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Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Aleksey Lunkin

PhD Program: Physics

Title of Thesis: Sachdev-Ye-Kitaev model in the presence of the quadratic perturbation

Supervisor: Assistant Professor Konstantin Tikhonov

Name of the Reviewer: Pavel Ostrovsky

I confirm the absence of any conflict of interest	**Date: 27-05-2022**

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

The PhD project by Aleksey Lunkin is devoted to the Sachdev-Ye-Kitaev (SYK) model — a zero-dimensional model of randomly interacting Majorana fermions. SYK model attracts a great theoretical attention in recent years since this is one of a very few examples of a solvable strongly interacting model that allows to study correlation effects far beyond the standard Fermi liquid theory. In the work by Lunkin, the solvable dispersionless model is augmented with the quadratic (in fermionic operators) perturbation, which allows to consider crossover phenomena between the strongly correlated regime and conventional Fermi liquid. In addition, finite temperature effects are taken into account and the complete two-parameter phase diagram of the system is developed.

The theses consist of Introduction, three main chapters, Conclusion, and two Appendices. The first chapter contains a general review of the unperturbed SYK model and its solution. This chapter serves introductory purposes and is based on the results from the works by Kitaev and by Bagrets, Altland, and Kamenev. The model is reformulated in the language of a path integral and then after certain transformations is reduced to the effective sigma-model action for a time-dependent reparametrization function. This effective action is based on the presence of a large parameter in the model — the total number of fermionic states. The Green functions of the original fermions are generated by a nonlocal in time source fields. To overcome this nonlocality problem, the action is further transformed by introducing an additional integration variable into an effective quantum-mechanical model with exponential potential — the Liouville quantum mechanics. This representation allows to construct explicit integrals for average product of the Green functions.

In the second chapter, the Hamiltonian of the model is extended by a random quadratic term. This introduces a finite dispersion of the fermions and allows to study the crossover from the Fermi-liquid to the strongly correlated regime. Quadratic term in the Hamiltonian is taken into account as a small perturbation that leads to corrections to the long-time asymptotics of the average Green function. It was found that an SYK to Fermi liquid phase transition occurs when the quadratic perturbation exceeds a certain threshold value. In the language of the effective Liouville quantum mechanics, this phase transition is related to the appearance of a bound state in the Hamiltonian.

The same problem with quadratic perturbation is generalized to the case of finite temperatures in the third chapter. The Liouville quantum mechanics is rederived in this case in a more general form and then is treated with the help of adiabatic approximation. It is shown that with increasing temperature, the effect of quadratic perturbation is suppressed. As a result, the perturbed SYK model exhibits a crossover from the Fermi-liquid regime to the strongly correlated behavior at higher temperatures. The complete phase diagram of the system is constructed showing the boundaries between Fermi-liquid, correlated, and fluctuating regimes of the model.

The work of Aleksey Lunkin is based on two publications in Physical Review Letters and has a very high scientific value. The subject of the work — SYK model — is a hot topic of the contemporary theoretical physics. The obtained results are important contributions in the field that are recognized and acknowledged by the community. The work is accomplished with the help of a variety of modern theoretical approaches including diagrammatic technique, functional integration, geometrical analysis. Aleksey Lunkin has demonstrated deep understanding of the underlying physical processes by using and developing relevant approximations (such as perturbation theories in different parameters and adiabatic approximations) to study all possible limiting cases of the model. The validity of the results is beyond any doubts since they were presented at several scientific conferences and seminars, published in the leading peer-reviewed physical journal, and are actively cited.

At the same time, the work could be improved further if the author addresses the following remarks:

1. The exact solution of the original SYK model provides average products of the Green functions. For the perturbed model studied in the theses, one averaged Green function is considered. It would be easier for the reader to understand the physical consequences of the perturbation and the very essence of the terms "Fermi-liquid" vs. "strongly correlated" regime, if some physically observable quantities have been also discussed. So far it is a bit unintuitive to judge the properties of the model solely by the asymptotics of its Green function.

2. In solving the SYK model, disorder average of the partition function is performed instead of averaging the free energy. This is motivated by assuming the temperature is above the glass transition. It seems appropriate to provide some further explanations of this point and also to give an estimate of the lower temperature bound. What kind of effects should be expected below the glass transition temperature?

These critical statements by no means diminish the scientific value of the work and do not question the validity of the obtained results. The presented work does conform to all the requirements for the PhD theses and Aleksey Lunkin definitely deserves the PhD title with the highest evaluation.

Provisional Recommendation

☒ *I recommend that the candidate should defend the thesis by means of a formal thesis defense*

☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*

☐ *The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*