Thesis Changes Log

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PhD Program: Petroleum Engineering

Title of Thesis: Asymptotic models of coupled geomechanics/fluid mechanics phenomena of hydraulic fracture growth

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The thesis document includes the following changes in answer to the external review process.

Dimitry Chuprakov

1. For derivation of pressure dependent leak-off in a near tip region author uses 1D diffusion equation. How 1D flow in a near-tip region can be justified, which, seems, must exhibit 2D flow more likely?

Section 3.2.4.4 of the present thesis is devoted to the discussion of the applicability limits of 1D mechanism. Here, we say, referring to paper [1], that 1D approach can be used in the "large velocity limit" that in terms of physical parameters is written as $\lambda \gg \ell_d = c/V$, where λ is the size of the circulation zone, ℓ_d is the diffusion lengthscale ($\ell_d \sim \sqrt{ct}$ and $t \sim \lambda/V$), c is the diffusivity coefficient, and V is the fracture tip propagation velocity. In other words, 1D mechanism is approximately valid when the pore pressure perturbation introduced by a hydraulic fracture is located inside the boundary layer of size ℓ_d that is thin compared to the characteristic lengthscale of fluid pressure change along the part of the fracture where the fluid exchange process is important, i.e., λ. Using the maps provided by Figure 3-14, one can identify where the proposed fracture tip solution is valid. E.g., when $\chi/\zeta^3 = 0.1$ the condition $\lambda \gg \ell_d$ is satisfied for all analyzed χ. Similarly, in the case of $\chi/\zeta^3 = 1$ and $\chi/\zeta^3 = 10$, 1D approach is correct for $\chi < 0.1$ and $\chi < 5$, respectively, where $\chi = Q'\sigma_o' E'/(K'\sqrt{V})$, $\zeta = E'\left(M'Q'\sqrt{V}\right)^{1/3}/K'$. Moreover, using table 3.5, one can compare dimensionless length of the circulation zone with $\ell_d \approx 0.01\zeta^6$ and find out that the condition $\lambda \gg \ell_d$ is satisfied for the low-permeability formation case.

When the pressure wave spreads inside the formation on distance much larger than the circulation zone length, i.e., $\lambda \ll \ell_d$, it is necessary to compute the near-tip region solution with 2D pressure-dependent leak-off. By comparing it with the 1D model outcome, it is possible to identify how exactly the 2D mechanism of the fluid exchange impacts on the semi-infinite crack properties.

In the present thesis, 1D mechanism is applied to simplify the problem formulation. In the case of 1D fluid exchange, the fracture tip solution, i.e., opening, and net pressure profiles, is governed by two parameters (dimensionless leak-off and leak-in numbers) in the normalized form. However, three parameters (dimensionless permeability, pore pressure, and velocity number) control the solution when the pressure-dependent leak-off is two-dimensional resulting in much more complex exploration of the problem parametric space where we would like to highlight domains where Carter's leak-off (1D, pressure-independent) provides non-accurate outcome.

2. On page 27, the parameter ct, which is conventionally denoted in literature as a *total compressibility*, is called fluid compressibility in the thesis. If this is indeed only a fluid component of compressibility as

written, then why author neglects a rock skeleton compressibility component, and what t index stands for in such case?

In the present thesis, c_t means the total compressibility of the saturated porous reservoir, i.e., it equals the sum of pore fluid and rock formation compressibility. The terminology is corrected throughout the manuscript.

3. Semi-infinite fracture is introduced for convenient representation of a near-tip region of any finite fracture. As author works with both geometries, I would expect that author discusses which region of a radial fracture corresponds to the region of validity of the semi-infinite fracture analog.

One can discuss this issue based on the example of a radial hydraulic fracture model with Carter's leak-off. The semi-infinite hydraulic fracture describes its properties near the tip. By considering the near-tip region model, it is possible to determine three limiting propagation regimes, and the conditions of their realization can be written in terms of the comparison of the transition length scales $\ell_{mk}, \ell_{m\tilde{m}}, \ell_{\tilde{m}k}$ with radius R: storage–viscosity regime ($\max(\ell_{mk}, \ell_{m\tilde{m}}) \ll R$), leak-off-viscosity regime ($\ell_{\tilde{m}k} \ll R < \ell_{m\tilde{m}}$), and toughness ($\min(\ell_{mk}, \ell_{\tilde{m}k}) > \ell$). In the first case, the semi-infinite hydraulic fracture corresponds to a boundary layer of thickness $d = \max(\ell_{mk}, \ell_{m\tilde{m}})$ adjacent to the moving front of the parent fracture provided that $d \ll R$. The effects linked with the rock toughness and leak-off exhibit inside the tip boundary layer, while m-vertex solution, i.e., far-field asymptote of the boundary layer solution, captures the behaviour of the outer solution near the tip. In other words, the boundary layer solution matches the outer solution for the parent fracture in the range of distances from the tip, $d \ll x \ll R$, where both solutions are represented by m-asymptote. In the second case, the semi-infinite hydraulic fracture corresponds also to a boundary layer but its thickness equals $d = \ell_{\tilde{m}k}$. It is characterized by the dominance of leak-off, while the effect of the rock toughness exhibits inside the boundary layer. Here, along the spatial domain $d \ll x \ll R$, the outer solution near the fracture tip matches the far-field asymptotic behaviour of the boundary layer solution, where both solutions possess m̃-asymptote. Finally, in the third case, rock toughness dominates in the energy dissipation. There is not boundary layer solution structure near the fracture tip, and the finite (radial) fracture solution is characterized by k-asymptote (LEFM asymptote) at the tip.

In the numerical model of a radial hydraulic fracture, it is assumed that the near-tip region model is valid along two spatial elements adjacent to the moving front (Figure 3-16). The closest to the front element is called the "tip element", and using the fracture tip model, we compute its width and the fluid volume participating in the fluid exchange process along the tip element. Moreover, the fracture tip model should be also applicable in the penultimate element (also known as the "ribbon element") since based on the fracture opening value in the ribbon element, the tip propagation velocity is determined via eq. (3.35). In the present model, the length of a spatial cell equals 1/50 of the fracture radius in the considered time moment (now, this value is mentioned in the manuscript in Section 3.3.3). In other words, the semi-infinite fracture model is valid along the distance R/25 from the moving front. Generally, the mesh size should be determined as a result of the convergence study in which different cases from fine mesh to coarse mesh are considered, and the model outcomes are compared with known solutions, e.g., with the analytical solutions in the limiting propagation regimes. The cell size value used in the present thesis was taken from paper [2] devoted to the radial fracture model with Carter's leak-off.

4. Estimates of errors due to the PDL effect look diverse and vague in different parts of the work. For example, on page 91, in the description of the most severe effect of PDL with ψ=0.1, the maximum error for the width is 38%, for the radius it is 61 %, and 13% for opening and pressure. At the same time, page 98 speaks about 5% for ψ=0.1, which is much smaller. In conclusions, author talks about 10% deviation for realistic parameters corresponding to field values.

In Section 3.3.4.3 (containing page 91), we performed a comparison of the dimensionless solutions corresponding to the pressure-dependent leak-off and Carter's leak-off in order to identify values of the governing parameters ϕ and ψ for which the pressure-dependent effects are important. We considered $\psi = 10^{-5}$ and $\psi = 0.1$, computed the relative differences between crack properties in two cases and plotted them in the form of color maps in the coordinates (τ, ϕ). Then, the results were summarized in Table 3.6, where we wrote the average and maximum deviations, which, in the case of $\psi = 0.1$ reach 61% for radius and 38% for maximum opening.

In Section 3.3.4.5 (containing page 98), we computed values ψ_α depending on τ and ϕ starting from which the ratio V_{PDL}/V_{inj} (pressure-dependent leak-off volume correction and injected volume) is above

5% meaning that the PDL impact on the problem solution becomes noticeable. In other words, here, we provide a simplified approach which will help a reader to determine whether pressure-dependent leak-off is important or not: $(\tau, \phi) \rightarrow \psi_\alpha = \psi_\alpha(\tau, \phi) \rightarrow \psi$ vs $\psi_\alpha \rightarrow$ if $\psi > \psi_\alpha$ PDL effect should be included into the model, else *($\psi < \psi_\alpha$) Carter's leak-off can be used.*

In Conclusions, the metric is related to the example for field parameters (given in Section 3.3.4.1), where the relative difference between fracture efficiencies in PDL and Carter's cases reach 10 %.

5. In Conclusions, it is very desirable to see quantitative estimates of such errors. It has many qualitative conclusions, e.g., "*the yield stress potentially results in notable deviations of the fracture parameters from the outcomes of the zero-yield stress model*". Such "*notable deviations*" could be either subjective or depend on the case. I suggest writing conclusions in a more measurable sense.

Since the impact of the analyzed fluid mechanics phenomenon strongly depends on the values of the input parameters, it is difficult to summarize quantitatively the results using just several values. In the main body of the thesis, we provided various maps built the dimensionless variables supplemented by tables using which a reader can easily estimate the importance of a hydrodynamical effect for the interested cases and decide whether it is necessary to implement this phenomenon into his/her hydraulic fracture model or not. However, in Conclusions, we can mention the quantitative value obtained in the example for field parameters (a particular case). We have added additional sentences into the manuscript expanded the mentioned phrase:

For example, in the impermeable rock case, we obtained 9 % relative difference between the crack radius values at the end of injection period in the case of Herschel–Bulkley fluid and power-law fluid; this metric for the maximum opening property equals 20 %. When the leak-off with Carter's coefficient

$C' = 3 \cdot 10^{-5}\ m/\sqrt{s}$ *was introduced into the model, the deviation for radius comprises 3 %, while for the fracture opening near the wellbore, it is about 16 %.*

6. The model is not simple and will require quite a bit of work for implementation of the PDL effect into existing simulators. Is it possible to elaborate a convenient workaround or approximation of the PDL effect, for example, by the modification of the Carter leak-off coefficient?

Pressure-dependent fluid exchange rate (Appendix A.1) depends on fluid pressure which is a function of time and distance from the source. Moreover, pressure profile is an unknow fracture property, i.e., it is a part of the problem solution. That is why, it is rather difficult to put into a radial fracture model with Carter's leak-off some effective value of Carter's coefficient which allows one to approximately reproduce the solution with the pressure-dependent leak-off without preliminary estimates.

Firstly, it is required to mention that in Sections 3.3.4.3 and 3.3.4.5 we identify regions of the problem parameter space where the solution corresponding to the pressure-dependent fluid-exchange reduces to the outcome of the model with Carter's leak-off, namely, it is the domain with large values of the leak-off number ϕ and large time moments τ. Here, the radial fracture model with Carter's leak-off can be applied with the same value of Carter's coefficient as it is taken in the pressure-dependent case. In turn, in the opposite domain, the discussed solutions differ significantly. In the case of large ϕ, perhaps, the effective Carter's coefficient will allow obtaining similar fracture dynamics during the propagation period where pressure-dependent leak-off influence on the fracture evolution is tangible. However, this solution does not tend to the correct behaviour at large time moments. When the leak-off number ϕ is small, pressure-dependent and Carter's solutions are very different along the considered time span, and one can use the approach with effective value of Carter's coefficient.

I suppose that it is possible to solve the inverse problem of finding the effective Carter's coefficient. E.g., we can determine the effective leak-off number $\tilde{\phi}$ in the model with Carter's leak-off which provides approximately the same fracture efficiency that of in the model with the pressure-dependent leak-off characterizing by the leak-off number ϕ and leak-in number ψ. Further, it is required to analyze different values of ϕ and ψ leading to a database containing the accordance: $\phi, \psi \rightarrow \tilde{\phi}$. Next, a regression problem should be solved. I suppose that this approach is time consuming; however, it will allow utilizing the well-known radial crack model with Carter's leak-off to estimate the crack properties corresponding to more realistic physics.

7. In the dissertation, author decided to omit derivations of key equations and solutions. Instead, he included references to the published papers, e.g., Eqns. (2.5) - (2.6) on page 28, Eqn. (3.1) on page 38, Eqn. (3.16) - (3.17) on page 50. The most interesting for me numerical model of radial HF (e.g., Eqn.

(3.34) on page 78) is referred to the paper for details. The dissertation is a self-dependent work, without limitation in size (as opposed to some journals) and can freely contain all necessary derivations used in the work for convenience of a reader.

Final version of PhD thesis contains derivations of equations (2.5) and (2.6) in Section 2.1.2.3. Derivation of formula (3.1) is written in Appendix A.1, while the details of the numerical scheme applied for calculating the solution for a radial hydraulic fracture with pressure dependent leak-off are described in Appendix A.2. Expansions near the vertex solutions m *and* $\widetilde{m}$, *equations (3.16) - (3.19), are derived using the approach provided by paper [3] that is why, in the manuscript, the final expressions are written only. I suppose that a reader can easily use these formulas without knowing certain details, which, in turn, he/she can look through in [3] at desire.*

Gennady Mishuris

1. In field applications, HF surfaces are rather rough. Does the roughness impact the flow regime transition inside the fracture channel from laminar to turbulent?

The roughness of the crack surface strongly affects the fracture characteristics when the turbulent flow dominates inside the crack channel. The authors of papers [4-6] considered PKN/radial/KGD fractures driven by pure water turbulent flow in channel with rough surface, derived rough and smooth turbulent limiting propagation regimes, and analysed problem solution evolution between them. Since the friction factor in turbulent flow in rough channel is higher than in the smooth channel, the crack radius should be even smaller, while maximum opening is even larger compared to the laminar model outcomes than the crack model with smooth channel provides. Mentioned behaviour can be observed in papers [6, 7].

The focus of Chapter 4 of the present thesis is aimed at the slickwater fracturing case for which the approximation of the experimental data [8], suitable for the implementation into the numerical model, is available for the smooth pipe flow only (in work, we recalculate the results for pipe flow to the plane channel flow). That is why the analyses of the turbulent-laminar flow in the crack channel with rough surface was not included in the thesis. Moreover, it is impossible to derive semi-analytically the limiting propagation regimes in which turbulent flow occupies approximately the entire fracture channel if friction factor is not governed by the power-law function of flow Reynolds number.

However, the examination of a radial crack with rough surface is among plans for the further continuation of the work performed during PhD study. It is possible to approximate Virk's experiments on slickwater flow in a rough pipe and implement such dependencies into a hydraulic fracture model. One can assume that the behaviour of the crack properties in the compared models, i.e., rough vs. smooth crack channels, will be qualitatively similar to the pure water case analysed in [6, 7].

2. The author has applied two numerical approaches, identified as accurate and approximate, when he analyses an impact of the fracturing fluid yield stress on a radial crack propagation. However, the comparison between those numerical solutions is omitted. I believe such comparison (with clear estimation of the errors) would be a valuable addition for practitioners.

Validation of an approximate approach using in the model for a radial crack driven by Herschel–Bulkley fluid is written in Appendix A.6. Here, I compare the proposed solvers in terms of radius, maximum opening, pressure at the half radius, and efficiency for different values of flow behaviour index ($n = 0.3, 0.75, 1$) *and*

1. *yield-stress number* ($\psi = 10^{-10}, 10^{-5}, 1, 10^{5}$) *for the impermeable formation case, i.e.,* $\phi = 0$,
2. *leak-off number* ($\phi = 10^{-20}, 10^{-10}, 10^{-5}, 1, 10^{5}, 10^{10}$) *and yield-stress number values* $\psi = 0$ *and 1.*

The results are presented in the form of average and maximum relative differences between accurate ("numer") and approximate ("appr") solutions, $\delta_A = |A_{numer} - A_{appr}|/|A_{numer}|$, *where A is one of the fracture properties enumerated above, computed for a considered time span.*

Aleksey Vishnyakov

1. I really recommend that the novelty and the significance of the work as a separate section: what is actually done that was not done before and how it affects the science in general and the practice of hydraulic fracturing. This will greatly benefit the thesis. If there is a sense to introduce the “statements brought to defence” this should be done as well.

In the final version of the thesis, introduction chapter is divided into two sections. The first one includes a literature review consisting of the following items: (i) description how the complexity of the numerical models of hydraulic fracture growth has been developed over time; (ii) enumeration of the large variety of the near-tip region models; (iii) description of various radial hydraulic fracture models. In items (ii) and (iii), we would like to outline what has been published in literature relatively to the examination how various fluid and solid mechanics phenomena impact on the propagation of a semi-infinite and radial fractures. The second section is devoted to the objectives of each study, their novelty compared to the existing literature sources considering similar physics in a hydraulic fracture propagation problem, and question of the significance of the work, namely, how a potential reader can utilize the presented results for the determination of the cases for which the interested fluid mechanics phenomenon strongly impacts on the crack evolution and where the constructed models can be applied, e.g., fracture tip models can be embedded into a finite fracture model as a propagation criterion, and radial crack models can be used as a benchmark solution for numerical simulators, i.e., for their verification.

2. How the outcome of the modelling can be experimentally verified? Either by means of smaller laboratory experiments of even from the results of actual hydraulic fracturing practice?

I think that it is possible to validate the constructed radial crack models using laboratory experiments. In lab conditions, one can organize the mechanical experiment in such a way that a penny-shaped hydraulic fracture is formed. During the experiment, we can measure fluid pressure at the source, track the fracture footprint, i.e., radius position, via acoustic signal, and measure rock deformations within time which can be used for calculating the crack aperture. As a result, one can compare the measured radius, maximum opening, and pressure near the source with the corresponding properties provided by the numerical model. For the verification of the radial fracture model with the pressure-dependent fluid exchange, we should be able to create a hydraulic fracture in a permeable rock sample. Regarding the radial crack model driven by turbulent-laminar flow, we should inject fluid at large rate leading to the turbulent flow regime realization during the initial period of the propagation. The last model, i.e., penny-shaped crack driven by Hershel-Bulkley fluid, requires a laboratory experiment with viscoplastic fluid. It is required to highlight that the laboratory experiment should be a model [9], i.e., we should create the conditions at which various physical processes neglected in the numerical model do not realize, e.g., presence of the cohesion zone, lag saturated by vapour, damage zone ahead of the fracture. In other words, if some physical effect dominates in the lab experiment but is not accounted for in the model, we cannot obtain similar fracture properties provided experimentally and numerically. However, sometimes, the physical processes which dominate in lab conditions are not manifested in the field conditions where the fracture size is considerably large. I am not sure that field data is suitable for the numerical model validation. Usually, model calibration is performed using the field data, e.g., determination of Carter’s coefficient via interpretation of the leak-off test, and the estimated value is utilized for the subsequent fracture design. It is crucial to apply the model whose assumptions are close to real field conditions; otherwise, the interpretation outcomes will be incorrect. E.g., in the work [10], the authors performed PKN crack model calibration on the field data. The authors highlighted that using the fully laminar model, one can estimate Carter’s coefficient incorrectly if the fracture was created by fully turbulent flow. It happened because the laminar model overpredicts crack radius and underpredicts pressure.

The paragraph devoted to the validation of the constructed numerical models using laboratory experiments is added into Conclusions section.

3. The author mentions foams as fracturing fluids. It is true the foams are non-Newtonian (of course) and can be described with the Herschel-Bulkley formalism, but the foams have many features, like inherent instability, Poisson ratio and compressibility, which are out of the intervals the author continues.

Section 5 considers radial hydraulic fracture driven by the fluid with Hershel-Bulkley rheology. According to this rheological model, the fluid properties are described by three parameters: yield stress τ_0, consistency index M and flow behaviour index n. We suggest that during fracture propagation these parameters do not change, and this assumption allows performing not only numerical calculations but also analytical derivations of the limiting propagation regimes.

However, the foam rheology is more complex, and, according to literature, e.g., [11-12], the flow behaviour index and consistency index depend on the foam quality Q and pressure p. We should mention when the foam rheology is governed by the power-law model, Hershel-Bulkley model is also applicable, since power-law rheological model is a particular case of Hershel-Bulkley model. Pressure inside the fracture channel varies in space and time, while the foam quality can also alter within time [13]. Consequently, M and n are functions of time and distance from the source. When the foam quality Q is approximately constant, and fluid pressure inside the fracture channel alters slightly along the fracture and over time, we can carry out simulations using the constructed radial fracture model by taking $n = n(Q, \sigma_o)$ and $M = M(Q, \sigma_o)$, where σ_o is the far-field confining stress. We suppose that the model outcomes will demonstrate correct qualitative effects linked with the impact of the foam rheology on the hydraulic fracture propagation. Nevertheless, fracture properties corresponding to the correct foam rheology, i.e., $n = n(Q, p)$ and $M = M(Q, p)$, can differ. Since in the radial fracture model, the momentum conservation equation is taken from the steady-state channel flow, i.e., the fluid flow inside the fracture channel is assumed to be quasi-steady-state, we can account for variations of M and n without modifications of the problem formulation outlined in the thesis; however, the closure relations for $n = n(Q, p), M = M(Q, p), Q = Q(t)$ should be supplemented.

Foam is also compressible liquid meaning that the equation of state should be embedded into the model. On the one hand, based on the work [14] and my examination of hydraulic fracture propagation due to injection of super-critical CO_2, fluid compressibility does not play an important role because fluid pressure inside the fracture varies slightly within time. On the other hand, in the foam case, compressibility can influence on the fracture characteristics more significant because the equation of state is more complex $\rho = \rho(p, Q, \rho_g, \rho_l)$, i.e., density can also depend on foam quality, density of gas and liquid phases. I think that additional analyses are required to answer precise to the question about the importance of accounting for the fluid compressibility in the problem of a hydraulic fracture driven by foam.

Brice Lecampion

1. I feel that some future perspectives are lacking in the final Conclusions of the thesis – what in the eyes of the candidate are the important problems to be tackled in the field of hydraulic fracture mechanics in the near future?

The paragraph devoted to possible directions for future research in the numerical modelling of hydraulic fracture growth is added into Conclusions section:

Finally, we would like to provide possible directions for future research in the numerical modelling of hydraulic fracture growth related to the topics discussed in the thesis. In recent times, examination of a hydraulic fracture arrest and closure in a permeable formation after injection shutdown has become popular. E.g., in the work [15], the authors considered the arrest of a radial crack, while in paper [16], the investigation includes not only the arrest but also the recession dynamics of a deflating radial fracture. In these studies, the fluid exchange process between the fracture and ambient permeable formation is taken in the form of Carter's leak-off law. However, fluid exchange plays an important role after injection stopping and has a significant impact on hydraulic fracture dynamics during the arrest and recession stages. That is why the consideration of a hydraulic fracture arrest and closure after shut-in accounting for more sophisticated pressure-dependent mechanism of the fluid exchange is of interest. Next, one can mention the problem in which natural hydraulic fractures are formed inside the subducting oceanic slab due to metamorphic dehydration reactions [17]. Dehydration can lead to pore fluid pressure buildup over the minimum principal stress and result in natural hydraulic fracturing. In this case, the fracture driving mechanism is the influx of pore fluid into the fracture channel from the surrounding porous media until the buoyancy force influence becomes significant. The influx should be described by the pressure-dependent leak-in mechanism in the numerical models. Further, in Section 3.2.4.3 we enumerated several limitations of the constructed fracture tip model with pressure-dependent leak-

off, e.g., application of 1D mechanism for the fluid-exchange description and sameness of fracturing and pore fluids. We suppose that in the future research, it is possible to remove these restrictions and analyze how 2D pressure-dependent leak-off and presence of fracturing and pore fluids with different properties influence on the propagation of hydraulic fractures with semi-infinite and finite (e.g., radial, KGD, PKN) geometries. Let us move on to the topics related to the fluid flow inside the fracture channel, namely, non-laminar flow and non-Newtonian fracturing fluid rheology. We can propose that it will be interesting to analyze a hydraulic fracture driven by turbulent-laminar slickwater flow accounting for the roughness of the crack channel surface. It can be done via an approximation of Virk's experiments on slickwater flow in a rough pipe [8]. Next, we should mention the problem of waterless hydraulic fracturing which requires careful examination. In this reservoir treatment the utilized fracturing fluid is either foam or supercritical CO_2 or liquefied N_2, i.e., compressible liquid with complex rheology. The last proposition is the study of a hydraulic fracture propagation accounting for the proppant transport inside the fracture channel, i.e., the flow of slurry governing by non-Newtonian rheological model.

Sergey Stanchits

1. Evgenii carried out modelling of fluid flow, taking into account two different crack models: semi-infinite and penny-shaped. How close do these idealized models correspond to the real hydraulic cracks created in the field conditions? Where can each of these models be applied?

Semi-infinite hydraulic fracture model accurately describes the near-tip region of a finite fracture and helps resolve the contribution of physical processes realizing in the vicinity of the crack front to its movement. Using the fracture tip model, one can determine a finite fracture front location for each time instance. For that purpose, the near-tip region model should be numerically implemented into a module for the growth of a finite fracture in the form of a so-called tip element, used to match the fracture opening in the near tip zone between the global numerical solution and the local near-tip asymptote and invert for the local fracture front velocity. In literature, the approach based on "tip logic" was applied inside the penny-shaped fracture model, KGD model, enhanced PKN model and more complex planar crack models based on the enhanced Pseudo3D and Planar3D approached. In Section 3 of the present thesis, we built the fracture tip model accounting for the pressure-dependent fluid exchange and then embedded it into a radial hydraulic fracture model. As a result, the proposed fracture tip models (Sections 3 and 4) can be applied as a propagation criterion in various finite fracture models.

The second crack model utilized in the thesis is a penny-shaped hydraulic fracture model. It is an example of a finite fracture which allows obtaining demonstrable results linked with the fluid mechanics phenomena under consideration. A radial crack occurs in nature, e.g., during the initial propagation period, hydraulic fracture growing due to the injection from the point source has penny-shaped geometry until the influence of heterogeneity of rock mechanical properties and/or buoyancy force become significant. Moreover, the penny-shaped hydraulic fracture model can be used as a benchmark solution for the numerical simulators of more realistic (complex) fracturing problems including the same physics. In other words, if the numerical simulator for modeling the hydraulic fracture propagation accounts for pressure-dependent leak-off or laminar-turbulent flow inside the fracture channel or fracturing fluid with Herschel-Bulkley rheology, one can apply the radial crack models constructed in the present thesis for its verification.

2. Is it possible to give at least a few examples of hydraulic fractures, in which the consideration of the fluid exchange related to the fluid pressure inside the fracture is particularly important?

One can give an example of a problem in which natural hydraulic fractures are formed inside the subducting slab due to metamorphic dehydration reactions. Dehydration can lead to pore fluid pressure buildup in excess of the minimum principal stress and result in natural hydraulic fracturing. In this case, fracture driving mechanism is the influx of pore fluid into the fracture channel from the surrounding porous media until the buoyancy force influence becomes significant. During numerical modelling, this influx should be described by the pressure-dependent leak-in mechanism.

3. Evgenii implemented two numerical approaches for the Herschel–Bulkley rheological model of the penny-shaped fracture: accurate and approximate. Why were two numerical algorithms proposed? What are the limitations of each approach? Which one can be considered best?

We proposed two numerical algorithms for calculating the solution for a radial crack driven by Herschel–Bulkley fluid. The first approach is a direct numerical solver allowing one to compute the fracture characteristics accurately, and it is based on Gauss-Chebyshev quadrature and Barycentric Lagrange interpolation techniques. The second approach helps us to construct the simplified approximate solution based on the full-crack continuation of the near-tip region asymptote and the global fluid balance equation.

In addition to calculating the accurate solution, the first method has been used for tuning parameter λ inside the simplified method and for its validation. We have also performed the quantitative analysis of the plug zone formation inside the fracture channel using the accurate approach. The computation process via the accurate solver is not fast especially near the leak-off and yield stress dominated regimes, in which it is slow or even breaks down due problems with convergence.

In turn, the simplified approach is computationally efficient and allows us to rapidly calculate the problem solution for any values of the input parameter, i.e., using the approximate method, we can simulate the fracture propagation corresponding to large leak-off and/or large yield stress values. Overall, rapid approximate solution is more beneficial to perform estimations for the whole problem parametric space which include massive calculations, which was one of the primary goals of Chapter 4. From a practical point of view, it will be easier for a reader to implement the simplified approach.

4. Some important details are omitted in the manuscript, such as the derivation of the pressure-dependent leak-off rate and how this fluid exchange mechanism fits into the numerical algorithm for a radial hydraulic fracture. They are given as references, but I recommend Evgenii to add them into the thesis for the completeness.

Derivation of the relation governed the pressure-dependent fluid exchange rate is given by Appendix A.1. Most important features of the numerical scheme applying in the model for a radial crack with pressure-dependent leak-off are outlined in Section 3.3.3, while the details of the discretization of governing equations are presented in Appendix A.2.

5. Finally, I would recommend Evgenii to add to his thesis, for example, in the Conclusions section, some ideas regarding how the developed models can be validated using laboratory and/or field data. I assume that verified models may be of a higher value for possible industrial application than unverified ones.

The paragraph devoted to the validation of the constructed numerical models using laboratory experiments is added into Conclusions section:

Constructed radial crack models can be verified using laboratory experiments. In lab conditions, mechanical experiment can be organized in such a way that a penny-shaped hydraulic fracture is formed. During the experiment, one can measure fluid pressure at the source, track the fracture footprint via acoustic signal, and measure rock deformations within time which can be used for calculating the crack aperture. As a result, it is possible to compare the measured radius, maximum opening, and pressure near the source with the corresponding properties provided by the numerical model. For the verification of the radial fracture model with the pressure-dependent fluid exchange, we should be able to create a hydraulic fracture in a permeable rock sample. Regarding the radial crack model driven by turbulent-laminar flow, we should inject fluid at large rate leading to the turbulent flow regime realization during the initial period of the propagation. The third model, i.e., penny-shaped crack driven by Hershel-Bulkley fluid, requires a laboratory experiment with viscoplastic fluid. It is required to highlight that the laboratory experiment should be a model [9], i.e., we should create the conditions at which various physical processes neglected in the numerical model do not realize, e.g., presence of the cohesion zone, lag saturated by vapour, damage zone ahead of the fracture. In other words, if some physical effect dominates in the lab experiment but is not accounted for in the model, we cannot obtain similar fracture properties provided experimentally and numerically. However, sometimes, the physical processes which dominate in lab conditions are not manifested in the field conditions where the fracture size is considerably large. In turn, field data can be used for model calibration, e.g., determination of

Carter's coefficient via interpretation of the leak-off test. Thereafter, the estimated value is utilized for the design of a hydraulic fracture which can be created in the same rock formation. It is crucial to apply the model whose assumptions are close to real field conditions; otherwise, the interpretation outcomes will be incorrect. E.g., in the work [10], the authors performed PKN crack model calibration on the field data, and they highlighted that using the fully laminar model, one can estimate Carter's coefficient incorrectly if the fracture was created by fully turbulent flow. It happened because the laminar model overpredicts crack radius and underpredicts pressure.

Dmitry Koroteev

1. Talking practical application of his study, I would like to see some kind of ranking of the importance of the studied effects for the actual success of HF job in various geological formations with various HF fluids, proppants etc. This would make a better fit for the petroleum engineering direction of Evgeny's thesis and may act as some kind of the recommendations list for advanced HF designers.

In the present thesis, we examined the influence of three fluid mechanics phenomena on hydraulic fracture propagation. Among them are pressure-dependent fluid exchange between the fracture and ambient permeable formation, laminar-turbulent flow inside the fracture channel, and non-zero fracturing fluid yield stress. Since these effects are quite different, I think that it makes sense to consider them separately. For each fluid mechanics effect, a reader would like to find out in which cases, i.e., rock and fluid properties, injection rate, etc., it is crucial to account for the hydrodynamic phenomenon inside the numerical model for a hydraulic fracture growth.

Let us consider an example of a permeable formation. In a hydraulic fracture model, we can apply most common Carter's law, i.e., pressure-independent mechanism, for the leak-off description. However, more sophisticated pressure-dependent mechanism should govern the fluid-exchange process. In Section 3, we determined the value ranges of the input parameters of a radial crack model for which the effects linked with the pressure-dependent leak-off impact on the fracture evolution significantly. Using Figure 3-25 given in Section 3.3.4.5, a reader can estimate the importance of the pressure-dependent fluid exchange for the required values of input parameters. The procedure can be outlined as: $(\tau, \phi) \rightarrow \psi_\alpha = \psi_\alpha(\tau, \phi) \rightarrow \psi \ vs \ \psi_\alpha \rightarrow if \ \psi > \psi_\alpha$ *PDL effect should be included into the model, else* $(\psi < \psi_\alpha)$ *Carter's leak-off can be used. Next, results of Section 3.3.4.3 will help to estimate quantitively the difference between the solution corresponding to the pressure-dependent fluid exchange and the one corresponding to Carter's leak-off law.*

Further, we move on to the case of fracturing fluid with low viscosity. Fluid injection should be carried out with large rate leading to the turbulent flow regime realization inside the crack channel. Using the results presented in Section 4.3.6, a reader can estimate how long turbulent flow impacts on the crack growth for certain values of the input parameters. If the considered case is located out of the domain of the parameter space where the solution is fully-laminar, hydraulic fracture design should be performed accounting for the turbulent flow inside the fracture and its transformation to laminar flow in the vicinity of the front. E.g., in the work [10], the authors performed PKN crack model calibration on the field data (pressure at the wellbore and crack size from microseismic data). The authors highlighted that using the fully laminar model, one can estimate the Carter's coefficient incorrectly if the fracture was created by fully turbulent flow. It is happened because the laminar model overpredicts crack radius and underpredicts pressure.

Finally, when fracturing fluid has non-zero yield stress, a reader has a question whether it is still possible to use the power-law fluid model, or the yield stress has a significant impact on the crack properties. Using the maps shown in Section 5.5.2, a reader can determine how far inside the parameter space the interested case is located from the regimes characterized by the dominance of the yield stress. If the required case is close to the applicability domains of T *and* $\tilde{T}$ *regimes, it is necessary to carry out the fracture design accounting for the correct fluid rheology, i.e., use Herschel–Bulkley rheological model which includes the yield stress and non-linearity of shear stress.*

The reflections discussed here is added into Section 1.2 of the final version of the thesis.

Jean Desroches

1. Though I am inclined to believe it, as the full numerical solution is being used to benchmark the approximate solution, I would have liked to see if the computed behaviour at the fracture tip does indeed match the corresponding asymptotic solutions. I strongly believe that a numerical solution should be benchmarked with an analytical solution prior to using it to benchmark an approximate solution.

Appendix A.3 presents the comparison between the numerical solution based on Gauss-Chebyshev quadrature and Barycentric Lagrange interpolation techniques and analytical asymptotic solutions near the fracture tip (viscosity, leak-off, and toughness asymptotes).

2. Similarly, I would have appreciated a check of the output of the radial model developed for studying the effect of turbulent flow with known solutions for the case of laminar flow, just to be complete.

The verification results of the numerical approach based on Gauss-Chebyshev quadrature and Barycentric Lagrange interpolation are shown in Appendix A.3 on the example of a radial fracture driven by laminar flow of Newtonian fluid in a permeable rock. Here, we demonstrate the comparison of the algorithm outcomes with several reference solutions (numerical and semi-analytical).

3. The only add-on that I would mention here is the request to add a nomenclature per chapter, which is always useful, but particularly here as the same symbol may take different meanings from one chapter to the next.

The nomenclature is added into the thesis.

4. Should the side $\tilde{o}k$ of triangle $m\tilde{o}k$ be green, like in the zero-storage case, or not (Figure 3-2)?

This side is shown by dashed black line because in the $m\tilde{o}k$-face solution, the intermediate asymptote $\tilde{o}$ is never realized. According to the discussion in Section 3.2.2.2, $m\tilde{o}$-edge solution does not exist leading to the absence of $\tilde{o}$ asymptote and the emergence of $\tilde{o}k$-edge solution when the leak-off number equals 0, and leak-in number is large ($\zeta \to \infty$).

5. Why the error is that large around the K vertex? (Figure 3-21)

Applicability domain of the storage-toughness limiting propagation regime (K-vertex) of a radial crack with Carter's leak-off is located in the zone of parametric space characterized by low values of the leak-off number $\phi < 10^{-16}$ and large values of the dimensionless time $\tau > 10^{6}$. When ϕ values are so small, Carter's leak-off term in equation (3.21) does not contribute to the fluid exchange rate at all. However, we have the pressure-dependent correction in equation (3.21) which becomes important already at small values of the dimensionless leak-in number ψ. E.g., at the map for $\psi = 10^{-5}$, we observe the largest difference in efficiency values between the solution with the pressure-dependent leak-off and the one that uses Carter's leak-off model in the discussed part of the parametric domain (Figure 3-21). The deviation increases when the leak-in number equals $\psi = 0.1$ (Figure 3-23). Moreover, in Section 3.3.4.5, we performed simplified analyses of the pressure-dependent leak-off influence including the determination of the critical values of the dimensionless leak-in number as a function of the dimensionless time and leak-off number, i.e., $\psi_\alpha = \psi_\alpha(\tau, \phi)$, starting from which, the pressure-dependent leak-off effects cannot be neglected. Based on the analyses, we obtained that for $\phi < 10^{-16}$ and $\tau > 10^{6}$, ψ_α is about 10^{-8} meaning that deviation of the radial crack solution with the pressure-dependent leak-off from the solution with Carter's leak-off in that zone is tangible for $\psi > 10^{-8}$ supporting the results shown in Figures 3-21 and 3-23.

6. You may want to highlight somewhere that there is no transitional regime (or that it is lumped with the turbulent portion) (Figure 4-1).

Figure 4-1 is modified. Here, it is highlighted that in the vicinity of boundary between laminar and non-laminar flow ($x = \lambda$), the flow regime is transient, and, afterward, it becomes fully turbulent.

7. The mixture of numerical computation and analytical results is unclear (p. 133).

A clarification sentence is added into this paragraph:
In expressions (4.41) for radius $R_t(t)$, maximum opening $w_t(0,t)$, and pressure at the half radius $p_t(R_t/2,t)$, coefficients are determined from the fitting of the full numerical solution by analytical dependencies (4.32) supplemented by formulas for the length scale L_t and small parameter ϵ_t (4.39).

8. Why is there a kink? It looks unphysical - though it may be real for the mathematical formulation. It might deserve a comment. (Figure 4-8)

The kink is a result of pressure evolution representation, namely, here, the ratio of pressure values near the wellbore in the turbulent-laminar p and fully laminar p_{lam} case is depicted. However, p(t) corresponding to the pure water fracturing and $p_{lam}(t)$ (shown in Figure 4-9) are rather smooth. But p_{water}(t) (red line) tends to $p_{lam}(t)$ more sharply compared to $p_{slick-water}(t)$ (blue line).

9. For figure 4-8 and 4-9, I would very strongly suggest that you add efficiency, as it is needed to understand where the "permeable" case sits; furthermore, efficiency is a primary concern for field cases.

Figure with efficiency evolution supplemented by description is added into the manuscript:
Figure 4-10 demonstrates how the fracture efficiency η changes over time in the permeable reservoir case. Here, we present turbulent-laminar solutions for slickwater (dashed blue line), and pure water (dashed red line) fracturing and compare them with η obtained in the fully laminar model (solid grey line). One can notice that both turbulent-laminar solutions become indistinguishable from fully laminar solution after 100 seconds of fluid injection. The main differences are observed during the first few seconds of radial crack propagation: here, the fracture efficiency in the turbulent-laminar model is greater than that of in the fully laminar model. Moreover, crack efficiency corresponding to slickwater solution is smaller than the similar characteristic is the pure water case.

10. Is this "hump" becoming a-physical, or not? (Figure 5-3)

Approximate solutions for the yield-stress dominated regimes (T and $\tilde{T}$) are inaccurate as it can be noticed from panel (c) in Figure 5-3. It is a result of utilized opening profile approximation which was initially proposed for smaller values of $\bar{\delta}$ and λ corresponding to Newtonian fluids. Pressure profile was computed via the integral given by eq. (5.22). I suppose that the unphysical pressure behaviour near the wellbore in the form of "hump" is a result of the inaccurate opening profile.

11. You show same R, same η, but larger width and pressure, which should result in larger overall width profile; that can't be the case if R and η are the same. Why (I guess a rather different width profile, that might be worth showing?)

Solution for a permeable rock case is recalculated and now corresponds to formation permeability equals 1 mD. In the new version, the difference in radius and efficiency between Herschel–Bulkley fluid case and power-law fluid case becomes distinguishable, namely, radius is smaller and efficiency is larger when fracturing fluid has Herschel–Bulkley rheology, and relative differences comprise 3% and 8%, respectively.

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