Thesis Changes Log

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PhD Program: Computational and Data Science and Engineering

Title of Thesis: Automatic noise and artifacts removal from biomedical signals and images using tensor completion

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The thesis document includes the following changes in answer to the external review process.

I am grateful to the Defense Jury members for their reviews and comments that helped me to adjust and improve the thesis.

- Fixed mathematical formulations and notations in Chapter 1 and Chapter 2.
- Provided definition and relationship between the tensor tubal rank, the tensor multi-rank, and the tensor nuclear norm under t-SVD framework in Chapter 1.3
- Discussion on challenges of tensor completion methods for MRI data has been presented in Chapter 1.6.
- Added motivation for sparsity constraints on core tensors in Chapter 4.
- Provided details on the smoothing strategy in Chapter 5.3.2
- Included results for noisy image reconstruction and compared with neural network methods in Figure 4-9 and Figure 6.7.
- Provided motivation for using t-SVD for reconstruction in Chapter 6.1.
- Added new experimental results for MRI data compress sensing algorithms in Chapter 6. PSNR and RMSE values presented in Figure 6-6.
- Corrected errors in some algorithms and added new algorithm in Chapter 6.
- Figures and tables have been adjusted for better visibility.
- Corrected many grammatical and typographical errors.

Reply to specific comments

- It may be more reader-friendly if the motivation of sparse representation of tensor ring core can be explained in details. Is there any physical background for such a representation?

Response: Thank you for the comment.

In general, tensor ring decomposition is not unique. Moreover, tensor completion is a very ill conditioned optimization problem, thus some regularization is necessary in order to obtain stable, relatively simple and meaningful representation. Sparsity constraints is one such regularization technique. In other words, sparse representations have two main purposes: They are a form of regularization that pushes as many parameters as possible to exact zeros. Furthermore, sparsity leads to simpler tensor models by learning what parameters can be dropped, lowering their total number. In addition, sparse optimization methods are useful for signal processing problems that involve large, noisy, or incomplete data sets, where finding a simple and meaningful representation is challenging. Inspired by successful methods that introduced a sparsity prior on the columns to solve the failure of low-rankness in regularizing structural matrix completion tasks and another method that extends the matrix case to higher-dimensional tensors and considered structural missing data along each mode using the TT framework. In our proposed model, the underlying tensor is regularized by a low-TR-rankness prior to exploiting the inter-fibers/slices correlations, and its fibers are regularized by a sparsity prior under dictionaries to exploit intra-fibers correlations.

- In chapter 6, t-SVD based low rank approximation is used to MRI Motion Artifact Reconstruction. Why do you prefer this decomposition? Compared with some tensor networks, why do you choose t-SVD for it?

Response: The t-SVD model is relatively simple and its associated learning algorithm is quite efficient. One of my main objective was to investigate several tensor decomposition models and select the most promising one. It is known that unlike the Tucker and Canonical Polyadic Decompositions, the t-SVD has similar properties to the classical SVD. More precisely, the truncated t-SVD provides the best low tubal rank approximation in the least-squares sense for any invariant tensor norm. This was one motivation to use the t-SVD in our formulation. On the other hand, experimental results reported in the literature show the efficiency and performance of the t-SVD in many applications, such as tensor completion and tensor denoising. This was another motivation to utilize the t-SVD in our work. It is worth mentioning that several fast algorithms have been proposed to decompose a tensor into the t-SVD format and this facilitates the utilization of the t-SVD for real-world big data processing.

- What is the advantage of cross tensor approximation to the other random tensor approximation (e.g., random tensor t-SVD)?

Response: Randomized t-SVD is also a fast approach for computing a low tubal rank approximation. The main benefit of the cross approximation is that we need access to only a part of the underlying data tensor, while the randomized t-SVD multiplies the original data tensor with a random tensor, which means the method needs all components of the tensor. This computation could be very expensive, especially when we are dealing with big data tensors.

- The author uses tensor singular value thresholding. Its calculation depend on the parameter beta, see formula (7) in the Appendix. However, it is not clear from the results of the experiments whether they are robust w.r.t. this value. Any comments on this issue?

Response: The tensor singular value thresholding is a generalization of the matrix singular value thresholding, which was proposed for recovering tensors with missing elements. For the tensor singular value thresholding algorithm, the parameter β is defined as λ/μ_k, where λ is a penalty parameter and μ_k is a regularization parameter used at k-th iteration. So, the parameter β is not fixed and is dynamically changed at each iteration. Please note that the parameter μ is updated via formula $\mu_{k+1} = \min(J\mu_k, \mu max)$. The parameter λ should be carefully selected as using bigger values result in poor approximation and reconstruction. On the other hand, a smaller value takes too long to converge even though reconstruction is good.

- In (2.1) the min may not exist for the CP decomposition. Some comment about this would be appropriate.

Response: Thank you for the comments. For CP decomposition the best rank approximation may not exist. Therefore minimization problem is generally an ill-posed one. However, incorporating a coherence constraint in the minimization problem helps to overcome this ill-posedness. Moreover, the CP decomposition of tensors with order higher than two, is often unique under mild assumptions. This constitutes one of the attractive properties of the CP decomposition. Constraints such as orthogonality and non-negativity ensure the existence of the minimum of the optimization criterion used.