

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Ivan Sechin

PhD Program: Mathematics and Mechanics

Title of Thesis: Quantum R-matrix identities and integrable systems

Supervisor: Professor Anton Zabrodin

Co-supervisor: Dr. Andrei Zotov, Steklov Mathematical Institute, RAS

Name of the Reviewer: Nicolai Reshetikhin

I confirm the absence of any conflict of interest	Date: 30-09-2022
---	-------------------------

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense. If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

In his PhD thesis Ivan Sechin obtained important results in the theory of classical and quantum integrable systems and in particular at the interface with algebraic structures such as the associative Yang–Baxter equation.

The thesis contains four chapters. The first one is the brief introduction, summarizing the content of the dissertation and the main results obtained. The second one introduces the family of generalized interacting integrable tops — systems which generalizes both integrable tops and integrable Calogero–Moser systems. The construction of Lax pair and classical r-matrix structure of these systems is based on the quantum R-matrices satisfying the quadratic algebraic identities mentioned above, it is shown that every solution of Associative Yang–Baxter Equation (which is unitary and skew-symmetric) defines a new integrable system. In the third chapter these results are extended to the relativistic level, in this chapter systems generalizing both relativistic integrable tops and integrable Ruijsenaars–Schneider systems are constructed. Lax pairs for such systems is introduced, however. The fourth chapter is devoted to the quantization of such generalized interacting tops. The quantum dynamical algebra related to these quantum the interacting tops and the classical r-matrix structure are obtained. In the special elliptic case, the quadratic algebra based on the RLL-relations is constructed, this quantum algebra generalizes both the Sklyanin algebra and the elliptic quantum group.

The methodology of this dissertation is based on careful analysis of solutions to various Yang-Baxter equations. Algebraic relations on the quantum R-matrices are considered as the non-commutative versions of the Fay identity — the quadratic relation, which provides commuting family of integrals for the classical integrable systems in questions because they provide the Lax pairs. One can say that in this approach quantum R-matrices are considered as the building blocks of Lax operators for the purely classical systems. These methods allow to construct a wide class of new integrable systems, generalizing both known integrable systems of interacting particles and integrable top systems. The methods work not only for the Lax pairs, and the new classical and quantum R-matrix structures are also constructed.

One point that I did not quite understand from the thesis is to which extend Liouville integrability (or superintegrability) of these systems is proven.

The thesis is based on three publications in Russian and international journals, including the best mathematical physics journal in Russia (TMPh) and Q1 journal (JHEP).

This these is a work of very high quality, based on significant scientific results and on solid publications.

Provisional Recommendation

I recommend that the candidate should defend the thesis by means of a formal thesis defense
