Skoltech
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Thesis Changes Log

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PhD Program: Physics

Title of Thesis: Dynamics of exceptional states in many-body systems

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The thesis document includes the following changes in answer to the external review process.

"Introduction" and "Preliminary remark" section 2.1 were modified to improve consistency as requested by one of the committee members. In particular, on page 25, a paragraph was added explaining why we need to introduce special Lyapunov exponents for periodic trajectories. The text of the paragraph as below:

Usually, if a system has a positive maximal Lyapunov exponent $\lambda_{\max}$, it means that the almost all randomly selected trajectories in the system are unstable. The periodic trajectories, however, require special consideration. In particular, as we will show below, there are certain cases when a chaotic system possesses a stable periodic trajectory. The stability of such a trajectory cannot be described with the maximal Lyapunov exponent $\lambda_{\max}$. For this reason, we will introduce a special ``periodic'' Lyapunov exponent λ_p, describing the stability of the periodic trajectories.

In order to address the question, "Why is the motion periodic?" section 2.3.2 (Periodic trajectories) was expanded. Pages 28-29.

…The one-spin trajectory $\mathbf{S}_{\mathrm{p}}(t)$ is limited to a two-dimensional manifold - the surface of a unit sphere $|\mathbf{S}_{\mathrm{p}}| = 1$. On a two-dimensional manifold a phase-space trajectory normally cannot avoid closing onto itself, which means that it becomes periodic. The corresponding many-spin trajectory, to be denoted as $\mathcal{S}_{\mathrm{p}}(t)$, also becomes periodic.

Since all spins during this dynamics point in the same direction, the projections of $\mathbf{S}_{j-1}$ and $\mathbf{S}_{j+1}$ in Eq.(2.6) can be replaced by those of $\mathbf{S}_j$, and, as a result, the periodic one spin trajectories $\mathbf{S}_{\mathrm{p}}(t)$ can be computed with the help of one-spin Hamiltonian

$$\mathscr{H}_{\mathrm{p}} = -J(S^x_{\mathrm{p}})^2 - J(S^y_{\mathrm{p}})^2 + h\,S^x_{\mathrm{p}} + h\,S^y_{\mathrm{p}}\,. \quad (2.7)$$

Initial conditions (2.2) correspond to zero energy, indeed if $\mathbf{S}_p(0) = (0,0,1)$ then $\mathscr{H}_{\mathrm{p}} = 0$. The quadratic equation $\mathscr{H}_{\mathrm{p}} = 0$ in terms of S^x and S^y, together with the condition $|\mathbf{S}_{\mathrm{p}}| = 1$, defines an energy shell, which is represented by a line on the unit sphere. In Figure 2-2 we plot one-spin trajectories $\mathbf{S}_p(t)$ for different values of J. As expected $\mathbf{S}_p(t)$ closes onto itself, and thus becomes periodic.

In order to address the question: "What happens with Lyapunov exponents in the vicinity of separatrix" new figure was added (page 35):

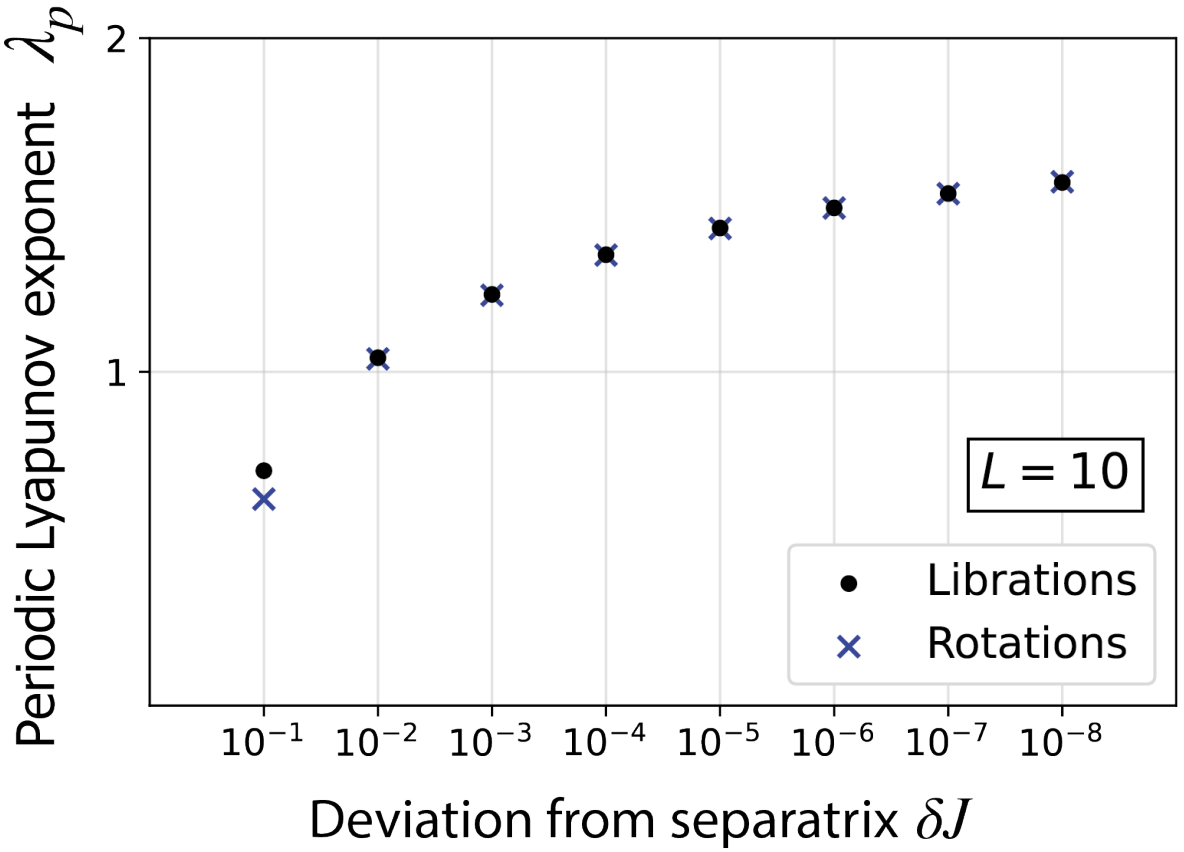

Figure 2-4: Lyapunov exponent λ_p in the vicinity of separatrix $J = J^* \pm \delta J$. The value of separatrix is approximately $J^* \simeq 1.1504059085$. Minimal value of δJ is equal to 10^{-8}. Values $J = J^* - \delta J$ correspond to the regime of librations, while $J = J^* + \delta J$ correspond to rotations.

The following text answering the above question was added (page 37):

To analyze the behavior of periodic Lyapunov exponents near separatrix, we have computed them for different small violations from J^* such that $J = J^* \pm \delta J$, see Fig. 2-4. This figure shows that λ_p growth in the vicinity of separatrix, yet this growth is slow.

Although we cannot compute λ_p on the separatrix or arbitrarily close to it, we expect that it has a finite value. The following argument supports this expectation. Points in the phase space move with limited speed; therefore, $\lambda_p(J)$ must be confined from above for all finite J, including J^*. Also, from Fig. 2-4, we can see that in the vicinity of separatrix, the Lyapunov exponent is an even function of perturbation, such that $\lambda_p(J^* - \delta J) = \lambda_p(J^* + \delta J)$. This fact hints that λ_p shall be continuous in the point $\lambda_p(J^*)$.

The standard algorithms of calculation of Lyapunov exponents do not work in the close vicinity of separatrix $J = J^*$. Indeed, as we approach separatrix closer, we need to take smaller values of initial perturbations $\delta\mathcal{S}$. In real calculations, however, we are always limited by selected machine precision (the smallest difference between two numbers which computer recognizes). Therefore, in practice, we cannot approach separatrix arbitrarily close.

In order to address the question about the behavior of entanglement entropy of a fully polarized state, a new figure was added (page 54):

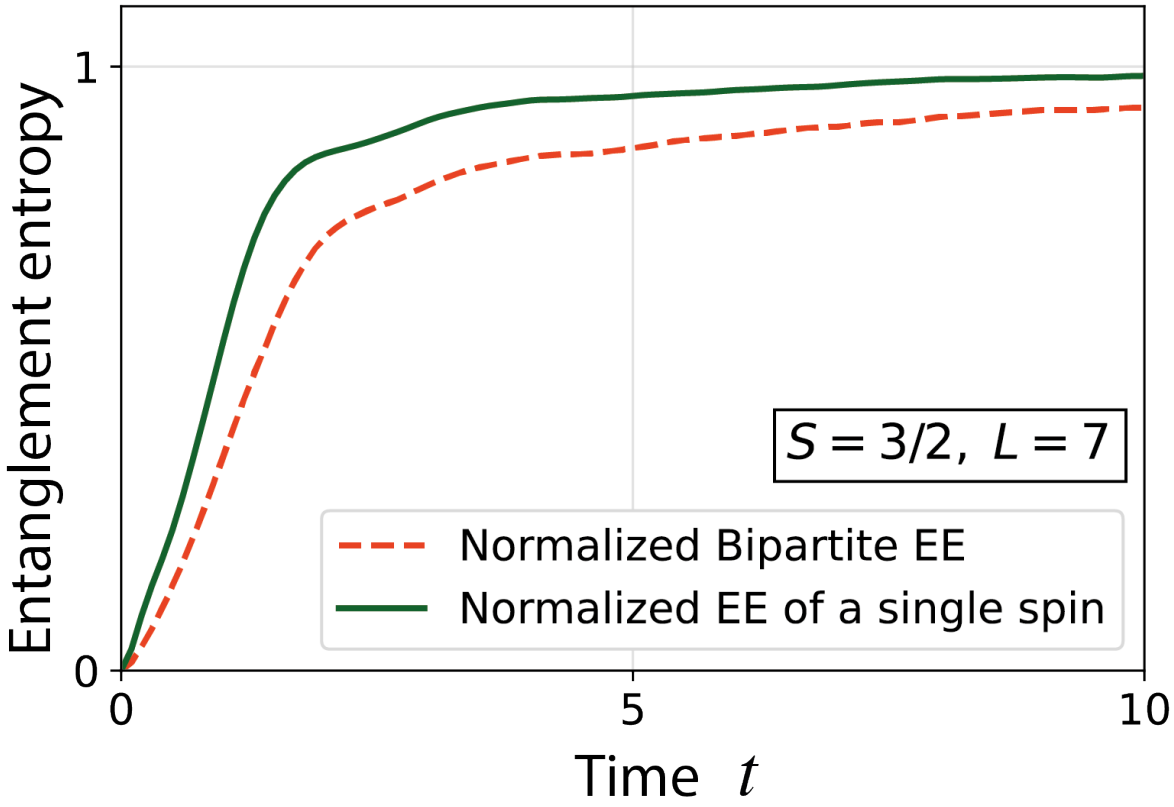

Figure 2-16: Time-evolution of entanglement entropy for initial $|\Psi^{\mathrm{up}}\rangle$ state. Entanglement is normalized by the quantity $\log \mathscr{M}$, where $\mathscr{M}$ is total number of states in subsystem. For a single spin $S = 3/2$, $\mathscr{M} = 4$, for the chain of three spins (bipartite division) $\mathscr{M} = 4^3$.

The following text answering the above question was added as a new subsubsection 2.4.4 (page 53):

In section 2.4.2 we used the value of entanglement entropy for the eigenstates of the system as the test of ETH. The entanglement entropy of the system as such was not the principal concern of the present investigation: instead we could have used the expectation value of a different physical observable, for example, the expectation value of S_1^x. Yet, in this subsection, for the sake of completeness, we present the time-evolution of entanglement entropy for initial state $|\Psi^{\mathrm{up}}\rangle$.

In the Figure 2-16, we plot normalized entanglement entropy $\mathscr{E}(|\Psi(t)\rangle)/\log \mathscr{M}$, where $\mathscr{M}$ is the dimensionality of the Hilbert space of subsystem. We investigated the entanglement entropies of a single spin and of a half of the chain ($\frac{L}{2}$ spins for even L, and $\frac{L-1}{2}$ spins for odd L). In both cases we see that $\mathscr{E}(|\Psi(t)\rangle)/\log \mathscr{M}$ slowly converges to some saturation value, which in its turn close to 1. Initially $\mathscr{E}(|\Psi(t)\rangle)/\log \mathscr{M}$ exhibits linear growth, which is slowed down after some time.

In order to address the question: "Expand on the measure of classical like behavior / coherence length. Why does it measure quantumness?" On the page 89 after formula (4.6) the explanation was expanded as follows:

Now we need to choose some physical quantity, to quantitatively characterize the quantumness of the heavy particle. For a set up described above the most natural quantity is the coherence length [86,87].

$$l(\rho^S) \equiv \sqrt{\frac{2\sum_{ij=1}^{L} |\langle i|\rho^S|j\rangle|^2 (i-j)^2}{\sum_{ij=1}^{L} |\langle i|\rho^S|j\rangle|^2}} \quad (4.6)$$

Here $\rho^S = \mathrm{tr}_B |\Psi\rangle\langle\Psi|$ is a reduced density matrix of the particle obtained from a pure state $|\Psi\rangle$ of the closed particle-gas system and $\langle i|\rho^S|j\rangle = \langle\Psi|a_j^\dagger a_i|\Psi\rangle$ is its matrix elements in the position basis. The quantity (4.6) has a measure of length and depends on off-diagonal elements of the density matrix. It effectively measures the spatial extension of a superposition of localized states, ranging from 0 for a particle localized on a single site to $(L-1)$ for a highly non-classical state of the form

$$|\Psi_Q\rangle = (1/\sqrt{2})(a_1^\dagger + a_L^\dagger)|F\rangle, \quad (4.7)$$

where $|F\rangle$ is some state of N fermions. If the coherence length is equal to zero, all the off-diagonal elements of the density matrix ρ^S are equal to zero. Larger values of coherence length correspond to larger values of off-diagonal elements in the density matrix ρ^S. Larger values of $|i-j|$ for non-zero off-diagonal elements imply longer coherence length.