutralize gravity torque by keeping system COM, \( \mathbf{x}_{\text{com}} \in \mathbb{R}^3 \), under suspension point. Let us define the model COM location in inertial space as \( \mathbf{x}_{\text{com}} = f_{\text{com}}(\gamma) \), then our control goal is to keep horizontal components of this vector, i.e., \( x_{\text{com}} \) and \( y_{\text{com}} \), as zeros.

3. The last task is injecting additional damping to the elbow (3-th) joint of the robotic arm. This task is introduced for further calculation convenience.

The second task indirectly ensures that unmeasured pendulum joints are close to zero when the oscillation damping controller is applied. Only in this case, we can state that the first task is fulfilled in the world frame since the utilized model is based only on the reduced state \( \gamma \).

In real mission, the robotic arm is under remote operator control [Lee et al., 2020], so the second task should have less priority [Dietrich et al., 2013], and it should not interrupt the first (main) one. Moreover, all control tasks should be structurally feasible at the same time. The last task has the least priority.

5.3.3 Controller derivation

To regulate the defined state \( \gamma \), in this chapter we adapt the Hierarchical impedance-based Whole-Body Controller [Coelho et al., 2021, Dietrich et al., 2021, Ott et al., 2011, Ott et al., 2015], see Figure 5-6. As it can be seen from the diagram, as input we set the desired position of the end effector, \( \mathbf{x}_e^{\text{des}} \), while the desired position of the system COM, \( \mathbf{x}_{\text{com}}^{\text{des}} \), is set to zero during the whole operation. The whole-body impedance-based controller produces the desired generalized torques for the robot joints, \( \tau_m \), and quasi joints of the platform, \( \tau_p \). Since the robotic arm is torque-controlled, it directly follows the desired command and provides the end
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Figure 5-6: Hierarchical whole-body control diagram.

effector position, $x_e$, as an output. The platform with integrated winches, on the other hand, is kinematically controlled. To this end, we first transform desired generalized torques $\tau_p$ to the required motion along platform quasi joints, $x_{s,p}^{des}$, by virtue of admittance interface (subsection 2.2.1) and further, we use the Inverse Kinematics (IK) to map it to the desired cable lengths. In parallel, $x_{s,p}^{des}$ returns back to the controller model in order to update the location of the resulted system COM. In the following subsections, all aforementioned sub-blocks will be considered in detail.

5.3.3.1 Hierarchically decoupled dynamics

Following the task space definition presented in subsection 5.3.2, we can define task velocities:

$$\dot{x}_e = J_1 \dot{\gamma},$$

$$\dot{x}_{com} = J_2 \dot{\gamma},$$

$$\dot{q}_{m,3} = J_3 \dot{\gamma},$$

(5.9)

here $\dot{x}_e \in \mathbb{R}^6$ is the body velocity of the end effector, $\dot{x}_{com} \in \mathbb{R}^3$ is the vector of translational velocities of model COM, $\dot{q}_{m,3} \in \mathbb{R}$ is the velocity of the robotic arm elbow joint. Besides, $J_i = \frac{\partial f_i(\gamma)}{\partial \gamma}$ for $i \in \{1, 2, 3\}$, i.e., $J_1 \in \mathbb{R}^{6 \times 10}$, $J_2 \in \mathbb{R}^{3 \times 10}$, and $J_3 \in \mathbb{R}^{1 \times 10}$ are corresponding Jacobian matrices that have a full row rank during the operation.

Adapting approach presented in [Dietrich and Ott, 2019] for redundant robots, let us first introduce the augmented Jacobian for $i$-th task by stacking all priority
5.3. Hierarchical impedance-based whole-body controller

task Jacobians up to $i$-th as follows:

$$J_{aug}^i(\gamma) = \begin{bmatrix}
J_1(\gamma) \\
\vdots \\
J_i(\gamma)
\end{bmatrix}.$$

Then, augmented task velocity $V \in \mathbb{R}^{10}$ is related to the joint velocities as follows:

$$V = \begin{bmatrix}
\dot{x}_e \\
\dot{x}_{com} \\
\dot{q}_{m,3}
\end{bmatrix} = \begin{bmatrix}
J_1(\gamma) \\
J_2(\gamma) \\
J_3(\gamma)
\end{bmatrix} \dot{\gamma}; \quad (5.10)$$

Expressing the task velocities through the vector of joint velocities plays a vital role in the whole-body framework. To avoid interference between tasks, we need to decouple the task velocities at the acceleration level. To this end, dynamically consistent null-space projectors [Khatib, 1987] can be used:

$$N_i(\gamma) = \begin{cases}
I, & \text{for } i = 1 \\
I - J_{i-1}^{aug}(\gamma)^T J_{i-1}^{aug}(\gamma)^{M+}, & \text{for } i = 2, 3.
\end{cases} \quad (5.11)$$

Here, the superscript $(.)^{M+}$ represents a dynamically consistent pseudoinverse [Die-trich et al., 2015] that is defined as:

$$J_i(\gamma)^{M+} = M_i(\gamma)^{-1} J_i(\gamma)^T (J_i(\gamma)M_i(\gamma)^{-1} J_i(\gamma)^T)^{-1}.$$ 

By utilizing (5.11), we can dynamically decouple original Jacobian matrices in (5.9):

$$\tilde{J}_i(\gamma) = J_i(\gamma) N_i(\gamma)^T.$$

Then, we can formulate corresponding hierarchically decoupled task-space velocities as follows:

$$\begin{bmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{bmatrix} = \begin{bmatrix}
\tilde{J}_1(\gamma) \\
\tilde{J}_2(\gamma) \\
\tilde{J}_3(\gamma)
\end{bmatrix} \dot{\gamma}. \quad (5.12)$$
Here, \( \mathbf{\nu} \) indicates the vector of the task space velocities in dynamically decoupled coordinates, and \( \mathbf{J}(\gamma) \in \mathbb{R}^{10 \times 10} \) is the extended Jacobian matrix that maps generalized velocities to the task velocities \( \mathbf{\nu} \). Note that the \( \mathbf{J}(\gamma) \) is invertible due to complemented 3–rd task.

The new set of coordinates allows to receive hierarchically decoupled motion dynamics of the system (5.8):

\[
\Lambda(\gamma)\dot{\mathbf{\nu}} + \mathbf{\mu}(\gamma, \dot{\gamma})\mathbf{\nu} = \mathbf{J}(\gamma)^{-T}(\mathbf{\tau}_\gamma + \mathbf{\tau}_{\text{ext}, r} - \mathbf{g}_r(\gamma)),
\]

where \( \Lambda(\gamma) = (\mathbf{J}M_r(q)^{-1}\mathbf{J}^T)^{-1} = \text{diag}(\Lambda_1, \Lambda_2, \Lambda_3) \) contains decoupled task inertias, and \( \mathbf{\mu}(\gamma, \dot{\gamma}) = \mathbf{J}^{-T}(\mathbf{C}_r(\gamma, \dot{\gamma}) - M_r(q)\mathbf{J}^{-1}\mathbf{J})\mathbf{J}^{-1} \) is transformed Coriolis-centrifugal matrix.

The task hierarchy can be intuitively observed by expressing the relation between augmented task velocities \( \mathbf{V} \) and hierarchically decoupled task-space velocities \( \mathbf{\nu} \) [Dietrich and Ott, 2019, Garofalo and Ott, 2020]. Inserting (5.10) in (5.12), we can receive the following:

\[
\mathbf{\nu} = \mathbf{J}(\gamma)\mathbf{J}_{\text{aug}}^T(\gamma)^{-1} \mathbf{V}.
\]

Then, mapping \( \mathbf{T}(q) \) is the lower-triangular matrix that projects the original task velocities one-to-one to the decoupled task velocities, i.e., \( \mathbf{T}_{i,j} = \mathbf{I} \) for \( i = j \), and ensures that the lower-priority level velocities do not disturb higher-priority ones, i.e., \( \mathbf{T}_{i,j} = 0 \) for \( i < j \):

\[
\mathbf{T}(q) = \begin{bmatrix}
    \mathbf{I} & 0 & 0 \\
    \mathbf{T}_{21} & \mathbf{I} & 0 \\
    \mathbf{T}_{31} & \mathbf{T}_{32} & \mathbf{I}
\end{bmatrix}.
\]

Dimensions of each element \( \mathbf{T}_{i,j} \) correspond to the size of the original and decoupled task spaces.

5.3.3.2 Control law

Since all tasks exploit the same DOFs, a hierarchical whole-body controller is applied in order to ensure that the main task is fulfilled without being disturbed by the
second and third. The control law is formulated as follows:

\[
\mathbf{r}_\gamma = \begin{bmatrix} \mathbf{r}_p \\ \mathbf{r}_m \end{bmatrix} = \mathbf{g}_e(\gamma) + \mathbf{r}_\mu + \sum_{i=1}^{3} \mathbf{J}_i^T \mathbf{F}_{i,ctrl} \tag{5.14}
\]

with

\[
\mathbf{r}_\mu = \sum_{i=1}^{3} \left( \mathbf{J}_i^T \left( \sum_{j=1}^{i-1} \mathbf{\mu}_{i,j}\mathbf{v}_j + \sum_{j=i+1}^{3} \mathbf{\mu}_{i,j}\mathbf{v}_j \right) \right),
\]

where \( \mathbf{F}_{i,ctrl} \) corresponds to the control force for the \( i \)-th task and \( \mathbf{r}_\mu \) is applied in order to decouple task dynamics at the velocity level. It compensates for the off-blockdiagonal submatrix of the Coriolis-centrifugal matrix \( \mathbf{\mu}(\gamma, \dot{\gamma}) \), i.e., \( \mathbf{\mu}_{12} \in \mathbb{R}^{6 \times 3}, \mathbf{\mu}_{13} \in \mathbb{R}^{6} \). Thus, all task control forces \( \mathbf{F}_{i,ctrl} \) are expressed through all joint torques of the system in such a way that tasks are conducted without interference according to hierarchy. This aspect establishes the foundation of the hierarchical whole-body control framework.

Thus, by applying (5.14) to (5.13), we receive fully decoupled dynamics equations as follows:

\[
\Lambda_i(\gamma)\mathbf{\dot{v}}_i + \mathbf{\mu}_{i,i}(\gamma, \dot{\gamma})\mathbf{v}_i = \mathbf{F}_{i,ctrl} + \mathbf{J}_i(\gamma)^T \mathbf{r}_{ext,i}
\]

for \( i \in \{1, 2, 3\} \).

With structural feasibility of all tasks and absence of external torques, the proposed control law (5.14) guarantees the asymptotic stability of the equilibrium point for the main task and conditional stability for the second and third tasks maintaining the overall passivity, as shown in [Dietrich et al., 2016, Ott et al., 2015].

### 5.3.3.3 Impedance control in the task space

To accomplish the desired tasks and achieve compliant behaviour on all hierarchical levels, the Cartesian impedance control is exploited [Caccavale et al., 1998, Zhang and Fasse, 2000]:

\[
\mathbf{F}_{1,ctrl} = \begin{bmatrix} -R_e^T \mathbf{K}_{pe} \mathbf{p}_e \\ -2R_e^T \mathbf{E}(\eta_e, \mathbf{e}_e)^T \mathbf{K}_{oe} \mathbf{e}_e \end{bmatrix} - \mathbf{K}_{D_e} \mathbf{\dot{e}}_e,
\]

\[
\mathbf{F}_{2,ctrl} = -\mathbf{K}_{pe} \mathbf{p}_e - \mathbf{K}_{D_c} \mathbf{\dot{x}}_{com},
\]

\[
\mathbf{F}_{3,ctrl} = -\mathbf{K}_{D_3} \mathbf{\dot{q}}_{m,3},
\]
Chapter 5. Hierarchical whole-body control with the winch-based actuation and kinematically redundant manipulator

Here, $F_{1,ctrl}$ is the body wrench applied to the end effector, $F_{2,ctrl}$ is the force of the secondary task before being projected in the null space of the first one, i.e., the components of the secondary task force that collide with the main task will be nullified after projection, and $F_{3,ctrl}$ is the damping torque at the 3-th joint of the robotic arm. Moreover, the matrices $K(\cdot)$ are positive definite gain matrices (subscripts $p$, $o$, $d$ stand for positional stiffness, orientational stiffness, and damping, respectively), $K_{D_{3}}$ is the damping gain, $R(\cdot), p(\cdot)$ are the rotational and positional errors between the desired and current pose of the corresponding operational spaces (subscripts $e$ and $c$ stand for the end effector and COM), see the detailed definition of error elements in paragraph 2.1.1.1.6. Error corresponding to the first task is expressed in the tool frame, while the error of the second task is presented in the world frame. $\eta(\cdot)$ and $\epsilon(\cdot)$ are the scalar and vector parts of a quaternion representation of $R(\cdot)$, and the matrix $E(\eta(\cdot), \epsilon(\cdot)) = \eta(\cdot)I_3 - \hat{\epsilon}(\cdot)$.

Taking into account the control force definition (5.15), formulated control law (5.14) can be applied directly to our system for the execution of all control tasks. However, while the robotic arm is torque controlled, the winch servos are position-controlled. Therefore, additional transformation is required.

5.3.3.4 Admittance interface

As it was mentioned in subsection 3.3.4, each winch contains an embedded motion controller and takes the cable length as the input. In order to operate the whole system at the torque level, the admittance interface is adapted [Dietrich et al., 2016, Iskandar et al., 2019]. It takes commanded forces in quasi joints as input to a virtual system with desired dynamics and produces the corresponding displacements as outputs which are further processed through IK for final cable length calculation. The exploitation of such an interface implies utilizing a high gain motion controller for the winch-based actuation, which can perfectly realize desired admittance dynamics despite external and internal (e.g., due to coupled dynamics) disturbances.

Passing $\tau_p$ through the admittance interface with desired dynamics, the corresponding displacement of the platform, $x_{s,p}^{des}$, is described by:

$$M_{adm}\ddot{x}_{s,p}^{des} + D_{adm}\dot{x}_{s,p}^{des} = \tau_p, \quad (5.16)$$

where $M_{adm} \in \mathbb{R}^{3 \times 3}$, $D_{adm} \in \mathbb{R}^{3 \times 3}$ are positive inertia and damping diagonal matrices describing desired dynamics of the system, respectively.
Assuming that \( x_{s,p} \approx x_{s,p}^{des} \), the overall system dynamics (5.8) can be rewritten:

\[
\begin{bmatrix}
M_{adm} & 0 \\
M_{pm}(\gamma) & M_m(\gamma)
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_{s,p} \\
\ddot{q}_m
\end{bmatrix}
+ \begin{bmatrix}
D_{adm} & 0 \\
C_{pm}(\gamma, \dot{\gamma}) & C_m(\gamma, \dot{\gamma})
\end{bmatrix}
\begin{bmatrix}
\dot{x}_{s,p} \\
\dot{q}_m
\end{bmatrix}
+ \begin{bmatrix}
0 \\
g_m(\gamma)
\end{bmatrix} = \begin{bmatrix}
\tau_p \\
\tau_m
\end{bmatrix}.
\]

(5.17)

Here, the term \( \tau_{ext} \) is omitted since we do not measure external wrench applied to the system and assume that the designing controller is robust enough to cope with external perturbations. To estimate this term, the force-torque sensor at the end effector or tension force sensor in the rigging cables can be installed [Haddadin et al., 2017, Iskandar et al., 2021, Lu et al., 2005]. \( M_{pm} \) and \( C_{pm} \) are the inertia and Coriolis couplings between platform and manipulator which are corresponding submatrices of \( M_r(\gamma) \) and \( C_r(\gamma, \dot{\gamma}) \).

Following the [Dietrich et al., 2016], the control law (5.14) is extended with additional term to remove coupling effect between manipulator and platform in (5.17) due to admittance dynamics:

\[
\tau_\gamma = g_r(\gamma) + \tau_{\mu} + \tau_{comp} + \sum_{i=1}^{3} \bar{J}_i^T F_i,ctrl,
\]

(5.18)

where

\[
\tau_{comp} = \begin{bmatrix}
0 \\
M_{pm}(\gamma) \ddot{x}_{s,p} + C_{pm}(\gamma, \dot{\gamma}) \ddot{q}_m + g_m(\gamma)
\end{bmatrix}.
\]

It is worth noting that acceleration and velocities for \( \tau_{comp} \) term can be estimated from (5.16) without direct measurement.

It can be seen that in the admittance interface dynamics (5.16), there is no stiffness term. Indeed, the presence of the virtual spring would bring the system toward zero configuration as soon as the control force is removed, while we want the system to stay in the configuration it reaches even after releasing the control force.

As a final step, the IK for the rigging cable suspension should be defined. It takes as the input the displacement, \( x_{s,p}^{des} \), and provides cable lengths as the output.
5.3.3.5 Inverse kinematics

In order to control quasi joints, \( \mathbf{x}_{s,p} = [x_c, y_c, z_c]^T \), via the lengths of the cables, the IK for the rigging cable suspension is formulated based on subsection 2.1.3. As shown in Figure 5-7, when pendulum joints are zeros, the following vector loop can be constructed:

\[
\mathbf{l}_{i}^{\text{des}} = \mathbf{r} + \mathbf{R}_{w}^{p}(\mathbf{d} + \mathbf{a}_i). \quad (5.19)
\]

Here, \( \|\mathbf{l}_{i}^{\text{des}}\| \) is the length of the \( i \)-cable. The constant vector \( \mathbf{a}_i \) represents the location of the \( i \)-th cable start point, \( W_i \), relative to the platform GC, point \( C \), see subsubsection 3.3.4.1. Vector \( \mathbf{r} = [0, 0, z_c]^T \) connects the suspension point \( A \) with point \( K \) which indicates the system COM, point \( G \), projected at the platform plane, see Figure 5-3b. A displacement between \( K \) and \( C \) is defined by vector \( \mathbf{d} \). The vectors \( \mathbf{a}_i \) and \( \mathbf{d} \) are located in the plane of the platform. The rotation matrix \( \mathbf{R}_{w}^{p} \) represents an orientation of the platform via roll and pitch (\( \varphi, \theta \)) angles of the vector \( \mathbf{q}_{s,p} \). Since it is in our interests to keep \( \varphi, \theta \) angles as zeros, we impose \( \mathbf{R}_{w}^{p} = \mathbf{I} \) such that \( \mathbf{R}_{w}^{p}\mathbf{d} = [x_c, y_c, 0]^T \). Thus, for the desired platform displacement \( [x_c, y_c, z_c]^T \), we calculate only such cable lengths that guarantee zero tilting angles and ensures the horizontal COM components under the suspension point. It is possible because inverse kinematics represents a fully determined nonlinear system of equations (5.19), which can be solved for any desired quasi-state.

Figure 5-7: Inverse Kinematics for the rigging cable suspension.
5.3. Hierarchical impedance-based whole-body controller

5.3.4 Simulation study

The main goal of this subsection is to validate the ability to regulate the multi-DOF system COM using the whole-body framework\(^5\).

The Luca Dynamics environment (subsection 2.1.4) calculates a system COM location and centroidal momentum matrix \(A = [A_p, A_i]^T\) [Garofalo et al., 2015, Orin and Goswami, 2008] which relates linear and angular momenta with joint velocities \(\gamma\). Then, for linear momentum \(p\) we can formulate the following:

\[
p = A_p \dot{\gamma} = (m_m + m_p) \dot{x}_{com},
\]

where \(\dot{x}_{com}\) is the velocities of the system COM.

Utilizing (5.9), the \(J_2\) can be found as follows:

\[
J_2 = \frac{A_p}{m_m + m_p}.
\]

Then, adapting the control law (5.18) for only one task we receive:

\[
\tau_\gamma = g + J_2^T F_{com};
\]

\[
F_{com} = \mathcal{PD}(x_{com} - x_{com}^{des}).
\]

Here, \(\mathcal{PD}\) indicates the proportional-derivative operation on the error of the \(x_{com}\) location.

In order to validate the control law, we conducted a simulation in MATLAB Simulink. To this end, the model corresponding to the state \(\gamma\) was utilized. It contains 3 translational DOFs of the platform and 7 rotational DOFs of the robotic arm. As desired location of the COM the point \(x_{com}^{des} = [0.224, -0.3, 0.49]^T\) was chosen\(^6\). As a result, the controller generates forces applied to the platform and torques applied to the robotic arm joints, see Figure 5-8a. As shown in Figure 5-8b, the system COM location converges to the desired values by utilizing joint torques of the whole system within 4.3 seconds for the not properly tuned controller. As we will see in the following chapter, the possible COM displacement of the experimental setup presented in chapter 3 is roughly 1.5 times lower for critical configuration with horizontally stretched arm.

\(^5\)The task hierarchy will be evaluated in the experimental studies in the following section.

\(^6\)Our final control goal is to nullify the first two components of this vector, but here we use non-zero desired values just to validate approach.
Chapter 5. Hierarchical whole-body control with the winch-based actuation and kinematically redundant manipulator

Figure 5-8: Results of the simulation study on the whole-body control framework performance.

(a) Control forces applied to the platform (top) and torques applied to the arm joints (bottom)

(b) Location of the system COM

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Chapter 6

Experimental investigation of the proposed control strategies

In this chapter, in order to validate the performance and functionality of the cable-suspended aerial manipulator concept regulated by the developed control algorithms, an extensive experimental investigation is carried out. In particular, we examine the behavior of the system:

- without any actuation through FFT analysis,
- controlled by the Optimal Oscillation Damping Controller (OODC),
- controlled by the Hierarchical impedance-based Whole-Body Controller (HWBC),
- controlled simultaneously by two controllers.

As a demonstrator, the crane-stationed SAM platform, see chapter 3 is utilized in two different setups: indoor and outdoor. Several experiments are conducted with the platform hanged to the ceiling in the laboratory. Depending on the controller and experimental conditions, various performance criteria are specified.

6.1 Experimental setup description

6.1.1 The demonstrator SAM

Developed cable-suspended aerial manipulator SAM, see Figure 6-1, is utilized in the experimental studies. The platform orientation and angular velocities are retrieved from the onboard IMU, the end effector pose is calculated using forward kinematics
applied to the robotic arm joint sensor data, and the platform translational motion due to winch-based actuation is calculated using the forward kinematics applied to the collected cable length data. Exploited settings and weight of the system are specified in Table 6.1. The data logging frequency is different for each experiment, therefore it is specified directly in each experiment description as $\nu_{data}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propeller-based actuation control loop</td>
<td>200</td>
<td>Hz</td>
</tr>
<tr>
<td>Winch-based actuation control loop</td>
<td>200</td>
<td>Hz</td>
</tr>
<tr>
<td>Robotic arm control loop</td>
<td>1000</td>
<td>Hz</td>
</tr>
<tr>
<td>IMU raw data frequency</td>
<td>400</td>
<td>Hz</td>
</tr>
<tr>
<td>Platform weight</td>
<td>35.7</td>
<td>kg</td>
</tr>
<tr>
<td>Robotic arm weight</td>
<td>16.7</td>
<td>kg</td>
</tr>
</tbody>
</table>

Table 6.1: Settings and parameters of the SAM in the experimental sessions.

### 6.1.2 RMC laboratory

Several validation experiments were conducted in the RMC laboratory, see Figure 6-2a. In contrast to the crane-stationed environment, in the laboratory the SAM is hanged directly to the cable without the hook. Thus, the system behaves as a single spherical pendulum.
6.1. Experimental setup description

Suspension point

Direct connection (without the hook)

Winch

Chain (6 m)

Hook (18.5 kg)

Ground station

The SAM (55 kg)

(a) Laboratory environment

(b) Indoor environment

Figure 6-2: Experimental setups.

6.1.3 Indoor environment

The indoor experiment is located in the DLR Oberpfaffenhofen TechLab facility with integrated overhead crane Stahl ST50\(^1\), see Figure 6-2b. The main characteristics of the indoor setup which were utilized in the optimization problem for the control law (4.14) are presented in Table 6.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hook weight</td>
<td>18.5</td>
<td>kg</td>
</tr>
<tr>
<td>Chain density</td>
<td>2.85</td>
<td>kg/m</td>
</tr>
<tr>
<td>Length of the chain</td>
<td>6</td>
<td>m</td>
</tr>
<tr>
<td>Distance from the hook to the platform</td>
<td>2.2</td>
<td>m</td>
</tr>
</tbody>
</table>

Table 6.2: Weight-geometric parameters of the indoor setup.

\(^1\)https://www.stahlcranes.com
6.1.4 Outdoor environment

The outdoor environment is based on a mini crane Hoeflon C6\(^2\) and organized in DLR Oberpfaffenhofen site, see Figure 6-3a. The utilized crane can be transformed to the mobile platform on caterpillars, see Figure 6-3b, and is controlled remotely by the joystick. The main characteristics of the outdoor setup are presented in Table 6.3\(^3\). In contrast to the indoor environment, the outdoor experiment is challenging due to presence of wind, dust, different light conditions, and various obstacles. The wind reaches the value of 3-5 at the Beaufort scale being able to shake tree branches, straighten waving ribbon (see Figure 6-3c), shake the crane’s jib, i.e., the suspension point, and tilt the platform for 2-3 degrees around roll and pitch angles.

For the indoor and outdoor environments: the suspension point and the hook correspond to the two passive spherical joints; the hook structure has a passive DOF around the yaw, i.e., the hook itself can rotate around the hook base; the crane chain can twist around a vertical axis.

![Figure 6-3: Outdoor environment.](https://www.hoeflon.com)

2\(^{https://www.hoeflon.com}\)

3\(^{https://www.meteoblue.com\text{ on 13.03.2021-19.03.2021.}}\)
6.2 Frequency-based model validation

In subsection 4.2.1, we assumed to consider a crane-stationed cable-suspended aerial manipulator as a double pendulum. To see the validity of this assumption, we investigate oscillation modes (eigen- or natural frequencies) of the real hardware during the free motion after an external stimulation. To this end, we pulled the SAM to the side from the equilibrium point and then released to force it oscillating. Afterward, we applied the FFT to the collected onboard angular velocity in order to obtain a power spectrum and determine the eigenfrequencies of the system dynamics. As will be shown, in the resulted spectrum there exist two dominant frequency modes, and therefore, it is reasonable to model the system as a double pendulum. Based, on the oscillation frequencies, we calculated the optimal control gains for the control law (4.14) similar to the process described in subsection 4.3.5. Since in the experimental validation, we conduct tests utilizing the indoor and outdoor environments with different weight-geometric characteristics, we investigate them separately. Additionally, it is worth reminding that the obtained oscillation mode frequencies have much lower bandwidth in comparison with the rotational frequency of the spinning propeller (subsubsection 3.3.1.6), so there is no risk of a resonance.

### Indoor environment

Oscillation mode frequencies $\nu_{\text{slow}}$ and $\nu_{\text{fast}}$ were obtained by oscillating the indoor setup with no actuation, see Figure 6-4. Consequently, the corresponding cutoff frequency for the low-pass filter (4.13) is calculated $\nu_{\text{cutoff}} = 0.76 \text{ Hz}$.

In order to obtain a better understanding behind slow and fast oscillation frequencies, we introduce normal modes of the double pendulum, i.e., the motion where

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hook weight</td>
<td>36.86</td>
<td>kg</td>
</tr>
<tr>
<td>Length of the chain</td>
<td>5.6</td>
<td>m</td>
</tr>
<tr>
<td>Distance from the hook to the platform</td>
<td>1.9</td>
<td>m</td>
</tr>
<tr>
<td>Wind speed</td>
<td>10-35</td>
<td>$\frac{\text{km}}{\text{h}}$</td>
</tr>
</tbody>
</table>

Table 6.3: Weight-geometric parameters of the outdoor setup.
both masses move with the same frequency. In case, if both masses moves with $\nu_{\text{slow}}$ in exactly same manner, the double pendulum behaves as the single pendulum. On another hand, when both masses oscillates with $\nu_{\text{fast}}$, they produce an opposite motion, see Figure 6-5.

Based on conducted frequency analysis, the optimal gains for the OODC were
6.2. Frequency-based model validation

Figure 6-6: Relation between optimal control gains $K_v, K_\omega$ and parameter $\sigma$ for the indoor environment.

calculated as a function of the $\sigma$ parameter, see Figure 6-6.

**Outdoor environment**

Since in the outdoor environment we operate with tower-like crane (section 3.2), a free dynamics of the outdoor setup was investigated for various chain length, see Figure 6-7 (All data was collected with $\nu_{data} = 120.56$ Hz). As we can see for the short length of the crane chain $l_1 = 1.2$ m, the system behaves as a single pendulum. Time-domain angular velocity is presented as a sine signal, and corresponding frequency domain spectrum contains only 1 dominant peak for $\nu_{slow} = 0.4393$ Hz. As soon as the chain length exceeds the length of the second pendulum link (1.9 m), the system starts to demonstrate the double pendulum behavior. It fully conforms to the discussed theory, see subsection 4.2.3. It is worth mentioning that the linear density of the chain in mobile crane is less than for the overhead crane exploited in the indoor environment. To this end, the frequency domain contains only two peaks corresponding to the double pendulum motion without the wave-related frequency.

Due to internal damping in the double pendulum motion, the time transient signal is damped. However, since the damping value in the non-actuated system is very low, the signal demonstrates the periodic motion with decreasing amplitude. The peaks in the corresponding FFT represent the eigenfrequencies while the smear around the peak shows the sine wave decay. The slower the decay, the narrower the smear.
Chapter 6. Experimental investigation of the proposed control strategies

Figure 6-7: Angular velocity around $x_b$ axis (left) and corresponding single-sided amplitude spectrum (right) for various chain lengths $l_1$ in the outdoor setup. Amp stands for Amplitude.
The optimal control gains for the OODC in the outdoor setup were obtained for the various cable chain length $l_1$ depending on the parameter $\sigma$, see Figure 6-8. It is worth noting that for the short crane chain lengths, the linear gain $K_v$ has a concave shape.

![Figure 6-8: Relation between optimal control gains $K_v$ (left), $K_\omega$ (right) and parameter $\sigma$ for the various lengths of the first link $l_1$.](image)

### 6.3 Geometric yaw controller

In order to validate an applicability of the geometric yaw controller described in Appendix A, an experiment for the setpoint yaw regulation of the SAM system using propeller-based actuation in the outdoor environment was conducted. The most significant performance criteria include the settling time and steady-state error.

The selected control gains are shown in the Table 6.4. Here, $K_{p,yaw}$ is $k_R(3, 3)$, and $K_{d,r}$ is $k_\omega(3, 3)$, i.e., control gains corresponding to the yaw channel of the control law (A.1). The $\mathcal{PD}$ gains are selected to demonstrate a slightly over-damped behavior.

As it can be seen in Figure 6-9, during the experiment the SAM followed the desired set of yaw angles $[10, 40, 85, 10]$ degrees. The yaw angle of the platform

<table>
<thead>
<tr>
<th>Gain</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{p,yaw}$</td>
<td>21</td>
</tr>
<tr>
<td>$K_{d,r}$</td>
<td>29.28</td>
</tr>
</tbody>
</table>

Table 6.4: Geometric yaw controller gains.
Figure 6-9: Results of the experiment: geometric yaw control. At $t = 0$ s, the SAM is located in the initial configuration of 10 deg, while at $t = 60$ s, the SAM is turned around vertical axis to 85 deg. The data was collected with $\nu_{\text{data}} = 26.42$ Hz. Converged to the desired setpoint, while the angular velocity $r$ converged to the zero. The corresponding control torque $\tau_z$ of the (A.1) applied to the platform around the vertical axis is shown in the same figure (bottom plot).

Processing the results of the first step response of the platform, i.e., the turn from

Figure 6-10: Illustration of the experiment: geometric yaw control. Configurations of the SAM platform at $t = 0$ s and at $t = 60$ s are shown. The purple dashed circle denotes the location of the landing pad.
6.4 Optimal oscillation damping controller (OODC)

10 to 40 degrees, the settling time (5%) of 12.6 seconds is estimated, while steady state error did not exceed 1.2 degrees. Such an error can be reduced by deriving more precise dynamic model and extending the control law with an integral term.

Figure 6-10 depicts the SAM platform configuration before and after yaw rotation. Shown results validate that the SAM yaw channel can be successfully controlled with a geometric yaw controller.

6.4 Optimal oscillation damping controller (OODC)

To corroborate simulation results, extensive experimental validation is carried out for the OODC. Namely, six experiments are conducted for the actuated platform:

- case of the robotic arm motion in the outdoor environment,
- case of the external disturbances in the indoor environment,
- case of the suspension point motion in the outdoor environment,
- case of the varying chain length in the outdoor environment,
- case of the single pendulum behavior (short chain length) in the outdoor environment,
- case of the switching between single and double pendulum controllers in the outdoor environment.

Motivated by the simulative studies in (4.3.6), the optimal control gains are chosen for the $\sigma = 5e^{-6}$ and presented in the Table 6.5. The main performance criterion is a dampening time. It is worth reminding that the gains are selected

<table>
<thead>
<tr>
<th>Environment</th>
<th>Gain</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdoor setup</td>
<td>$K_{w,x} = K_{w,y}$</td>
<td>29.28</td>
</tr>
<tr>
<td>($l_1 = 5.6 \text{ m}$)</td>
<td>$K_{v,x} = K_{v,y}$</td>
<td>54.35</td>
</tr>
<tr>
<td>Indoor setup</td>
<td>$K_{w,x} = K_{w,y}$</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>$K_{v,x} = K_{v,y}$</td>
<td>48</td>
</tr>
</tbody>
</table>

Table 6.5: Optimal oscillation damping controller gains for $\sigma = 5e^{-6}$. 

128
optimally to keep the balance between performance and power consumption. So, the controller might response in a more rigid (fast) way if needed by varying the weighing matrices \( Q \) and \( R \) in (4.17).

### 6.4.1 Case of the robotic arm motion

In this experiment, we investigate the behavior of the SAM platform while the robotic arm performs a jerky motion. To this end, the robotic arm end effector was controlled by the operator to follow the trajectory depicted in Figure 6-11. During this motion, the joint velocities reached relatively high values of \( \frac{2 \pi}{s} \) as shown in Figure 6-12. It is worth noting that in the real manipulation mission, the arm does not move so fast. Investigation of the arm jerky motion helps us to analyze a robustness to uncertainties of the proposed controller.

As a result, the propeller-based actuation dampened all disturbances caused by robotic arm motion during roughly 8 seconds while keeping the desired yaw angle of 130.89 degrees, see Figure 6-13. The corresponding angular velocities of the platform converged to the zero. As shown in Figure 6-14, an OODC-generated wrench (4.14)
6.4. Optimal oscillation damping controller (OODC)

Figure 6-12: Results of the experiment: robotic arm motion. Robotic arm joint velocities.

Figure 6-13: Results of the experiment: robotic arm motion. Roll, pitch, and yaw angles of the platform are depicted on the left, and corresponding angular velocities are shown on the right.
includes the body forces $f_{b,x}$, $f_{b,y}$ and torques $\tau_{b,x}$, $\tau_{b,y}$ corresponding to the $x_b$ and $y_b$ axes, while the geometric yaw controller (A.1) generates the $\tau_z$ torque. All data was collected with $\nu_{data} = 33.13$ Hz.

### 6.4.2 Case of the external disturbances

In this experiment conducted in the indoor environment, the human operator applied external disturbance to excite the SAM by pulling and releasing the rope attached to the landing pad, while the controller was trying to dampen any oscillations. As shown in Figure 6-15, the controller quickly damped out the oscillation within 7 seconds, and the SAM gradually in underdamped manner converged to zero point keeping desired yaw as 25 degrees. Correspondingly, the angular velocities converged to zero.
6.4. Optimal oscillation damping controller (OODC)

![Graphs showing roll, pitch, and yaw angles with corresponding angular velocities]

Figure 6-15: Results of the experiment: applying of the external disturbances. Roll, pitch, and yaw angles of the platform are depicted on the left, and corresponding angular velocities are shown on the right. The data was collected with $\nu_{data} = 107.91$ Hz.

6.4.3 Case of the suspension point motion

In order to validate robustness of the OODC to the suspension point motion, we moved the crane with switched off (passive platform) and switched on (active platform) propeller-based actuation. Namely, the crane was turned roughly for 90 de-

![Images showing initial, middle, and final configurations]

Figure 6-16: Illustration of the experiment: suspension point motion. Crane direct (clockwise) motion around the vertical axis.
degrees around the vertical axis and back in such a way that direct motion was performed with slow speed while return motion was conducted fastly (roughly twice of the slow motion speed). The direct motion is depicted in Figure 6-16. The data was collected with $\nu_{data} = 122.06$ Hz.

### 6.4.3.1 Passive platform

Firstly, we turned the crane while the platform was switched off, see Figure 6-17. It can be seen, that after slow crane motion (green zone), the platform oscillation did not exceed 2 degrees around roll and pitch, while the fast motion (red zone) caused oscillation of 4-5 degrees around the same axes.

![Figure 6-17: Results of the experiment: suspension point motion for the passive platform. Roll, pitch, and yaw angles of the platform are depicted on the left, and corresponding angular velocities are shown on the right.](image)
6.4.3.2 Active platform

Secondly, we switched OODC on, and repeated the same experiment. It can be seen in Figure 6-18, that after the end of the slow and fast motion period, the controller dampened all oscillations within 9 seconds while keeping yaw angle close to the desired at 35 degrees. At the same time, oscillations around roll and pitch reached maximum 2.2 degrees.

Figure 6-18: Results of the experiment: suspension point motion for the active platform. Roll, pitch, and yaw angles of the platform are depicted on the left, and corresponding angular velocities are shown on the right.

6.4.4 Case of the varying chain length

In order to further investigate robustness of the OODC to the system uncertainties, i.e., evaluate the sensitivity to the varying parameters, we tuned the controller for the crane chain length \( l_1 = 5.6 \) meters and then lifted the hook up (yellow zone) by reducing the cable length till \( l_1 = 2 \) meters. Afterward, we repeated previous
Chapter 6. Experimental investigation of the proposed control strategies

(a) Lowered configuration (b) Lifted configuration (c) Lifted and turned configuration

Figure 6-19: Illustration of the experiment: varying chain length. The SAM lifting and direct (clockwise) rotation around vertical axis was performed by the crane.

experiment with turning suspension point clockwise with slow speed and counterclockwise with the fast speed around the vertical axis, see Figure 6-19. Moreover, in the middle of the fast (return) motion at \( t = 90 \) second, the crane was suddenly shut down and turned on again (dark blue zone), see Figure 6-20. In the end, the hook was lowered down (yellow zone) till original \( l_1 = 5.6 \) meters. So, it can be seen that even for not properly tuned controller, the performance remained reasonable, and OODC dampened oscillations caused by slow and fast motion within 9-13 seconds while keeping the desired yaw angle of 65 degrees.

6.4.5 Case of the short chain length

After reducing the length of the crane chain, i.e., setting it equal to the second link length, \( l_1 = 1.9 \) meters, we conducted the crane motion similar to described in subsection 6.4.3. First, the slow clockwise motion (green zone) was performed by crane around the vertical axis, after that the fast return motion (red zone) was conducted. The control gains for the control law (C.1) of the single pendulum are shown in Table 6.6. Here, \( K_{p,x} \) is the \( K_p(1, 1) \), \( K_{p,y} \) is the \( K_p(2, 2) \), \( K_{v,x} \) is the

<table>
<thead>
<tr>
<th>Gain</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{p,yaw} )</td>
<td>21</td>
</tr>
<tr>
<td>( K_{d,r} )</td>
<td>45</td>
</tr>
<tr>
<td>( K_{p,x} = K_{p,y} )</td>
<td>3.82</td>
</tr>
<tr>
<td>( K_{v,x} = K_{v,y} )</td>
<td>54.18</td>
</tr>
</tbody>
</table>

Table 6.6: Oscillation damping controller gains for the single pendulum case.
6.4. Optimal oscillation damping controller (OODC)

Figure 6-20: Results of the experiment: varying chain length. Roll, pitch, and yaw angles of the platform are depicted on the top, and corresponding angular velocities are shown in the bottom. The data was collected with $\nu_{data} = 31.67$ Hz.
\( K_v(1, 1) \), and \( K_{v,y} \) is the \( K_v(2, 2) \) for the control law (C.1). The data was collected with \( \nu_{data} = 35.5 \text{ Hz} \).

The results of the experiment with passive platform are shown in Figure 6-21, while results of the same experiment with the active platform are depicted in

**Figure 6-21:** Results of the experiment: single pendulum case of the passive platform. Roll and pitch angles of the platform are depicted on the left, and corresponding angular velocities are shown on the right.

**Figure 6-22:** Results of the experiment: single pendulum case of the active platform. Roll and pitch angles of the platform are depicted on the left, and corresponding angular velocities are shown on the right.
Figure 6-22. It is seen that for the passive platform after motion stop, the platform started to oscillate with slowly decaying rate, while the active platform dampened oscillations within 8 seconds.

### 6.4.6 Case of the switching controllers

The last experiment in the validation of the OODC is aimed at the investigation of the switching transient between OODC and oscillation damping controller for the single pendulum. During the experiment, the system started at the lowered configuration with \( l_1 = 5.6 \) meters with active OODC as shown in Figure 6-19 (a). Then, at \( t = 12 \) seconds, the system was lifted up till \( l_1 = 1.9 \) meters as shown in Figure 6-19 (b). At this moment, the controllers were switched between each other, so the control law (4.14) was switched to the (C.1). As it can be seen in Figure 6-23, the switching did not affect performance, and the total angle error along roll and pitch remained within 1 degree.

![Figure 6-23](image_url)

Figure 6-23: Results of the experiment: switching between single and double pendulum controllers. The data was collected with \( \nu_{\text{data}} = 39.65 \) Hz.
6.5 Hierarchical impedance-based whole-body controller (HWBC)

In order to investigate the winch-based actuation performance and validate proposed control law (5.18) for the HWBC, we conduct three experiments in the RMC laboratory:

- arbitrary motion of the end effector,
- pure vertical motion of the end effector,
- pick and place of the cage for the mobile robot.

In the first two experiments, the robotic arm motion is preprogrammed to follow a specific trajectory, while in the third one, it is controlled manually by the operator.

The SAM dynamics for the reduced state $\gamma$ is simulated onboard in the Luca Dynamics environment (subsection 2.1.4) which provides required task Jacobians $J_1$, $J_2$, and $J_3$, in real-time based on the model (5.8) with mass-inertia parameters presented in Table 6.7. In all experiments, the platform was manually stabilized before the start, and propellers were used only to keep the constant yaw angle without applying the oscillation damping. Since experiments are conducted in the laboratory, the system behaves as the single pendulum, so the initial manual

<table>
<thead>
<tr>
<th>Node</th>
<th>Mass [kg]</th>
<th>$I_{xx}$ [kg m$^2$]</th>
<th>$I_{yy}$ [kg m$^2$]</th>
<th>$I_{zz}$ [kg m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platform</td>
<td>30</td>
<td>4.2</td>
<td>4.2</td>
<td>8.4</td>
</tr>
<tr>
<td>Link 1</td>
<td>2.71</td>
<td>0.078</td>
<td>0.077</td>
<td>0.006</td>
</tr>
<tr>
<td>Link 2</td>
<td>2.71</td>
<td>0.031</td>
<td>0.005</td>
<td>0.031</td>
</tr>
<tr>
<td>Link 3</td>
<td>2.54</td>
<td>0.160</td>
<td>0.160</td>
<td>0.005</td>
</tr>
<tr>
<td>Link 4</td>
<td>2.50</td>
<td>0.033</td>
<td>0.005</td>
<td>0.028</td>
</tr>
<tr>
<td>Link 5</td>
<td>1.30</td>
<td>0.085</td>
<td>0.085</td>
<td>0.026</td>
</tr>
<tr>
<td>Link 6</td>
<td>1.57</td>
<td>0.004</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>Link 7</td>
<td>2</td>
<td>0.164</td>
<td>0.181</td>
<td>0.073</td>
</tr>
</tbody>
</table>

Table 6.7: Mass and inertia properties of the system utilized in the model.
6.5. Hierarchical impedance-based whole-body controller (HWBC)

<table>
<thead>
<tr>
<th>Controller</th>
<th>Gain</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first</td>
<td>$K_{Pe}$</td>
<td>230 · diag(1.2 1.2 1.7)</td>
</tr>
<tr>
<td>task</td>
<td>$K_{d,trans}$</td>
<td>20 · diag(1 1 1)</td>
</tr>
<tr>
<td></td>
<td>$K_{De}$</td>
<td>165 · diag(1 1 1)</td>
</tr>
<tr>
<td></td>
<td>$K_{d,rot}$</td>
<td>10 · diag(1 1 1)</td>
</tr>
<tr>
<td>The second</td>
<td>$K_{Pe}$</td>
<td>570 · diag(1 1 1)</td>
</tr>
<tr>
<td>task</td>
<td>$K_{De}$</td>
<td>560 · diag(1 1 1)</td>
</tr>
<tr>
<td>Admittance</td>
<td>$M_{adm}$</td>
<td>0.8 · diag(1 1 1)</td>
</tr>
<tr>
<td>interface</td>
<td>$D_{adm}$</td>
<td>1.6 · diag(1 1 1)</td>
</tr>
</tbody>
</table>

Table 6.8: Hierarchical impedance-based whole-body controller gains.

stabilization is enough to avoid significant oscillations during the experiments. The stiffness and damping matrices of the impedance controller (5.15) for two tasks as well as inertia and damping matrices of the admittance interface (5.16) are presented in Table 6.8. Here, $K_{d,trans}$ and $K_{d,rot}$ are submatrices of the damping matrix $K_{De}$.

It is worth noting that despite complete decoupling between control tasks guaranteed by the controller (5.18), the demonstrated performance is not expected to be perfect. There are several reasons for that:

- the controller relies on the accurate system dynamics model which is hard to obtain:
  - the real system might contain joint friction, backlash, and other uncertainties which are not considered in the model,
  - the real platform tilts which significantly affects the gravity term. Used model-based gravity compensation does not take this tilting into account,
  - the payload of the end effector brings additional uncertainty and should be added in the model as well, e.g., by utilizing FTS,
- the selected control gains are not optimal and can be further tuned for better performance,
- the impedance-based controller is a $PD$-based controller, so a steady-state
error might remain (increasing a translational stiffness and adding the intergral term should reduce such an error),

- the response of the system is slightly delayed due to use of the admittance interface,

- the intrinsic hardware contains computational error.

Nevertheless, reached performance should be enough to validate approach efficiency. Thus, the main performance criterion is fulfillment of both control tasks: regulating the end effector and nullifying the horizontal terms of the COM. Since the command to the end effector has the time-varied delay, only the value of the second task error (with constant desired value of zero) will be evaluated.

6.5.1 Arbitrary motion of the end effector

During the first experiment, the manipulator end effector moved along the preprogrammed trajectory which lay significantly beyond the common workspace of the SAM platform. The common workspace is located strictly below the platform, so any motion on the side of the SAM frame is considered as beyond the workspace. Such a motion should force the system to tilt due to displaced COM under the gravitational field. We conducted this experiment twice with switched winch-based actuation on and off in order to compare behavior. As a result, with operating winches, the end effector reached desired position while COM of the system remained under the suspension point keeping the platform horizontal. At the same time, disabling the winches led to the platform tilting, see Figure 6-24a.

As we can see, for the first control task, the proposed centralised whole-body controller with active winches demonstrated better performance for the translational motion and comparable performance for the rotational motion of the end effector, see Figure 6-25. It is worth mentioning that presented data, related to the robotic arm and platform translational motion, is extracted directly from the real-time model, which does not take into account the tilting of the platform due to gravity. So, the real performance of the system with switched winch-based actuation off is even worse for the first control task.

The second task, as we can see in Figure 6-26, is also fulfilled. With active winches, the platform kept horizontal orientation, while corresponding terms of the COM had close to zero values. Indeed, root mean square error (RMSE) for the
6.5. Hierarchical impedance-based whole-body controller (HWBC)

(a) Programmed motion of the end effector along the arbitrary trajectory
(b) Programmed motion of the end effector along the vertical axis

Figure 6-24: Illustration of the experiments: programmed motion of the end effector. Comparative behavior of the system following the same reference trajectory for the end effector with switched HWBC on (right) and off (left) is shown.

The second task based on the model data is 0.32 mm along $x$ axis and 0.35 mm along $y$ axis while maximum deviation from zero is 0.9 mm and 1.3 mm along $x$ and $y$ axes, respectively. So, the compensation of the gravitational torque was performed by the platform motion. When we switched HWBC off and repeated the experiment, the result was the opposite: the platform kept zero location while COM was moved. The data was collected with $\nu_{data} = 11.64 \text{ Hz}$.

### 6.5.2 Pure vertical motion of the end effector

Integration of the winch-based actuation brought additional redundancy to the manipulation. In order to reach desired position by the end effector, whole-body control distributes efforts among both, robotic arm and winch-actuated platform. To this
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Figure 6-25: Results of the experiment: programmed motion of the end effector along the arbitrary trajectory. First task: comparative behavior of the end effector motion programmed for the same trajectory with and without HWBC.

Figure 6-26: Results of the experiment: programmed motion of the end effector along the arbitrary trajectory. Second task: comparative behavior of the platform motion during programmed manipulator motion with and without HWBC.
end, robotic arm performs less motion than it would without winch contribution to reach the same point in the space. It allows to avoid singular configurations for the robotic arm in some directions, in particular, fully stretched configuration in the vertical direction.

To this end, the goal of the second experiment is to analyse changes in the manipulability of the robotic arm. Due to the fact, that usually the SAM performs operations below itself, the biggest manipulability degradation might be observed along the vertical motion. To this end, we performed preprogrammed jerky motion by cycling the position of the end effector up and down with switched winch-based actuation on and off, see Figure 6-24b. For both experiments, we calculated the manipulability ellipsoid for the arm based on (2.5).

As it can be seen in Figure 6-27, the system with active winches followed the commanded trajectory with less error, and a significant contribution was done by

![Graphs showing performance comparison](image)

Figure 6-27: Results of the experiment: programmed motion of the end effector along the vertical axis. First task: comparative behavior of the end effector motion programmed for the same trajectory with and without HWBC.
Chapter 6. Experimental investigation of the proposed control strategies

Figure 6-28: Results of the experiment: programmed motion of the end effector along the vertical axis. Second task: comparative behavior of the platform motion during programmed manipulator motion with and without HWBC.

Winch compensation Off—Winch compensation On

Figure 6-28: Results of the experiment: programmed motion of the end effector along the vertical axis. Second task: comparative behavior of the platform motion during programmed manipulator motion with and without HWBC.

It is interesting to notice, that due to internal robotic arm reconfiguration, the manipulability was growing for both cases, with switched winches on and off, see Figure 6-29. When system did not exploit the winches, the robotic arm moved significantly and had lower ellipsoid volume at the bottom configuration (when the arm is almost stretched, see the top picture in Figure 6-24b). At the same time at the top configuration, it had higher volume of the manipulability ellipsoid due to possibility to reconfigure itself in more ways. However, in this case the arm elbow was located in close proximity to the platform which might cause undesired collision.
6.5. Hierarchical impedance-based whole-body controller (HWBC)

Figure 6-29: Results of the experiment: programmed motion of the end effector along the vertical axis. Manipulability analysis: comparative robotic arm manipulability during programmed manipulator motion with and without HWBC versus Z end effector command. EV stands for the ellipsoid volume.

for certain end effector commands. The data was collected with $\nu_{\text{data}} = 12.55$ Hz.

6.5.3 Pick and place

In order to validate applicability of the total framework, the pick and place mission was conducted. The task was to pick and move the empty cage from the metallic case to the pipe. At the tip of the end effector, the 3D-printed hook was installed, that can pick up the special fixture attached to the cage. The snapshots of the mission are depicted in Figure 6-30. In this experiment, the end effector was controlled remotely by unprepared operator using joystick with force feedback. Such an approach is considered as real one for the system control, i.e., the shared control [Abbink et al., 2018, Coelho et al., 2021, Quere et al., 2020] when the operator is responsible for certain tasks, and the rest tasks are performed automatically. The data was collected with $\nu_{\text{data}} = 44.16$ Hz.

From the snapshots it can be seen that platform kept its own orientation close to the horizontal while height of the platform was adapting to assist the robotic arm: winches pulled the platform up and down in order to reduce stretching of the manipulator. In Figure 6-31, the commanded trajectory and real motion of the end effector is presented. The trajectory can be described by several steps:

- $t = 0-60$ s: initial positioning,
• $t = 60-80$ s: leveling the hook with the fixture at the cage,

• $t = 80-120$ s: preparing to pick up the cage keeping the hook parallel to the fixture,

• $t = 120-150$ s: picking the cage up and lifting it,

Figure 6-30: Illustration of the experiment: pick and place of the cage for the mobile robot. Snapshots demonstrate the process of picking the cage at metallic box and placing it at the pipe.

Figure 6-31: Results of the experiment: pick and place of the cage. First task: the translational motion of the end effector in accordance with operator joystick command.
6.5. Hierarchical impedance-based whole-body controller (HWBC)

- $t = 150-180$ s: the first attempt to put the cage on the pipe,
- $t = 190-200$ s: lifting the cage after unsuccessful attempt (the cage was placed with a wrong orientation with respect to the pipe),
- $t = 200-240$ s: preparing for the second attempt,
- $t = 240-260$ s: lowering the cage during the second attempt,
- $t = 260-290$ s: removing the hook from the fixture in horizontal plane while keeping $EE_z$,
- $t = 290-325$ s: bringing the hook below the platform center.

The second task performance is shown in Figure 6-32. As we can see, in real

![Figure 6-32: Results of the experiment: pick and place of the cage. Second task: the platform motion during joystick-controlled end effector motion. Admittance interface performance: motion of the platform in accordance with admittance interface command.](image-url)
mission with end effector control performed by operator, the HWBC fully provided a designed capabilities, i.e., the COM in horizontal plane was close to zero while the platform GC was shifted in the horizontal plane in order to compensate for the disturbing gravitational torque and in the vertical plane in order to assist the robotic arm motion. The RMSE for the second task is 0.13 mm along $x$ axis and 0.12 mm along $y$ axis while maximum deviation from zero is 0.85 mm and 0.77 mm along $x$ and $y$ axes, respectively. Also, Figure 6-32 demonstrates the behavior of the winch-based actuation in response to the input from the admittance interface. The motion of the platform was calculated by using the forward kinematics based on the logged cable lengths. We can see that during the real mission winch-based actuation provided a sufficient accuracy, i.e., good tracking performance with average delay of 0.39 s along $x_b$, 0.24 s along $y_b$, and 0.29 s along $z_b$ axis, by following the admittance interface command. The maximum delay for given admittance interface parameters reached 1.2 s. The obtained delay might be decreased by further tuning the admittance interface.

6.6 Integration of the OODC and the HWBC

In this section, we investigate the simultaneous operation of the designed two controllers, i.e., the OODC for the propeller-based actuation and the HWBC for the winch-based actuation and kinematically redundant robotic arm, for the stabilization of the SAM platform, as is depicted in Figure 3-29. To this end, two experiments are conducted in the outdoor environment:

- case of the robotic arm motion,
- platform null-space motion control.

In both experiments, the robotic arm and platform were controlled manually by the operator with a 2 DOF's joystick with force-feedback. It is worth discussing the mutual influence of the controllers. Since both actuations operate at the various bandwidths, there is no tangible mutual influence. Indeed, the winch-based actuation utilizes only slow robotic arm motion feedback while the propeller-based actuation uses IMU-based feedback. Additionally, it is worth mentioning that configuration of the robotic arm directly affects the overall system COM location, which ideally should be subject of the OODC. In our case, we consider COM displacement not significant, so OODC is constantly applied to the GC of the platform.
6.6. Integration of the OODC and the HWBC

6.6.1 Case of the robotic arm motion

In the first experiment, the operator manually controlled the trajectory of the robotic arm end effector while both controllers were switched on. Namely, following control tasks were performed: oscillation damping of the pendulum joints\footnote{It is worth mentioning that OODC affects the platform orientation indirectly through. The angular velocity data is omitted for the sake of brevity.}, compensation for the gravitational torque by utilizing the winches, and control of the end effector, see Figure 6-34. It can be seen from the plot, that the error in the Z-channel of the end effector motion was about 10 cm. This error is caused by additional payload at the end effector with weight (cage with dummy mobile inspection robot) of 6.5 kg, see Figure 6-33. This error might be reduced by increasing the stiffness gains in the Z-direction for the first control task of the HWBC or by adding the weight of the end effector payload to the model. The data was collected with a low average frequency $\nu_{data} = 1.5188$ Hz, so the logged signal are discontinuous (non-smooth). Moreover, the logging frequency was not constant, so some spikes might be observed.

6.6.2 Platform null-space control

High-redundancy of the system allows to integrate an additional control task while maintaining the high performance of the rest tasks, see subsection 5.3.2. For ex-
Chapter 6. Experimental investigation of the proposed control strategies

Figure 6-34: Results of the experiment: robotic arm motion for the case of simultaneous operating controllers. Top plot represents the joint velocities of the robotic arm. The next three plots present the orientation of the platform. The further 5 plots show the first and the second control task performance of the HWBC, i.e., the end effector position and horizontal displacement of the platform.
Spinning around yaw is easy to track by observing a footpad motion.

Figure 6-35: Illustration of the experiment: null-space control of the platform around the vertical axis. Snapshots demonstrate the change of the platform orientation around yaw while keeping the same pose of the robotic arm end effector. Purple dashed circle shows the particular footpad location.

ample, the platform itself might be controlled along the vertical axis, $z_p$ of the $\mathcal{F}_p$, while keeping end effector at the desired pose and zero gravitational torque. Such a capability might be useful in a number of cases, e.g., when we need to change a perspective of the onboard cameras or when we want to reduce the power consumption in the winch servos (longer rigging cable requires less torque at the motor for operation). To this end, we placed in the null-space two tasks: vertical motion of the platform and yaw motion of the platform. Both tasks are controlled in the same impedance-based manner (5.15) with control gains presented in Table 6.9.

In Figure 6-33 and Figure 6-35, the experimental scenario is shown. The SAM platform carried the cage with dummy mobile robot (the total weight of the end effector payload is 6.5 kg) by robotic arm. First, the operator manually commanded the platform to move up- and downwards while maintaining the rest configuration of the platform (the pose of the end effector is conservative as seen with respect to the shown part of the road). After that, the operator commanded to turn the platform around the yaw while keeping the rest configuration. Results of the experiment are depicted in Figure 6-36. As we can see in figure, while maintaining the end effector pose close to the original and keeping the system horizontal COM location close to zero, the platform was shifted first on 10 cm upwards and then on 16 cm downwards (red zone). After that, the platform was turned around the yaw for 31 degrees CCW, then for 48 degrees CW, and finally for 22 degrees CCW again (green zone).

It is worth mentioning that stiffness gains of the end effector were not high enough for such a heavy payload which was not declared in the model. As the result, in the $EE_z$ channel we can observe 8 cm difference between desired and real signal. The
Figure 6-36: Results of the experiment: null-space control of the platform. The first two plots represent the null-space motion of the platform along and around the vertical axis. The next two plots demonstrate the orientation of the platform around roll and pitch. The further 5 plots show the first and the second control task performance of the HWBC, i.e., the end effector position and horizontal displacement of the platform.
rest tasks demonstrated reasonable performance. Average frequency of the logging data was $\nu_{\text{data}} = 2.92$ Hz, so the logged signal is discontinuous (non-smooth).

### 6.7 Deployment of the foldable landing gear

As was mentioned in subsection 3.3.3, the three out of eight frame arms have the dual use. They serve as the frame arm during the operation, and as the landing legs

<table>
<thead>
<tr>
<th>Controller</th>
<th>Gain</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EE_z$</td>
<td>$K_{p,d}$</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>$K_{d,d}$</td>
<td>27</td>
</tr>
<tr>
<td>$Pl_{\text{yaw}}$</td>
<td>$K_{p,yaw}$</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>$K_{d,yaw}$</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 6.9: Null-space controller gains.

Figure 6-37: Illustration of the experiment: deployment of the foldable landing gear. Snapshots demonstrate the landed SAM ($t = 0$ s), the lifting of the passive SAM ($t = 13$ s), the start of the propeller spinning ($t = 16$ s), the lifting of the legs (to the arms) by the upward thrust while keeping the desired yaw utilizing the propellers located at the fixed 5 arms ($t = 30-42$ s), as soon as the legs are transformed to the arms and blocked by the locker mechanism the controller switches the oscillation damping (now shown) on, the lowering of the arms (to the legs) by gravity with damping by the thrust while keeping the constant yaw ($t = 42-58$ s), the passive SAM ready to be lowered by the crane ($t = 69$ s).
Table 6.10: State machine logic for the landing gear control.

during parking. During landing and taking-off, the legs are regulated by a finite-state machine which has two states for the locker mechanism: open and closed. At the open state, the leg can slide freely in the vertical plane, while at the closed state the locker mechanism fix the leg either in upper (arm) or bottom (leg) position. The snapshots demonstrating taking off and landing are shown in Figure 6-37. The logic behind the state machine is briefly depicted in Table 6.10.

Figure 6-38: State machine commands for the leg control.
During the taking-off, a transition of the leg to the arm is performed. In the beginning of motion \((t = 20 \text{ s})\), legs are at the open state, and slowly increasing thrust lifts them till the upper position. When the legs are fully lifted \((t = 52 \text{ s})\), the locker mechanism is switched to the closed state to fix the configuration, after which the thrust is reduced, see Figure 6-38a. During landing, the opposite procedure occurs with one additional step. Due to mechanical displacement of the locker groove and the servo shaft bar at the bottom position, after the main cycle a complementary command which aims to fit the shaft into the groove is exploited, see Figure 6-38b. At the bottom position, we turn the shaft of the locker mechanism to the closed state and fit it to the groove by shaking each leg with an amplitude around 5 degrees.

It is worth noting that time of the landing and taking off is strictly depending on the command and BLDC slew rate. For our settings, the taking off process takes approximately 60 seconds, while landing takes about 95 seconds.
"The reward of the young scientist is the emotional thrill of being the first person in the history of the world to see something or to understand something. Nothing can compare with that experience."

Cecilia Payne-Gaposchkin, 1977

Chapter 7

Field investigation in the industrial-like environment

As was highlighted in chapter 1, the cable-suspended aerial manipulation technology has a high practical potential to many industrial applications. The conducted research and engineering work on the development of the cable-suspension aerial manipulator was performed within H2020 AEROARMS project which the main goal is the integration of the academic research to the industrial applications.

To assess the developed technology performance, robustness, and operational safety, the experimental validation within the DLR Oberpfaffenhofen outdoor testing ground similar (mock-up) to the real industrial environment, described in subsection 6.1.4, is performed. Namely, three industrial-like experiments are conducted utilizing the SAM platform regulated by designed control strategies, i.e., OODC and HWBC:

- deployment of a mobile inspection robot at the remotely located pipeline,
- placing peg into a hole with a \(~1-2\) mm gap at the remotely located box,
- turning the valve at the remotely located spot.

Each experiment was preceded by a pre-flight check which included tests of the main operational blocks, i.e., sensors (IMU, cameras), actuation systems (propeller-based actuation, winch-based actuation, robotic arm), battery level, power and communication between on-board functional nodes and ground station.

It is essential to highlight that each experiment contains a free flight as well as the interaction with the environment. The presented results should demonstrate the
potential impact of the scientific advancements on the industry and the possibility of applying the cable-suspended aerial manipulator in commercial projects.

7.1 Deployment of the mobile inspection robot

7.1.1 Inspection crawler

Let us first introduce the mobile inspection robot that is exploited in the experimental session. Namely, the Tri-Wheel-Inspection-Crawler (TRIC), see Figure 7-1, from GE Inspection Robotics\(^1\) is utilized. Among its key features, it is worth highlighting the following:

- omnidirectional magnetic drives (three magnetic wheels) that allow the crawler to safely move along and around the convex and concave surfaces, i.e., outside and inside the pipe, see Figure 7-2,

- integrated electrical conductivity and ultrasonic testing sensors, visual inspection sensors, and wireless communication,

- ability to operate with the pipe diameter bigger than 300 mm,

- the robot can cross only the steps with a height of 0.5 mm,

- in operational mode, the robot moves approximately with the speed of \(25 \text{ mm/sec}\),

![Figure 7-1: Mobile inspection robot TRIC.](https://inspection-robotics.com/tric/)
• the weight of the crawler with an integrated battery is roughly 6.5 kg.

7.1.2 Deployment strategy

In order to provide safe delivery and retrieval of the mobile inspection robot TRIC to the remotely located pipe by the flying system, the special deployment cage is proposed. The deployment cage has the size roughly 2 mm bigger than the TRIC dimension in all horizontal directions. It contains a flexible rubber plate with integrated magnetic pads (for each wheel) to prevent the TRIC from falling out of the cage, i.e., to fix the robot during transportation. At the top of the cage, a fixture for the picking up by a tool, i.e., a hook, is attached, see Figure 7-3a.

The cage should be placed and stabilized on the top of the pipe during the deployment while the robot drives out of it. Considering the crawler speed mentioned

Figure 7-3: The crawler TRIC operates on the pipe (left) and simultaneously localizes itself (right).²

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²The snapshot is taken from the demo video on https://inspection-robotics.com/tric/.
above, it takes approximately 10 seconds for the robot to move in or out of the cage completely. So, during the 10 seconds, reliable interaction with the environment should be performed.

During the retrieval of the TRIC, the cage should be fixed in front of the robot. Indeed, when the TRIC drives on the rubber plate, it reduces the magnetic force between magnetic wheels and the pipe. In the case of the free-placed cage, the robot would just push it, see Figure 7-3b. In order to allow retrieval of the TRIC, a significant vertical force should be applied on the top of the cage, see Figure 7-3c. During the operation, this pressing should be performed by the robotic arm.

### 7.1.3 Mission description

We had to deploy the inspection crawler on the remotely-located pipe employing the SAM in the experimental mission. This experiment aimed to validate the cable-suspended aerial manipulator concept and was conducted at the early development stage (June 2019), so the oscillation damping controller for the single pendulum was utilized only (Appendix C) for the system stabilization.

#### Task details

The overall task is depicted in Figure 7-4a. Initially, the SAM is located in the
landing configuration at a start point. Then, the crane should lift it up to perform taking off. After that, the SAM should pick up the cage located close to the start point. Next, the crane should bring the platform with the cage to the target point, where the pipe is placed, close to the desired trajectory in order to overcome the obstacle on the way. Finally, the crawler should be deployed at the pipe. The retrieval of the crawler should be done in the opposite sequence.

The pipe is located on a platform roughly 8 meters above the ground and surrounded by different obstacles, i.e., a bar-based fence, a wall, see Figure 7-4b. Thus, the manipulation should be performed in a constrained narrow environment in close proximity to the obstacles. It is worth noting that to increase the difficulty of the mission, the most minor pipe diameter at which the crawler can operate was used.

**Ground station**

The ground station is deployed in the specially-equipped van, see Figure 7-5a. Mission operators, i.e., crawler operator, robotic arm operator, and perception oper-
7.1. Deployment of the mobile inspection robot

(a) Landed flying system  (b) Cage picking  (c) Cage lifting

Figure 7-6: The cage grasping.

ator, were conveniently engaging from it, see Figure 7-5b. Since the flight controllers function autonomously during the experiment, the flight control operator was outside with a red button ready to interrupt the mission in case of an emergency. Thus, control of the whole manipulation operation is performed with no direct sight to the manipulation area, i.e., pipe, or to the SAM platform, so all processes, including cage grasping, placement on the pipe, crawler deployment and retrieval, are conducted either autonomously or based on the sensor information. At the same time, the crane operator was outside as well and had direct visual feedback on the SAM transportation trajectory.

The robotic arm and crawler operators are provided with different tools for telemetry, haptic, audio, and visual feedback. The control station of the robotic arm teleoperator is equipped with multiple control panels for conveying the feedback information, see Figure 7-5c, including visual feedback from the monocular camera on the platform and 3D reconstructed visualization (virtual reality visualization) with depth information of the manipulation area [Lee et al., 2020] from the stereo camera attached to the end effector. Moreover, the force feedback is provided by the haptic input device (joystick). Therefore, the robotic operator could interact with the remote environment through the robot while receiving haptic information about it.

**Grasping the cage**

Initially, the SAM is located in the landed configuration, see Figure 7-6a. After lifting by the crane and transforming the landing legs to the frame arms (section 6.7), the SAM grasps the cage, which contains the crawler located close by. To this end, the SAM is moved by the crane so that the cage is conveniently placed inside the
Chapter 7. Field investigation in the industrial-like environment

(a) Lifting the SAM at the required height
(b) Overcoming the wall (top view)
(c) Overcoming the wall (side view)
(d) Lowering the SAM toward the pipe

Figure 7-7: The SAM transportation.

The onboard cameras detect the cage and estimate a relative pose of the cage with respect to the tool (the 3D printed hook). With this aid, the operator telemanipulates the tool and grasps the cage, during which the force feedback (weight of the cage with the crawler) also supports him to realize that the cage has been lifted off from the ground, see see Figure 7-6c.

Transportation to the target point

Once the manipulator has grasped the cage with the crawler, the next step is to transport the system to the target pipe location. To this end, the robotic arm has a fixed configuration during the transportation, so the platform actuation and crane perform the main work.

In the selected outdoor scenario, there are multiple obstacles that the SAM has to avoid until it reaches the target location. The crane is remotely controlled to avoid these obstacles, and in the end, the SAM platform is located above the target pipe, see Figure 7-7. It has to be mentioned that severe oscillations of the cage can result in the crawler falling off. Thus, the oscillation damping functionality of the SAM platform plays a major role during this period and cancels out even significant oscillations created during the crane motion.

It is worth noting that since oscillation damping controller for the single pendulum was utilized, the main motion during the transportation was performed by the crane, and the crane’s chain length during the whole period was kept short enough
7.1. Deployment of the mobile inspection robot

for excluding the double pendulum behavior. The chain length was only reduced in order to overcome the wall, see Figure 7-7c.

**Crawler deployment**

Deployment of the crawler requires precise placement of the cage on the pipe. To this end, the monocular camera detects the target pipe, and the orientation of the SAM is autonomously aligned to match it utilizing yaw control, see Figure 7-8b. The pipe’s tracking also helps visualize the relative position of the cage with respect to the pipe in the virtual reality environment presented to the operator. These two features support the operator to place the cage properly on the pipe to deploy the crawler safely. The crawler operator then remotely controls the mobile inspection robot to move out of the cage. During this period, the teleoperator pushes down the cage toward the pipe applying a vertical downward force (Figure 7-8c), so the crawler and the cage do not lose contact from the pipe, which might lead to the falling off of the crawler to the ground. Accurate force regulation for this contact maintenance is achieved because of the force feedback provided to the operator through the haptic joystick. Once the crawler is entirely out of the cage, the empty cage is lifted up, and the SAM goes back to the base station, see Figure 7-8d, while the crawler performs the pipe inspection, see Figure 7-9.

![Figure 7-8: The crawler deployment on the pipe.](image-url)
Crawler retrieval

As the crawler has completed the task, the empty cage is transported to the pipe by the SAM, see Figure 7-10a. The transportation of the SAM with an empty cage is similar to the described above. Once the cage is properly placed and pressed to the pipe, see Figure 7-10b, the crawler operator remotely controls it into the cage. During this action, he uses both: the front camera of the crawler as well as the monocular camera on the SAM platform to position and move the crawler.
properly into the cage. As the crawler is completely inside the cage, the robotic arm teleoperator pulls the cage up into the air, see Figure 7-10d.

The SAM landing

The SAM, along with the crawler, is taken back to the base station by the crane, where the teleoperator places the cage on the ground, see Figure 7-11a. Then, the landing process starts with reconfiguring the robotic arm from the operational to the parking position and lowering the legs, see Figure 7-11b. Finally, the crane puts the platform down till it touches the ground. After that, all propulsion units are switched off, see Figure 7-11c. It is worth noting that the crawler is supposed to inspect the pipe during the day, so it was extracted close to the night. It can be seen that at the end of the mission, the daylight is almost gone.

(a) Placing the cage with the crawler on the ground    (b) Lowering the legs    (c) Landed system

Figure 7-11: Landing of the SAM.

7.2 Peg-in-hole assembly

The peg-in-hole is a demonstrative robotic task that requires a physical interaction with an environment. Namely, this task implies placing the peg into the fixed hole by the robotic manipulator [Park et al., 2013]. The hole is slightly (1-2 mm) bigger than the peg dimensions, so the FTS sensor is utilized to "feel" the hole location by measuring the reaction wrench when the peg is touching the hole’s edges.
Chapter 7. Field investigation in the industrial-like environment

(a) Lowering the SAM toward the box  (b) Adjusting the distance between the platform and the box by null-space control of the platform along vertical axis

Figure 7-12: The setup for the peg-in-hole assembly.

In our setup, the peg tool and the box with the hole are manufactured from the wood, see Figure 7-12a. The SAM was hanged on the crane with a long chain, so both controllers for the platform, i.e., OODC and HWBC, were utilized to provide better performance and high safety. The custom-made 6 DOFs FTS was mounted at the end effector before the tool.

(a) Approaching the box with a peg  (b) Aligning the peg with a hole

(c) Inserting the peg into the hole  (d) Operation panel with camera feedback (left) and 3D visualization of the manipulation area (right) during the peg and hole aligning

Figure 7-13: The peg-in-hole assembly performed by the SAM.
7.3 Valve turn

In order to reduce the distance between the cage and the platform, winch-based actuation was utilized. Distance adjusting helped to avoid singularity in the robotic arm during the peg insertion. Although the crane could assist with this task, the regulation by winches allows the more precise motion, see Figure 7-12b. It is easy to observe the height adjusting by comparing the proximity of the SAM frame to the fence.

Peg insertion process is demonstrated by snapshots taken by GoPro camera, see Figure 7-13. It is worth mentioning that 3D visualization (Figure 7-13d) serves as an excellent assistant for the aligning peg and hole in all dimensions, see Figure 7-13b. As a result, due to the operation of the platform controllers, the base was almost motionless, and the peg was placed in the hole, see Figure 7-13c, smoothly.

7.3 Valve turn

The developed approach for the aerial manipulation has shown high efficiency in safety, performance, and reliability in the experiments above. Therefore, it was decided to apply the developed system to solve the real industrial problem in the inspection and maintenance. Namely, the turning of the remotely located ball-type valve with a lever handle should be performed by the SAM under the control of designed algorithms.

To this end, at the tip of the wooden peg, the wooden tip with a groove was additionally attached. The mentioned groove should be aligned with a valve lever
handle and then turned by the rotational motion of the robotic arm end effector while keeping its pose static. The standard valve was attached to a similar peg and placed in the wooden box hole. The general view of the setup is shown in Figure 7-14. Both controllers for the SAM platform, i.e., OODC and HWBC, were activated for this mission conduction.

The valve turning process is demonstrated by six snapshots taken by the GoPro camera, see Figure 7-15. As it can be seen, the valve was turned roughly for 90 degrees by the robotic arm end effector.

The robotic arm end effector pose during this mission is shown in Figure 7-16. Two vertical dashed green lines partition the whole process to the three zones:

(a) Initial valve configuration
(b) Approaching the valve by the groove of the tool
(c) Fitting the tool groove with a valve handle
(d) Turning the valve handle by rotation of the end effector for roughly 45 degrees
(e) Turning the valve handle by rotation of the end effector for 87.5 degrees
(e) Lifting the tool from the valve

Figure 7-15: The valve turn performed by the SAM.
7.3. Valve turn

Figure 7-16: Results of the valve turn mission. First task: the end effector pose manually commanded by the operator utilizing the joystick. Vertical green lines separate the period [19.9, 47] seconds during which the valve was turned.

1. [0, 19.9] seconds: lowering and aligning the tool groove with the valve handle,

2. [19.9, 47] seconds: turning the valve by changing the orientation of the end effector while keeping the end effector position,

3. [47, 74] seconds: lifting the tool from the valve handle.

It can be seen from the figure that besides the end effector yaw rotation for 87.5 degrees, from 69.5 to -18 degrees, there are roughly 5 degrees commands around the end effector roll and pitch. It might be explained by the tool frame orientation. The translational motion of the end effector during valve turn is almost constant.

The orientation of the platform and corresponding angular velocities are depicted in Figure 7-17. It can be seen that OODC well-performed its own duties by dampening oscillations while regulating the yaw toward the desired value. It is worth mentioning that a specific torque was applied to the valve handle by the robotic
Figure 7-17: Results of the valve turn mission. The platform roll, pitch, and yaw angles are depicted on the left, and corresponding angular velocities are shown on the right. Vertical green lines separate the period [19.9, 47] seconds during which the valve was turned.

arm end effector during the valve turn. Due to coupling between the manipulator and the platform, the latter reacted by turning around the yaw in the opposite direction for roughly 5 degrees. However, the yaw controller returned the platform back by utilizing the propeller-based actuation as soon as the valve turning was complete. Increasing \( PD \) gains and adding the integral term to the yaw control law would compensate for the aforementioned error. Moreover, it can be observed the static error of 1.5 degrees around the platform pitch, which normally should be compensated by HWBC. The error is caused by the unbalanced platform with non-symmetric weight distribution, which is not considered in the model.

Effect of the HWBC is demonstrated in Figure 7-18. It can be seen that despite the non-zero platform pitch angle, the integrated controller was regulating the system toward zero horizontal terms of the COM based on the symmetric platform
model (uniform cylinder). It was performed by shifting the platform along the admittance interface commands generated by HWBC utilizing the winch servos. It can be seen that in the vertical (Z) channel, the base kept its own height constantly, which means that the vertical COM projection was varying only due to the manipulator motion. Indeed, while the manipulator was lowered to the valve handle by 15 cm, the total COM was lowered for 3.7 cm. As soon as the manipulator was pulled up, the COM was raised as well.

The logging data was collected with the frequency of \( \nu_{data} = 20 \text{ Hz} \).
Chapter 8

Conclusion

In the last chapter, we provide a concise summary of the major results and contributions, main lessons learned during technology field investigation, existing limitations of the cable-suspended aerial manipulation concept as well as designed control strategies, and outlook on future work.

8.1 Summary

In today’s world, inspection and maintenance have become the subject of digitalization and robotization to make people’s work easier, safer, and more efficient. Robots serve as a tool to assist in tasks that include a high risk for human lives or a high probability of human error.

One such tool is the aerial manipulator. It is used to perform various operations that might require contact with the environment: high-altitude work (inspection, maintenance, repair), work in a hazardous (toxic) environment for people (collection soil samples), or transporting goods. The demand arises from the oil and gas, construction, chemicals, nuclear, and other industries, which imply strict requirements for the flying system’s safety and performance. To this end, the primary Research Question of this thesis was formulated as follows (chapter 1): how to enhance manipulation performance and operational safety for the aerial manipulation in industrial applications, e.g., valve turn, peg-in-hole assembly? The generated hypothesis that addressed the research question stated that an adequately controlled cable-suspended stabilizing platform equipped with a robotic arm can provide high performance and safety in aerial robotic manipulation.

In general, the control of an aerial manipulator with high precision is challenging
due to dynamic and static disturbances that affect the aerial base, such as wind, sensor signal noise, couplings with payload, as well as the COM displacement due to arm articulation or applied external forces from the environment. As a result, in contrast to the fixed-base manipulators, such systems can fluctuate in the air and are more sensitive to external factors. Therefore, the general control goal for the cable-suspended stabilizing platform in the scope of this thesis can be formulated as follows: the cable-suspended aerial system should keep the state of the platform close to the motionless to facilitate the manipulation process. In other words, we have to ensure a quasi-fixed base for the robotic arm to perform manipulation tasks precisely.

To dissect and test the aforementioned hypothesis as well as to satisfy the general control goal, three research objectives were defined:

RO1 To develop a robust control approach that can compensate for dynamic disturbances during transportation and manipulation of the cable-suspended platform without access to the system entire state and characterize its performance.

RO2 To develop a control approach to facilitate a manipulation performance of cable-suspended platform endowed with kinematically redundant robotic manipulator by compensation for static disturbances and evaluate its performance.

RO3 To assess manipulation performance and operational safety of the cable-suspended stabilizing platform through extensive experimental studies in a similar to the industrial environment.

In the following, a brief overview of each chapter contribution with respect to the defined research objectives is described.

The cable-suspended aerial manipulation is a sophisticated concept that is weakly covered by the research and engineering community. To this end, first of all, the design ideas and developments of the particular experimental demonstrator, a novel cable-suspended aerial manipulator SAM, were highlighted in chapter 3. The SAM is equipped with two actuations: fully-actuated propeller-based actuation and winch-based actuation, for stabilization and enhancing the manipulation performance. The first actuation contains eight propellers and can generate an omnidirectional wrench for the compensation of dynamic disturbances. The second one includes three winches, each of which can control the length of the rigging cable
connecting the suspension point, i.e., the hook’s frame and the flying platform. It allows controlling the platform displacement relative to the winch suspension point. Therefore, slow winches with integrated breaks compensate for the static deviations, e.g., the COM displacement. As the main carrier of the cable-suspended aerial system, the mobile crane is proposed. The carrier supports the weight of the SAM, and as a result, actuation systems should not compensate for the gravity. Consequently, the platform can have reasonable dimensions that allow working in a complex narrow environment. Additionally, it is worth highlighting the transformable landing gear that allows the SAM to have a larger workspace for manipulation without any restrictions created by the standard landing gear, e.g., skids. The main engineering contributions performed by the author in the scope of this chapter include formulation of the technical requirements for the actuation systems via conducting simulational studies, thrust identification of the propeller-based actuation, analysis of the force and torque envelopes produced by the propeller-based actuation, development of the CANopen stack for QNX Neutrino RtOS for communication of the central computer (FCC) with winch-based actuation, and integration of the real-time control framework with target hardware.

Investigation of the obtained concept of the cable-suspended aerial manipulator has brought into focus the need to develop the appropriate control strategies. Due to the cable suspension, the system exerted the manipulation-disturbing pendulum-like behavior. Therefore, the primary control goal deeply investigated and tackled in chapter 4 was set as the oscillation damping. Namely, the Optimal Oscillation Damping Controller (OODC) was designed, applied, and validated in simulation studies. This contribution is entirely devoted to the RO1. First of all, the system behavior was analyzed in detail. It turned out that the flying platform suspended to the crane’s hook and constrained utilizing the suspension crane’s chain evolves according to the spherical double pendulum model with a strong dependence on the pendulum weight-geometric parameters, including the hook weight and length of the chain of the crane. The most crucial challenge was the absence of information about the full state of the system since among sensors for the task, only the single onboard IMU was implied as available. Moreover, the pendulum can be controlled only indirectly by generating a damping wrench at the tip of the second link by the propeller-based actuation. As a solution for such a task formulation, the OODC was proposed and designed. The controller is based on constructing an estimator of the system angular velocities and minimizing linear-quadratic cost function that balances high system performance and low power consumption. The resulting controller
dissipates the oscillation with the desired balance between performance and power consumption. It is robust to the task because it does not require model parameters and utilizes only a reliable IMU sensor. Moreover, by virtue of the optimal gains, we can easily tune the gains for different operating conditions. Additionally, the global stability of the closed-loop system under the designed controller was shown.

Chapter 5 addressed the modeling and control of the winch-based actuation system in pair with the redundant robotic arm. The winch cabling suspension was modeled as a closed-chain dynamics and further transformed to the equal serial-chain dynamics, which is instantaneously feasible w. r. t. the defined holonomic constraints. This transformation allowed to access direct translational motion of the platform instead of the rigging cable length and to formulate two intuitive control tasks. The first one is to keep the end effector in the desired pose. The second one is to shift the system COM to compensate for the gravity-generated disturbing torque caused by the robotic arm weight. Thus, the vector connecting the suspension point and the overall COM of the platform (including the manipulator) is aligned with the gravity vector regardless of the robotic arm configuration, and the total weight does not generate the disturbing gravitational torque. In order to regulate both tasks, the Hierarchical impedance-based Whole-Body Controller (HWBC) was utilized. Based on the real-time model, it generates control torques for the robotic arm and platform. The controlled torque is directly used for the robotic arm and transformed through the admittance interface for the position-controlled winches. The selected impedance-based strategy provides the ability to interact with the environment compliantly and contributes to additional safety and effectiveness during the task execution. Thus, the central contribution of this chapter, modeling the complex constrained system dynamics with the following model transforming for the hierarchical whole-body controller, captures the RO2.

Extensive experimental validation and investigation of the aforementioned theoretical contributions was performed in the chapter 6. As the demonstrator system, the SAM presented in chapter 3 was utilized. Experiments were conducted in various conditions, including specially equipped indoor and outdoor environments exploiting the crane as the carrier. We started with a frequency-based model validation of the cable-suspended aerial manipulator to confirm the double pendulum behavior assumed in the control design. Further, OODC was tested in the various experiments, including compensation for external disturbances, robotic arm motion, and suspension point motion, i.e., the crane. The controller demonstrated the ability
to compensate for the strong oscillations within 7-8 seconds for selected gains. Additionally, the robustness of the tuned controller during a varying the chain length was studied.

To the best of author knowledge, in this thesis it was the first time when the winch-based cable-driven suspension was utilized for the aerial manipulation. So, analysis of benefits and performance validation for winch-based actuation as well as adapted HWBC were studied through a set of experiments. Driven by the controller, the system effectively maintained a horizontal orientation keeping horizontal terms of the COM at zero while the robotic arm performed various operations. Namely, the arm was commanded to move in the worst possible configuration within the accessible workspace that affects the COM location. According to the controller model data, the COM was kept close to zero with RMSE of 0.32 mm along $x$ axis and 0.35 mm along $y$ axis while maximum deviation from zero were 0.9 mm and 1.3 mm along $x$ and $y$ axes, respectively. Received results showed the applicability of the proposed actuation system as well as applied control framework to the real scenarios for implementing complex aerial manipulation tasks. At the end of the 6-th chapter, the workability of the SAM driven by two controllers, OODC and HWBC, simultaneously was validated.

The proposed concept of cable-suspended aerial manipulation contributes to state of the art in aerial manipulation field. The three experimental missions in the industrial-like environment were conducted in chapter 7 in order to emphasize the practical importance and viable commercial application of the proposed concept as well as designed control algorithms in a complex high-altitude environment. The first mission was devoted to the deployment of the heavy (6.5 kg) mobile inspection robot at the remotely-located pipe. The second mission demonstrated the assembly capabilities of the concept by performing the peg-in-hole assembly. Finally, the real industrial task, turning a valve with the lever handle, was performed. All missions were successfully completed by the operator without direct visual contact at the manipulation area, demonstrating the high safety and performance of the concept and the designed controllers. Received successful results within this and previous chapter allowed to reach the RO3.
8.2 Lessons learned

Based on the field investigation of the cable-suspended aerial manipulator performance, we came up with certain lessons about the applicability of this kind of technology in commercial tasks.

The suspension of the aerial platform helps to perform the manipulation tasks in terms of safety and performance. Indeed, the compact size allowed to operate in a narrow complex environment with obstacles without any collisions. Moreover, the platform could be fastly stabilized by the OODC so that the manipulation could be performed with precise positioning of the end effector tool. The oscillation damping is essential to perform a manipulation task using a suspended aerial system. It decreased the operational time for the operator to conduct missions. It is worth highlighting that the robust yaw controller is critical during the mission since manipulation with a spinning platform is hardly possible.

The HWBC allowed to keep the platform horizontal during the mission regardless of the robotic arm configuration. First of all, it made the manual manipulation process more convenient for the operator. Secondly, it helped to avoid the data jumps in the onboard sensors and perception system, including visual feedback for the operator. Thus, dual actuation confronted almost all types of disturbances that, in the end, made the operation more convenient, safe, and fast.

The required manipulation efforts in the industrial-like environment make the completely automated solution hardly possible for the considered tasks due to the presence of sudden disturbances in the outdoor environment (e.g., the wind) as well as due to the lack of high robustness of the sensors and perception system. The sharing control functionality that implies the use of both operator control and automation components makes the whole system safe, robust, and versatile. Moreover, the gained experience with aerial manipulation has shown that redundancy in the manipulator (in our case, it has 7 DOF) helps to avoid the singular configuration of the robotic arm and makes the operation conduction easier.

Exploiting the FTS attached to the end effector allowed to perform the interaction tasks with the environment more reliably, safely, and faster. Thus, during the deployment of the crawler, it was possible to press the cage toward the pipe with the required vertical force. At the same time, due to the FTS, it was possible to align the peg with a hole and feel the turning of the valve through the haptic feedback. It makes the operator experience similar to the case in which he performs the mission
personally by hand.

The performance of the cable-suspended aerial manipulation concept was validated with a mobile crane. The crane is a widely accessible platform for the industrial application areas, e.g., oil refinery or nuclear plant. The crane as the carrier has the following advantages:

- safe operation of the cable-suspended aerial manipulator: collisions with obstacles could be easily avoided, missions could be performed in narrow environments,
- high payload,
- wide availability of the system,
- no flight permissions is required,
- easier compliance with the safety regulations of the production plants.

8.3 Limitations

Despite the high efficiency and robustness of the investigated concept, the cable-suspension aerial manipulation has its own technological limitations. The main drawback of the concept includes an inability to operate above the platform. It restricts the use of the system in some applications, e.g., an inspection of the bridge bottom. Moreover, the operation of the presented platform SAM is not possible in the presence of copious dust or water drops since there is no water- and dust-proof integrated capabilities that might be required in the industrial environment.

Designed OODC does not consider the vertical crane jib oscillation, which might be induced by the sudden drop of the heavy payload or harsh environment contact. Also, the OODC is robust but sensitive to the cable chain length, which is unknown in general. So, some additional feedback from the crane to the aerial manipulator is desirable.

The performance of the HWBC is highly dependable on the system model precision. It means that each specific flying system should be manually adapted to the controller before start. In particular, the weight distribution within the platform and robotic arm dynamics should be well-known for calculating the gravitational torque precisely. Moreover, the feedback from the Force Torque Sensor (FTS) is required to
adapt the model during the interaction with the environment. Additionally, the efficient gain tuning rule for all control nodes in the scope of the sophisticated controller is required for better performance.

Finally, an optimization of the energy consumption by both actuation systems is required in order to keep the balance between high performance and battery level.

### 8.4 Future work

In the scope of one thesis, it is impossible to cover all aspects of the topic, so some of them will be addressed in future work.

Concerning the applications, deeper interactivity and immersion of the robotic arm operator into the operational environment would facilitate the mission execution. To this end, the virtual 3D environment that the operator uses at the display could be integrated into the virtual reality headset (glasses) to feel himself right in the operational spot while being in the office.

Moreover, for many industrial tasks, e.g., turning a gate valve with a circular handle, installing the nut while keeping the bolt, or other assembly operations where two components need to be controlled separately, two hands are desired. Therefore, the exploitation of two robotic arms on one platform provides human dexterity for complex tasks, especially for high-precision applications. So, the dexterous manipulation capabilities should be integrated and investigated. It is worth noting that due to the high payload capacity of the crane, the number of arms would not significantly affect the platform composition and dimensions.

In terms of the performance, a reliable and robust perception system is required. It would allow access to the system full state in real-time and perform not just more efficient oscillation damping but also the horizontal motion, whole-body control for all available DOFs, and other elaborated control strategies. It could be achieved by fusing the various sensor information, including onboard GPS, cameras, IMU, LiDAR. Moreover, it might help to avoid the necessity of getting any feedback from the crane carrier, i.e., chain length.

Additionally, it is worth investigating the redundancy of the propeller-based actuation. It contains eight propellers, while only seven are required for omnidirectionality. Thus, extra DOF could be used for fault tolerance or efficient energy consumption strategies. The energy cost estimation should be also performed for the winch-based actuation.
Furthermore, the performance might be increased on the actuator level by integrating the motor speed feedback from the ESC telemetry to the BLDC. It would allow keeping the motor thrust tolerant to the battery voltage level and motor characteristics, and, consequently, the performance of the proposed control schemes.

Finally, integration of the sensors for the winch cable tension measurement would allow the control of the whole system on the force-torque level for more quick and robust adaptation to any changes in the environment.
Appendices
Appendix A

Geometric yaw controller

The cabling of the suspended aerial manipulator does not put any constraints on the yaw motion. To this end, the control of the yaw channel might be performed in the same way as for the free rigid body. The rotational dynamics and kinematics of the rigid body, see subsubsection 2.1.1.3, can be described as follows:

\[ Y \dot{\omega} + \omega \times Y \omega = u, \]
\[ \dot{R} = R\dot{\omega}. \]

Let us define the rotational angle and velocity error as follows:

\[ e_R = \frac{1}{2}(R_d^T R - R^T R_d)^\vee, \]
\[ e_\omega = \omega - R^T R_d\dot{\omega}_d. \]

Then, the control law for the orientation of the free rigid body can be formulated as follows:

\[
\begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix} = -k_R e_R - k_\omega e_\omega + \omega \times Y \omega. \tag{A.1}
\]

Here, \( k_R \in \mathbb{R}^{3\times3} \) and \( k_\omega \in \mathbb{R}^{3\times3} \) are diagonal positive matrices of the proportional and derivative control gains, respectively.

The last term of the vector \( u \) can be used for the yaw control of the suspended aerial manipulator.
Appendix B

Spherical double pendulum modeling

By excluding yaw motion from the dynamic equations of motion for the spherical double pendulum, the state of generalized coordinates has the following form: \( \tilde{\mathbf{q}} = [q_1, q_2, q_3, q_4]^T \), where \( q_1 \) and \( q_2 \) are the roll and pitch angles of the hook motion in the world frame, \( q_3 \) and \( q_4 \) are the roll and pitch angles of the suspended aerial platform with respect to the frame attached to the hook and with axes parallel to the world frame.

Then, the location of the hook and platform in the inertial frame is defined as follows:

\[
\mathbf{r}_1 = \begin{bmatrix} x_h \\ y_h \\ z_h \end{bmatrix} = R_y(q_2)R_x(q_1) \begin{bmatrix} 0 \\ 0 \\ -l_1 \end{bmatrix}, \tag{B.1}
\]

\[
\mathbf{r}_2 = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} x_h \\ y_h \\ z_h \end{bmatrix} + R_y(q_4)R_x(q_3) \begin{bmatrix} 0 \\ 0 \\ -l_2 \end{bmatrix}. \tag{B.2}
\]

By taking derivative of (B.1) and (B.2) with respect to time, we can write down:

\[
\dot{\mathbf{r}}_1 = \mathbf{J}_1 \ddot{\mathbf{q}},
\]

\[
\dot{\mathbf{r}}_2 = \mathbf{J}_2 \ddot{\mathbf{q}}.
\]

Then, the inertia tensor can be formulated as follows:

\[
\mathbf{\ddot{M}} = m_1 \mathbf{J}_1^T \mathbf{J}_1 + m_2 \mathbf{J}_2^T \mathbf{J}_2.
\]
Appendix C

Oscillation damping controller for the single pendulum

The coordinates of the platform in the case of the single pendulum model are:

\[
r_p = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = R_w^p \begin{bmatrix} 0 \\ 0 \\ -l \end{bmatrix},
\]

where \( l = l_1 + l_2 \). Furthermore, the platform linear body velocities \( v_x^b \) and \( v_y^b \) are related with body angular velocities \( \omega_b = [p, q, r]^T \) as follows:

\[
v_p = \begin{bmatrix} v_x^b \\ v_y^b \\ 0 \end{bmatrix} = \begin{bmatrix} -lq \\ lp \\ 0 \end{bmatrix}.
\]

Then, the damping force of the body wrench might have the proportional-derivative (PD) form:

\[
f_b = \begin{bmatrix} f_{bx} \\ f_{by} \\ f_{bz} \end{bmatrix} = -R_w^b K_p r_p - K_v v_p. \tag{C.1}
\]

Here, \( R_w^b = (R_w^w)^{-1} \), \( K_p \in \mathbb{R}^{3 \times 3} \) and \( K_v \in \mathbb{R}^{3 \times 3} \) are positive diagonal gain matrices. As a result, a generated wrench should be composed of \( f_{bx} \), \( f_{by} \), and \( \tau_z \), with nullified rest components. Here, \( \tau_z \) is the control torque around the yaw axis received in Appendix A.
Appendix D

Derivation of matrices $A$, $S$, $B$, and $O$ for the HWBC

Matrix $A$

Projecting the equation (5.2) on the world frame axes, we receive the following:

\begin{align*}
Ox & : q_4 \cos(q_3) + q_7 \cos(q_6) = L \cos(q_5), \\
Oy & : q_4 \sin(q_3) + q_7 \sin(q_6) = L \sin(q_5).
\end{align*}

Here, $L = BE$ is the diameter of the platform base.

In order to obtain the matrix $A \in \mathbb{R}^{2 \times 6 + m}$, let us take a time derivatives of (D.1) and present it in the matrix form:

\[
\begin{bmatrix}
0 & 0 \\
-q_4 \sin q_3 & q_4 \cos q_3 \\
\cos q_3 & \sin q_3 \\
L \sin(q_5) & -L \cos(q_5) \\
-q_7 \sin(q_6) & q_7 \cos(q_6) \\
\cos(q_6) & \sin(q_6) \\
0_m & 0_m
\end{bmatrix}^T \dot{q} = 0.
\]

Matrix $S$

Let us split the received matrix $A$ as: $A = [A_δ, A_z]^T$. The first term $A_δ$ includes the components related to the vector of independent generalized coordinates $δ =
Appendix D. Derivation of matrices $A$, $S$, $B$, and $O$ for the HWBC

$[q_1, q_4, q_5, q_7, q_m]^T \in \mathbb{R}^{4+m}$, and the second one - dependent generalized coordinates $z = [q_3, q_6]^T \in \mathbb{R}^2$. As it was mentioned in (5.4), the matrix $S$ should comply with $S^T(q)A^T(q) = 0$. To this end, let us define it as $S = [I_{(4+m)\times(4+m)}, -A_z^{-1}A_\delta]^T \in \mathbb{R}^{(6+m)\times(4+m)}$ [My and Hoan, 2019], then:

$$
S^T(q)A^T(q) = \left[I_{(4+m)\times(4+m)} - A_\delta^T A_z^{-T}\right] \begin{bmatrix} A_\delta^T \\ A_z^T \end{bmatrix} = A_\delta^T - A_\delta^T A_z^{-T} A_z^T = 0.
$$

Matrix B

Projecting the equation (5.2) on the world frame axes, we can receive the forward kinematics for the point $C$ using two branches of the closed chain.

For the first branch:

$$
x_c = l_1 \sin(q_1) - q_4 \cos(q_3) + \frac{L}{2} \cos(q_5),
$$

$$
y_c = -l_1 \cos(q_1) - q_4 \sin(q_3) + \frac{L}{2} \sin(q_5).
$$

For the second branch:

$$
x_c = l_1 \sin(q_1) + q_7 \cos(q_6) - \frac{L}{2} \cos(q_5),
$$

$$
y_c = -l_1 \cos(q_1) + q_7 \sin(q_6) - \frac{L}{2} \sin(q_5).
$$

By squaring and adding the equations (D.2) and (D.3), we can express IK parameters with excluded $q_3$ and $q_6$ as follows:

$$
q_4 = \sqrt{\left(\frac{L}{2} \cos(q_5) + l_1 \sin(q_1) - x_c\right)^2 + \left(\frac{L}{2} \sin(q_5) - l_1 \cos(q_1) - y_c\right)^2},
$$

$$
q_7 = \sqrt{\left(\frac{L}{2} \cos(q_5) - l_1 \sin(q_1) + x_c\right)^2 + \left(\frac{L}{2} \sin(q_5) + l_1 \cos(q_1) + y_c\right)^2}.
$$

Taking derivatives of the (D.4) along the $q_p = [x_c, q_5, y_c]^T$, we can receive differential kinematics connecting the vector of independent generalized coordinates, $\delta$, and quasi-state vector of feasible motions, $\eta$, i.e.,[Jafari Harandi et al., 2019]:

$$
\dot{\delta} = B\dot{\eta}.
$$

1Splitting the matrix $A$ in shown way requires reorder of variables in the state as $[\delta, z]^T$. 187
Here, analytical Jacobian matrix $B$ can be expressed as:

$$
B = \begin{bmatrix}
I_{1 \times 1} & 0_{1 \times 3} & 0_{1 \times m} \\
0_{3 \times 1} & J_p & 0_{3 \times m} \\
0_{m \times 1} & 0_{m \times 3} & I_{m \times m}
\end{bmatrix},
$$

where $J_p$ includes mapping received from differentiating (D.4):

$$
J_T^p = \begin{bmatrix}
x \frac{L}{2} \cos(q_5) - l_1 \sin(q_1) & 0 & x \frac{L}{2} \cos(q_5) - l_1 \sin(q_1) \\
\frac{q_1}{2q_4} \frac{L(l_1 \cos(q_1 - q_5) - \sin(q_5)x_c + \cos(q_5)y_c)}{2q_4} & \frac{q_7}{2q_7} & \frac{L(l_1 \cos(q_1 - q_5) - \sin(q_5)x_c + \cos(q_5)y_c)}{2q_7} \\
y \frac{L}{2} \sin(q_5) + l_1 \cos(q_1) & 0 & y \frac{L}{2} \sin(q_5) + l_1 \cos(q_1) \\
\frac{q_4}{q_4} & 0 & \frac{q_7}{q_7}
\end{bmatrix}.
$$

In the case, when $q_1 = 0$, (D.5) can be rewritten as follows:

$$
J_T^p (q_1 = 0) = \begin{bmatrix}
x \frac{L}{2} \cos(q_5) & 0 & x \frac{L}{2} \cos(q_5) \\
\frac{q_1}{2q_4} \frac{L(l_1 \cos(q_5) - \sin(q_5)x_c + \cos(q_5)y_c)}{2q_4} & 1 & \frac{q_7}{2q_7} \frac{L(l_1 \cos(q_5) - \sin(q_5)x_c + \cos(q_5)y_c)}{2q_7} \\
y \frac{L}{2} \sin(q_5) + l_1 \cos(q_1) & 0 & y \frac{L}{2} \sin(q_5) + l_1 \cos(q_1) \\
\frac{q_4}{q_4} & 0 & \frac{q_7}{q_7}
\end{bmatrix}.
$$

**Matrix O**

To relate actuated torques $\tilde{\tau}_a$ with generalized torques $\tilde{\tau}$ of the system (5.6), let us define the Jacobian matrix $Y \in \mathbb{R}^{(3+m)\times(4+m)}$ which equals to the $B$ without the row corresponding to the non-controllable coordinate, i.e., $q_5$. Then, relation can be formulated as follows:
here $\mathbf{Y} \in \mathbb{R}^{(3+m) \times (3+m)}$ is Jacobian $\mathbf{Y}$ without column corresponding to the DOF that we do not aim to control, i.e., $q_5$. It is worth noting that $\mathbf{O}$ maps $\tau_{x_c}$ and $\tau_{y_c}$ without scaling and does not affect the $\mathbf{q}_1$ and $\mathbf{q}_m$ at all. Resulted $\tau_{q_5}$ in this case contains weighted sum of $\tau_{x_c}$ and $\tau_{y_c}$.


