Skoltech

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Akshay Vishwanathan

PhD Program: CDSE

Title of thesis: On quantum approximate optimization

Supervisor: Professor, Jacob Daniel Biamonte

Name of the Reviewer: Vladimir V. Palyulin

I confirm the absence of any conflict of interest	**Signature:** **Date: 17-04-2023**

Reviewer's Report
The manuscript "ON QUANTUM APPROXIMATE OPTIMIZATION" submitted as a PhD thesis by Akshay Vishwanathan is devoted to one of the variational methods of quantum computation, namely, Quantum Approximate Optimisation Algorithm. The thesis consists of 6 chapters and is based on 4 published first author Q1 papers two of them being Nature Index papers. This securely fulfils the requirements of the PhD thesis defence policy of Skoltech. The text consists of 3 introductory review chapters, 3 chapters describing the results obtained by the defendant, the conclusive chapter and an appendix. The first very short chapter part describes the quantum computation approach and the main definitions. The second chapter is devoted to the celebrated Ising model and the reduction of optimisation problems to Ising Hamiltonian. It starts with the discussion of P and NP computational complexity classes and continues with setting of the classical computational problem of Boolean satisfiability (k-SAT problem). The optimisation problem is a classical NP-hard problem. Then graph optimisation problems are described. The notion of densities of the above-mentioned problems are introduced. After that a few physics-inspired algorithms for minimising Ising Hamiltonians are mentioned. The next section, chapter 3, introduces the main approach used in the original parts of the thesis, the Quantum Approximate

Optimisation Algorithm. To that end the variational approach to Hamiltonian minimisation is described and the key notion of ansatz. Since QAOA is an approximate algorithm the performance metrics with the theorem about stability of approximations are discussed.

The chapter 4 is the first out three chapters with the original results. It is devoted to the underparametrisation effects in QAOA. The underparametrisation is closely tied to an effect named reachability deficits which is described in the beginning of the chapter. Then performance of QAOA is studied for the MAX-SAT problem. The worsening of the performance for the values of density exceeding the critical one is shown. The second half of the chapter shows the QAOA performance and reachability deficits for the graph problem. For simple cases analytical expressions are derived. More general combinations of parameters are explored numerically.

In the chapter 5 fares though the influence of concentration effects in QAOA. Analytical results are derived for the depth p=1,2 for the problem of variational state preparation and p=1 for the Sherrington-Kirkpatrik model. The theorem about the scaling for the parameter concentration in variational state preparation is stated and proven in the chapter. Then some of the cases not covered by analytical calculations are resolved numerically.

Finally, chapter 6 covers the estimation of circuit depth scaling in QAOA, i.e. it answers the question which depth is needed in order to approximate the solution with pre-set accuracy. The conjecture for the p*, the corresponding critical depth, is made in a form of a logistic function. The reasonable accuracy of this assumption is then illustrated numerically.

The conclusion summarises the findings and the novelty of the whole thesis.

Overall the thesis is very focussed and coherent. The depth and the level of the mathematical approaches are at the front edge of those used in the field. The manuscript presents a significant advancement of the field with the results already noticed and widely cited in the community. It highlights the ability of the author to implement the cutting-edge techniques and report the results in a written form, his deep knowledge of the pertinent literature and the capability to use it to produce trustworthy scientific results.

In the process of reading a few questions arose.

The obtained results disregard the influence of the noise on the performance. What would be typical sources of noise which could affect the performance of QAOA in the considered settings? Could the defendant speculate on whether the noise could change the obtained results in a qualitative way in terms of scalings?

In the second chapter the notions of clause and graph densities are defined. These are concrete examples of problem density. However the expression “problem density” is not used in the chapter. I believe that stylistically and for the benefit of potential readers it would be advantageous to introduce it there as well.

Besides I have spotted a few typos/needed minor corrections

The expression "up to" is consistently misspelled throughout the whole text and written as "upto". This needs a correction

The same happens with usage of "it's" instead of "its" throughout the text

Page 8 "Probelm"→ "Problem"

Page 35 "bitsting"→ "bitstring"

Page 38 "where-" a hanging hyphenation after a word

Page 71 "statistical likely"→"statistically likely"

Page 71 The first sentence in the second paragraph needs a grammar improvement

Page 73 two instances of "prameterized" typo

Page 83 sentence before Eq. (5.46) "approximate"→"approximated"

Page 88 sentence before Eq. (6.1) "describe"→ "described"

The first sentence in Conclusion section needs an improvement

Provisional Recommendation

☒ *I recommend that the candidate should defend the thesis by means of a formal thesis defense*

☐ ~~*I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*~~

☐ ~~*The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*~~