Jury Member Report - Doctor of Philosophy thesis.

Name of Candidate: Kirill Pavlenko

PhD Program: Mathematics and Mechanics

Title of Thesis: Quantum KdV charges, 2d conformal theories and eigenstate thermalization hypothesis

Supervisor: Associate Professor Anatoly Dymarsky

Date: 18-08-2022

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer’s Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

The dissertation is devoted to an interesting problem of studying of ETH (eigenstate thermalization hypothesis) in two-dimensional conformal field theory (CFT). Integrability of CFT is supposed to lead to a generalization of standard ETH dealing with the generalized Gibbs ensemble, which includes higher integrals of motion. Known results on simultaneous spectrum of
quantum KdV charges in CFT, were used to analytical study of this hypothesis. Using the WKB expansion of the spectra of KdV charges, some concrete results were obtained, thus confirming generalized (integrable) version of ETH.

The thesis consists of 7 chapters. The presentation is concise, clear and to the point. The results presented in the thesis, mainly follows five articles published by the author.

Second chapter is an overview of the classical theory of KdV equation and its finite zone solutions. In particular, a perturbative expansion of classical charges in terms of action variables has been derived.

Third chapter contains brief introduction to qKdV system in CFT and new results, which belong to the author, on large c (central charge) expansion of eigenvalues of local integrals of motion on primary states. The author extensively uses the so called ODE/IM correspondence proposed by Dorey-Tateo and Bazhanov-Lukyanov-Zamolodchikov. Withing this approach the spectrum of qKdV integrals of motion can be extracted from monodromy data of certain second order ordinary differential equation (Schroedinger equation). Large c corresponds to WKB expansion of this Schroedinger equation. This analysis together with some intuition coming from the classical theory enables to obtain large c expansion of eigenvalues for lower local charges. The same results have been confirmed using known analytic results for local charges and using the computer manipulation programs.

In the fourth chapter the knowledge of large c expansion of eigenvalues, obtained in chapter three, has been applied to generalized partition function. In the limit of infinitely large central charge the extensive part of free energy has been analytically computed. Also some information on 1/c corrections has been obtained.

Fifth chapter is devoted to pedagogical exposition of computation of modes of composite operators in the radial quantization approach. Although, this subject is well known and has been used by various authors, it has not been published elsewhere. In addition, chapter 5 contains important explicit expressions for zero modes of certain class of quasiprimary operators.

In chapters 6 and 7 ETH and generalized ETH in 2D CFT are studied. In particular, diagonal matrix elements of certain quasiprimary operators in the eigenbasis of qKdV system has been shown to express solely in terms of qKdV eigenvalues, at least in the leading order in 1/c. This should confirm generalized ETH in this limit.

In conclusion, I would like to note that the dissertation is devoted to a very interesting problem belonging to the intersection of thermodynamics and conformal field theory. The dissertation contains many new results in both areas. The dissertation is based on five articles by the author.
published in high-ranking journals. The author undoubtedly deserves a PhD degree.

**Provisional Recommendation**

- [x] I recommend that the candidate should defend the thesis by means of a formal thesis defense

- [ ] I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate’s thesis according to the recommendations of the present report

- [ ] The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense