"There are always times when those who do science need to pause and reflect ..."

Sven Treitel

Pitfalls and Challenges of Seismic Imaging and Inversion

Evgeny Landa

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OUTLINES

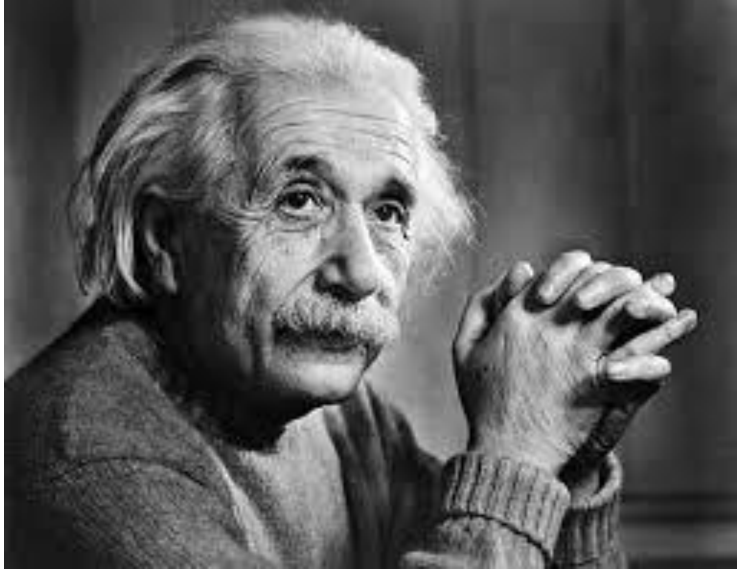
- **Pitfalls in seismic inversion**
- **Quantum seismic imaging: is it possible?**
- **Seismic diffractions - the abandoned stepchildren of traditional imaging**
- **Road ahead**

Inversion: mathematical formulation

Inversion, is a mathematical tool for interpreting ***indirect*** measurements, inferring properties of the Earth's interior from surface observations.

Forward problem: $$Data = Some\_function(Model)$$

Inverse problem: $$Model = Some\_function^{-1}(Data)$$



How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably appropriate to the objects of reality? ... In my opinion the answer to this question is, briefly, this: As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.

Albert Einstein

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Inversion - thinking backwards

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"Most people, if you describe a train of events to them will tell you what the result will be. There are few people, however that if you told them a result, would be able to evolve from their own inner consciousness what the steps were that led to that result. This power is what I mean when I talk of reasoning backward…"

Sherlock Holmes,
Sir Arthur Conan Doyle (1887)

Inversion: mathematical formulation

Inversion, is a mathematical tool for interpreting ***indirect*** measurements, inferring properties of the Earth's interior from surface observations.

Forward problem: $$Data = Some\_function(Model)$$

Inverse problem: $$Model = Some\_function^{-1}(Data)$$

- The pure mathematical community takes an analytical, "just-solve-the-equation" approach (***not that it is easy***!)
- The second community (geophysicists) focuses on **optimization-based approach**, which *may not* always give as much information as an analytical solution would
- Since stable analytical solutions to inverse problems generally do not exist **optimization *is better than nothing***

“Impolite” mathematics

Some problems can behave “impolitely”. Let us consider a linear system of equations:

$$x + 10y = 11$$
$$10x + 101y = 111$$

The unique solution is easy to find: $x=1; y=1$. Let us slightly change the right hand side of the first equation:

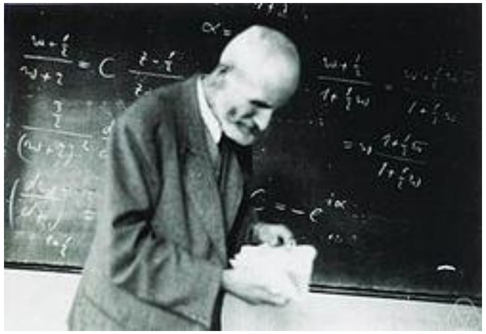
$$x + 10y = 11.1$$
$$10x + 101y = 111$$

Solution now is: $x = 11.1; y = 0.$

Small change of the input data led to sharp change of the solution.

***What practical value the solution of similar system can have? And the natural first answer - NONE*.**

Oscar Perron's paradox



- Suppose the largest natural number is **N**
- Then, if **N>1** we have **N² >N** contradicting the definition
- **Hence, the largest natural number is equal to 1!**
- We arrive at this absurd conclusion because we assumed that the largest natural number exists.

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Well-posed problems should satisfy the following conditions:

1. ***A solution exists***
2. ***The solution is unique***
3. ***The solution depends continuously on the data***

(Jacques Hadamard)

Verification of these conditions is often not a trivial task because the solutions obtained may not be a priori as absurd as in Perron's paradox. An apparently reasonable result can mistakenly create an illusion that the problem is solved.

Geophysical inversion

What doesn't it mean?

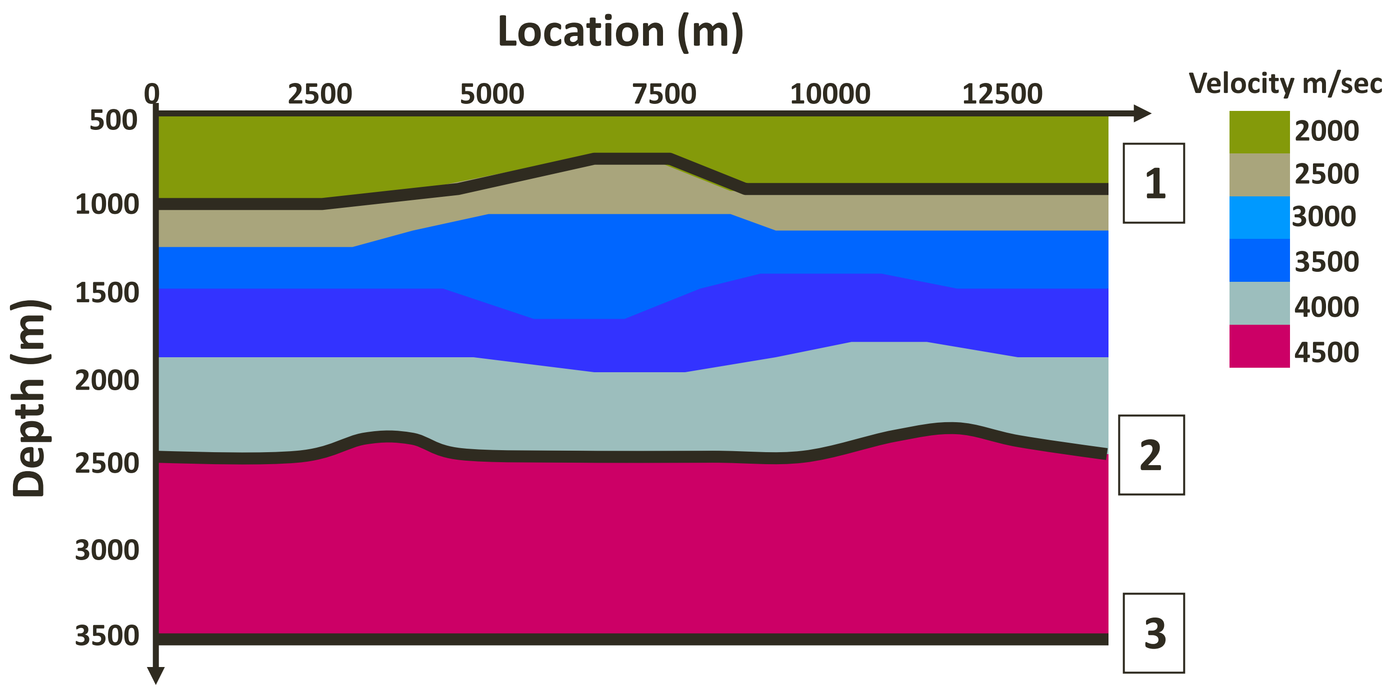
- **Inversion ≠ Data fitting**

"… a good fit is a necessary but by no means sufficient condition for success. By itself, a good fit does not guarantee that an inversion is correct. This occurs, in my opinion, more often than we would like to think".

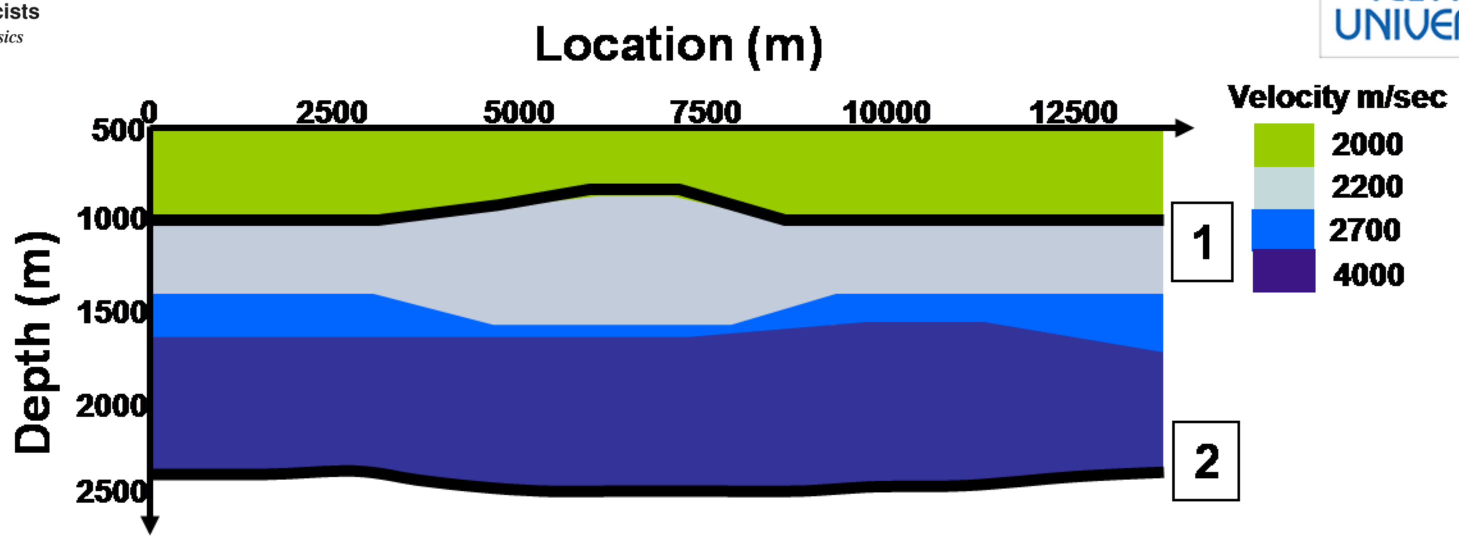
Sven Treitel

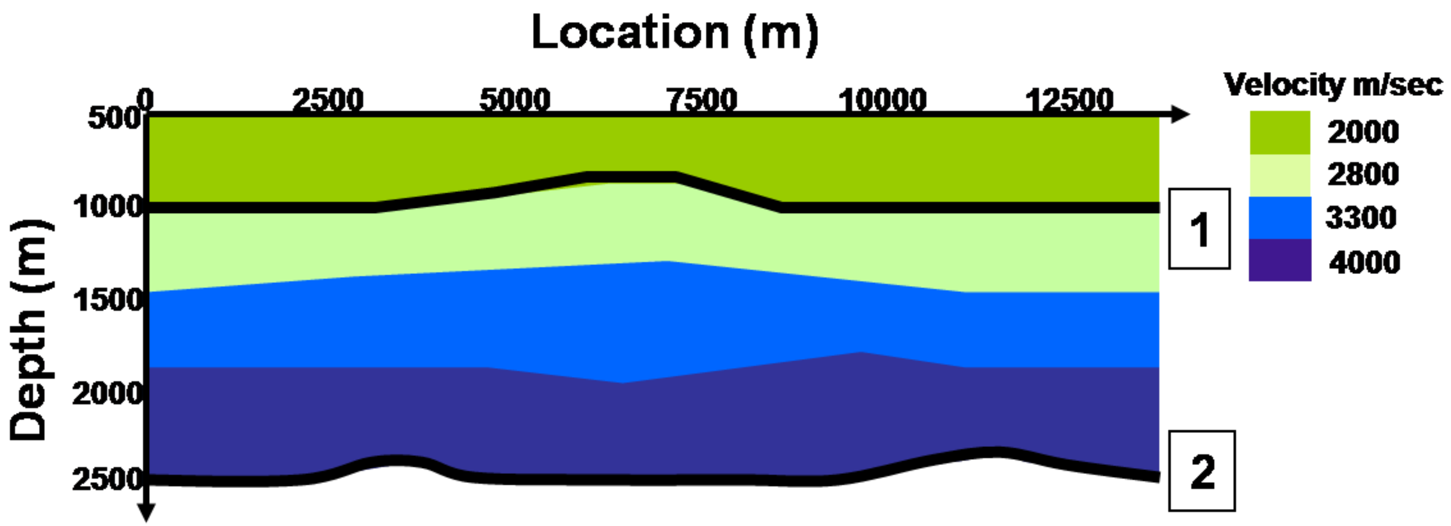
Non-uniqueness of the inverse kinematic problem

Three layer model



Two kinematically equivalent models





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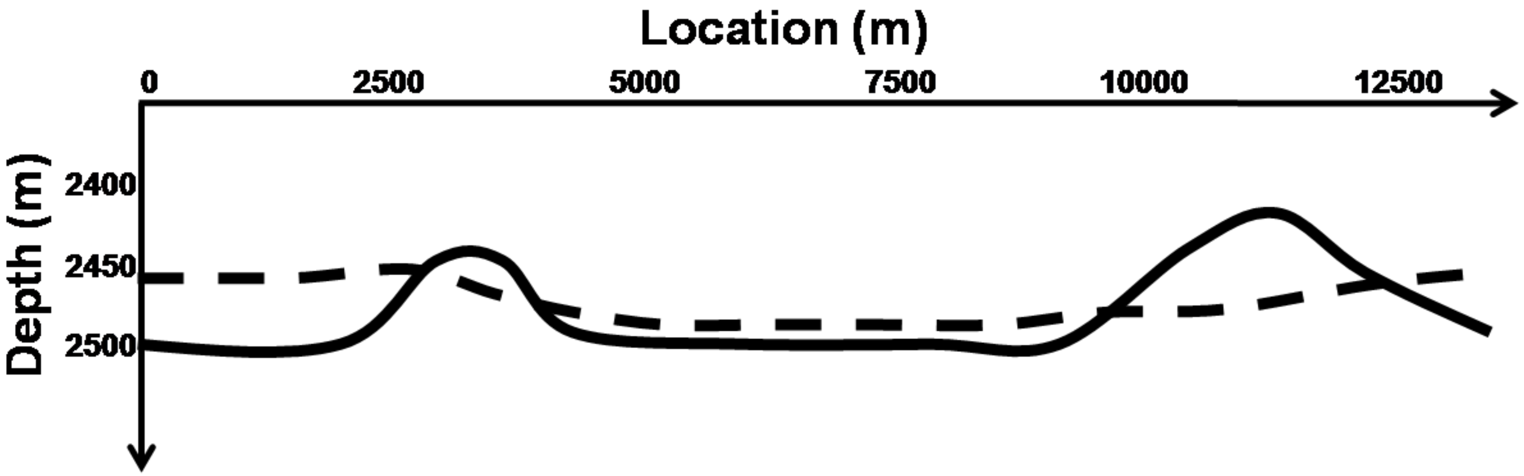
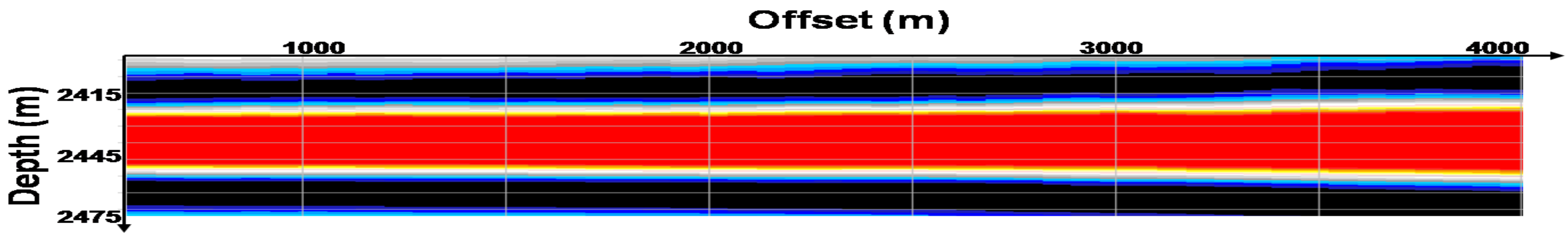
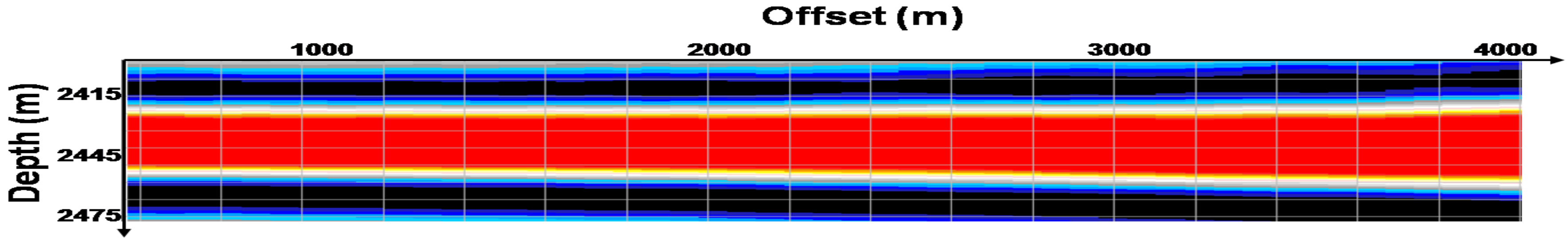
Difference between two models (zoom)



May be depth migration can help?

CIGs for two models





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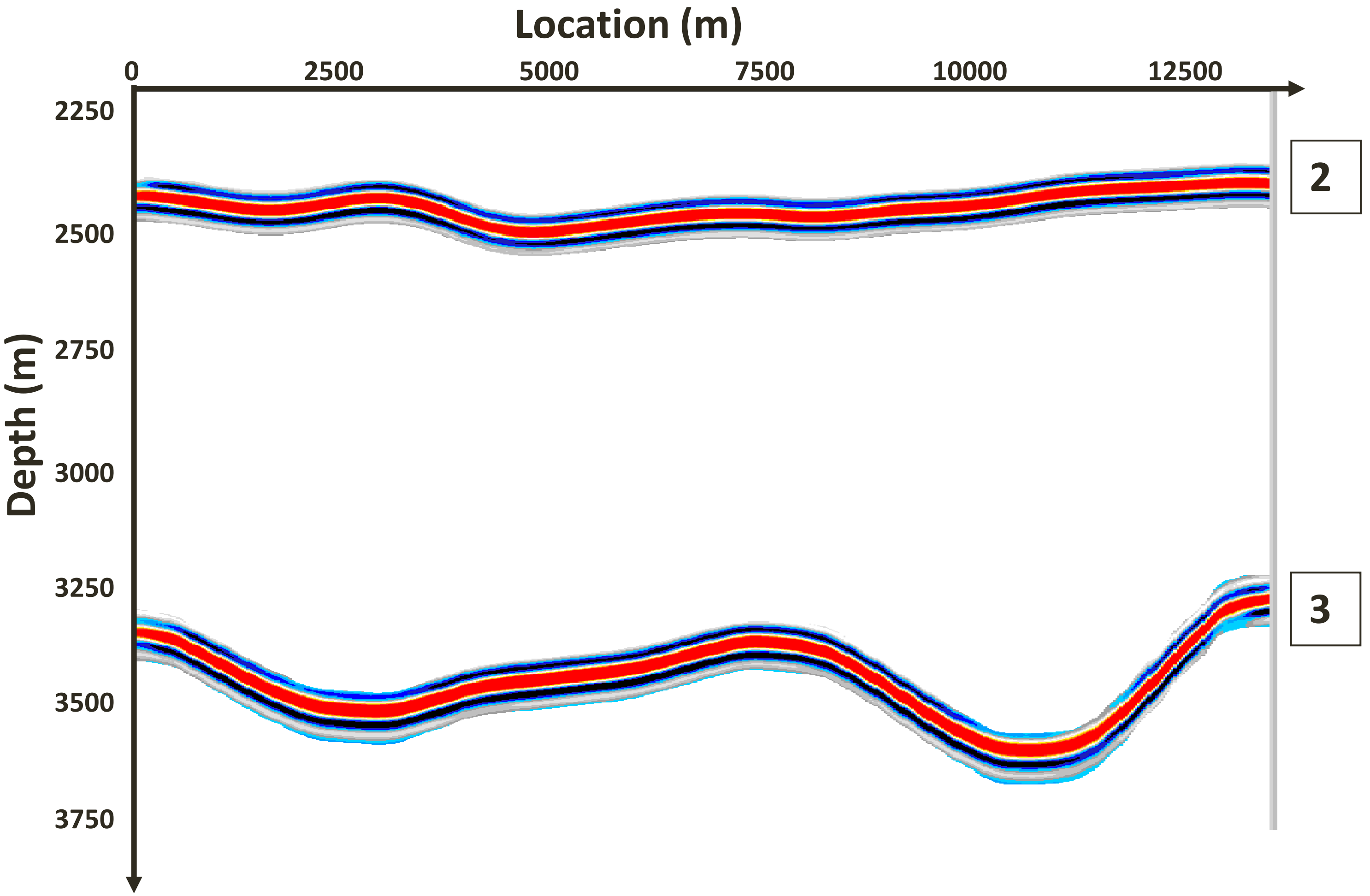
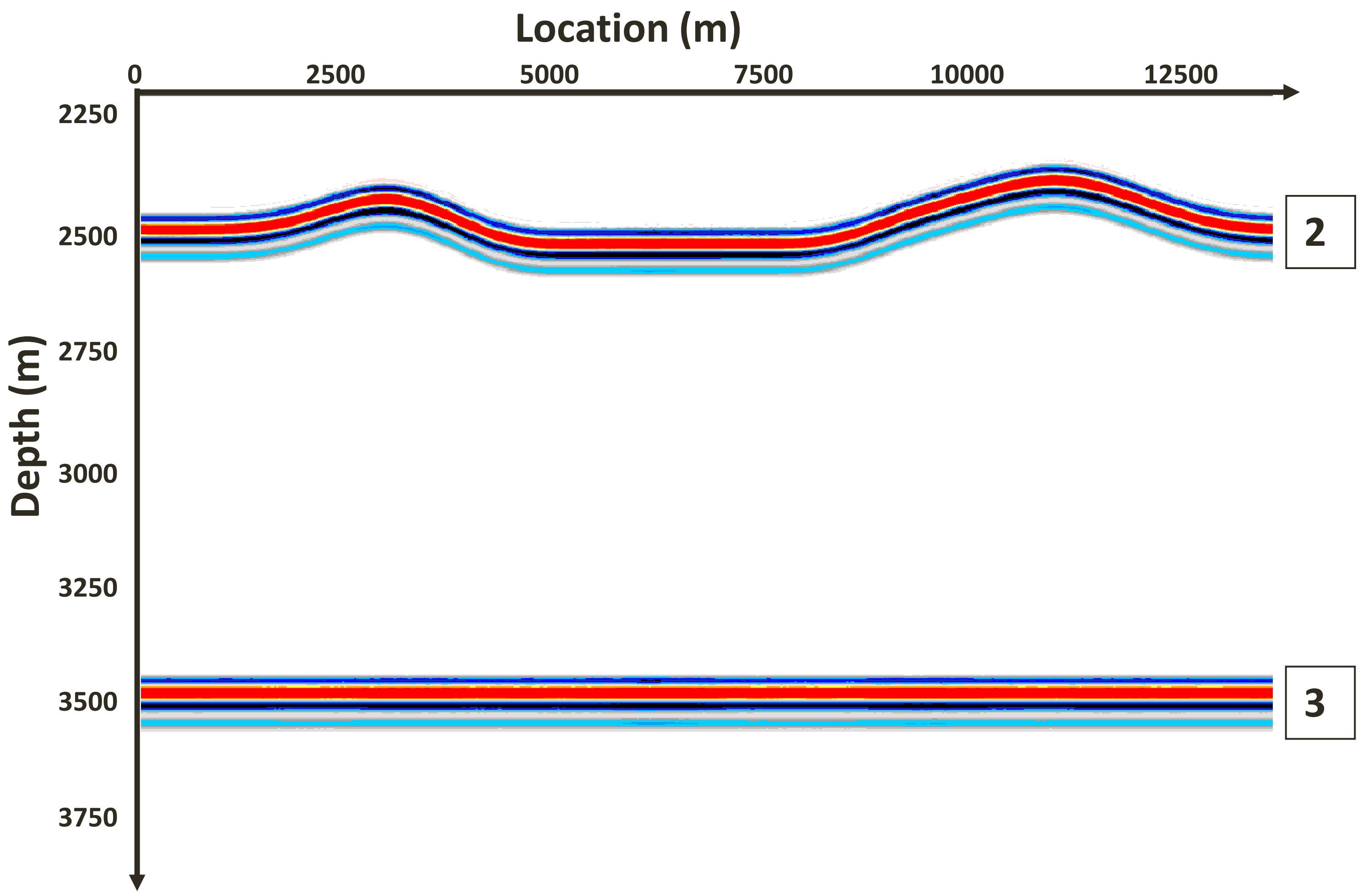
PSDM (model II)



PSDM (model I)


Location (m)
0
2500
5000
7500
10000
12500
Depth (m)
2250
2500
2750
3000
3250
3500
3750
2
3

Geophysical inversion

What does it mean?

- How much the solution allows for reconstruction of important characteristic of the subsurface

- At the same time the importance of these characteristics is a factor external to the inversion problem…

Few lessons

- Inversion based on the best fit of observed and calculated data may lead to construction of several subsurface models with significantly different geological meaning

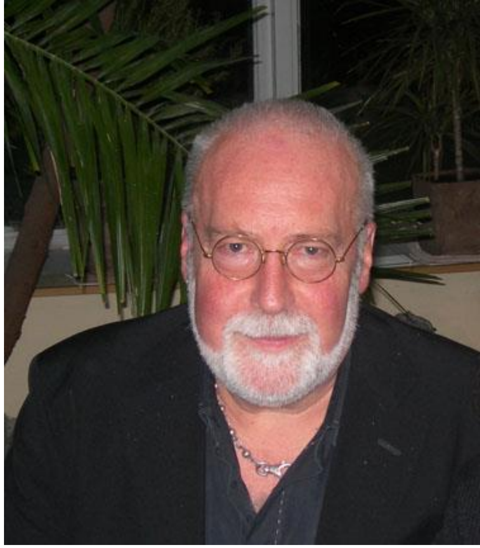
- An overburden model constructed by the best fit does not guarantee a correct solution for the deeper part of the model

- Refinement of the model parameterization may lead to a better fit but does not guarantee construction of a better subsurface model

- The question we should always keep in mind is “How correct and realistic is our seismic images” rather than the question “How well does it fit my data”

And what about FWI?

Tarantola took the view that the most general formulation of inverse problems can be obtained by using the language of probability and the ***Bayesian approach:***



- Bayesian approach requests *knowledge of the statistical properties of the model* as well as *the statistical properties of the data*
- According to the Bayesian approach, ***the data is used in inversion to constrain the a priori model***, and not the opposite as when ***the inversion is constructed from the data*** and the a priori model serves as a constraint.
- In practice our knowledge of statistical properties and a priori information are very poor: ***"It is difficult to use Bayes' theorem in seismic inversion and to be honest"***
- Our solutions are limited by the well known least squares method, assuming Gaussian distribution of noise

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JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 96, NO. B12, PAGES 20,289–20,299, NOVEMBER 10, 1991

Monte Carlo Estimation and Resolution Analysis of Seismic Background Velocities

ZVI KOREN,[1,2] KLAUS MOSEGAARD,[3] EVGENY LANDA,[4] PIERRE THORE[4]
AND ALBERT TARANTOLA[5]

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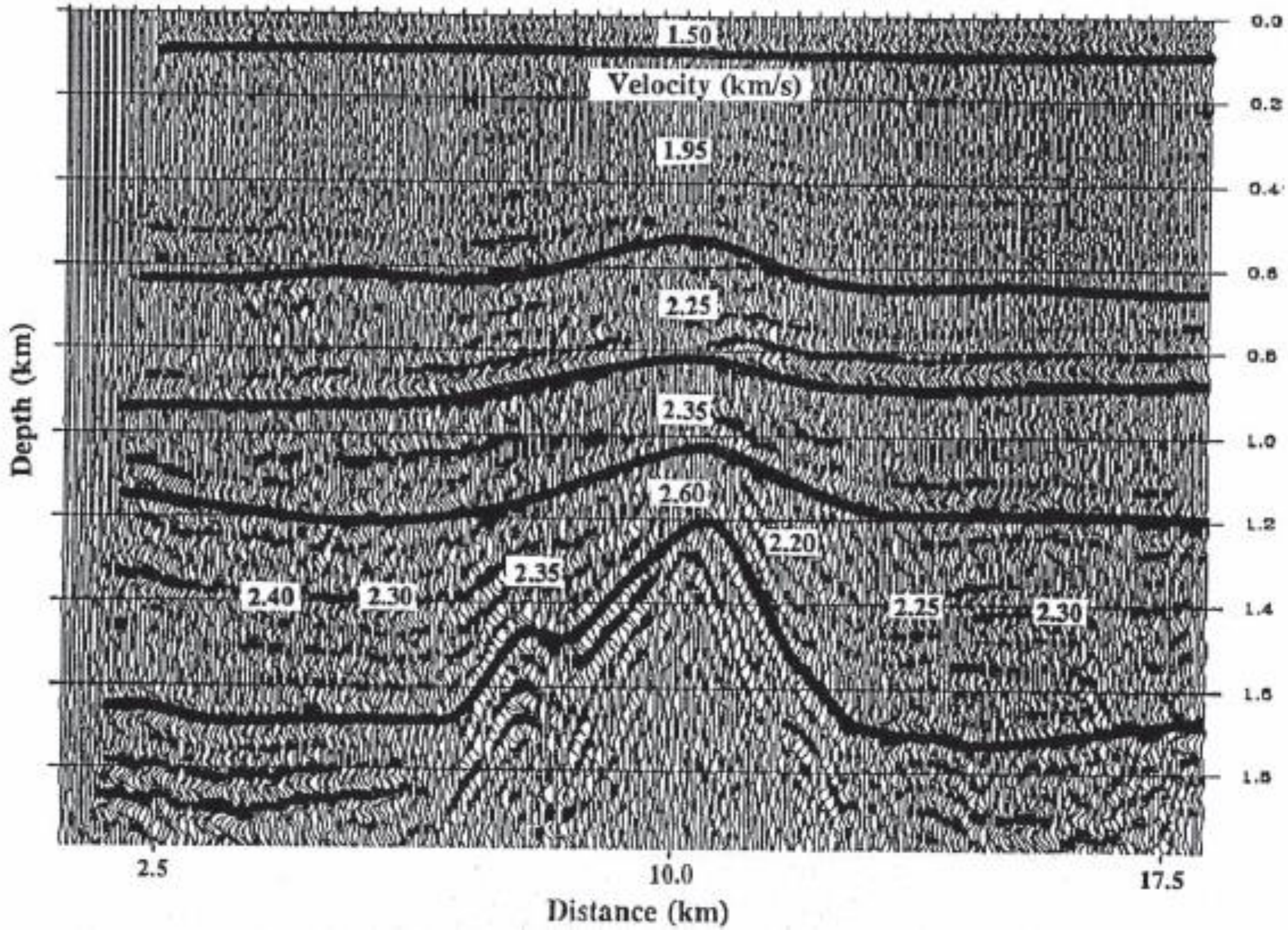

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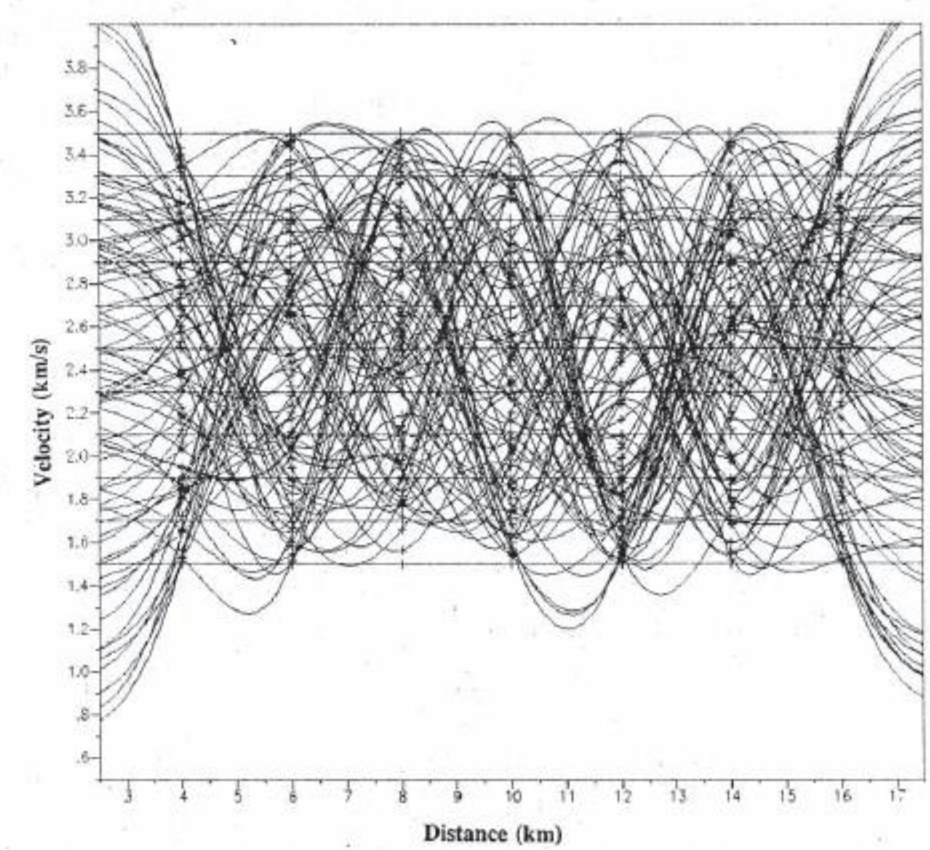

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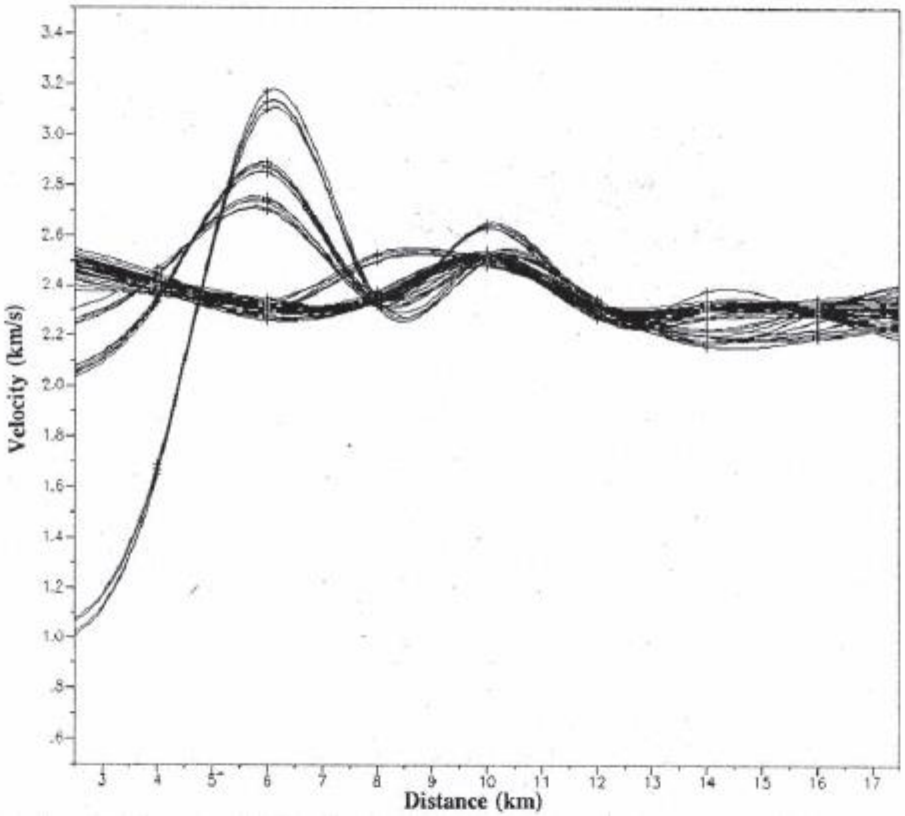

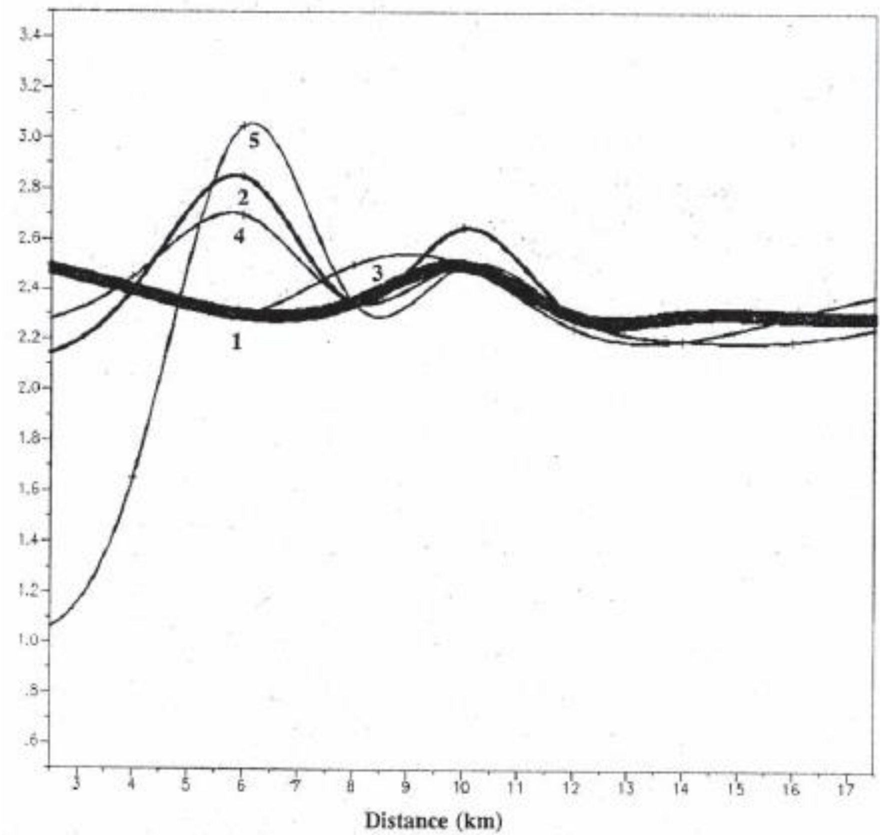

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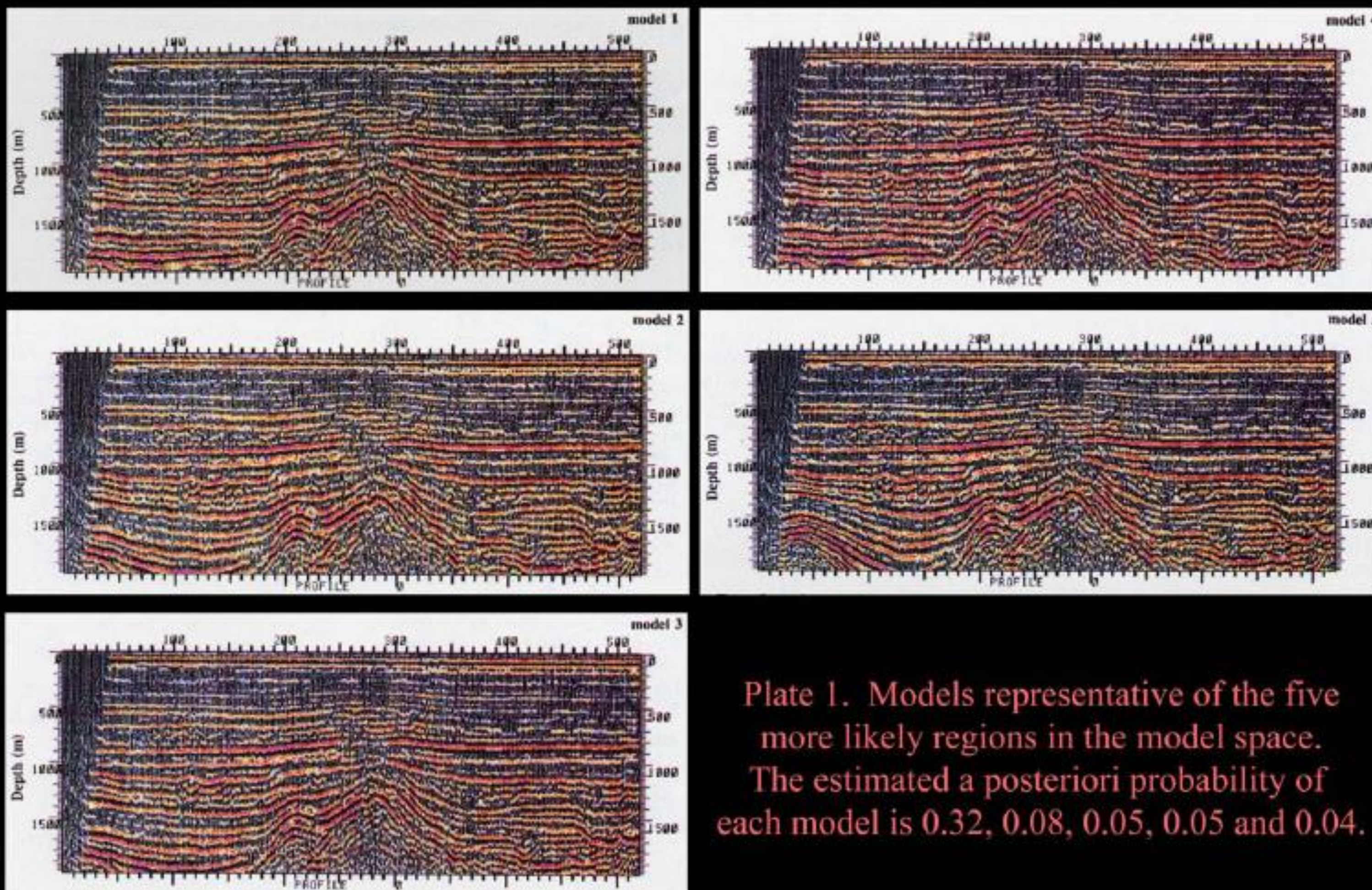
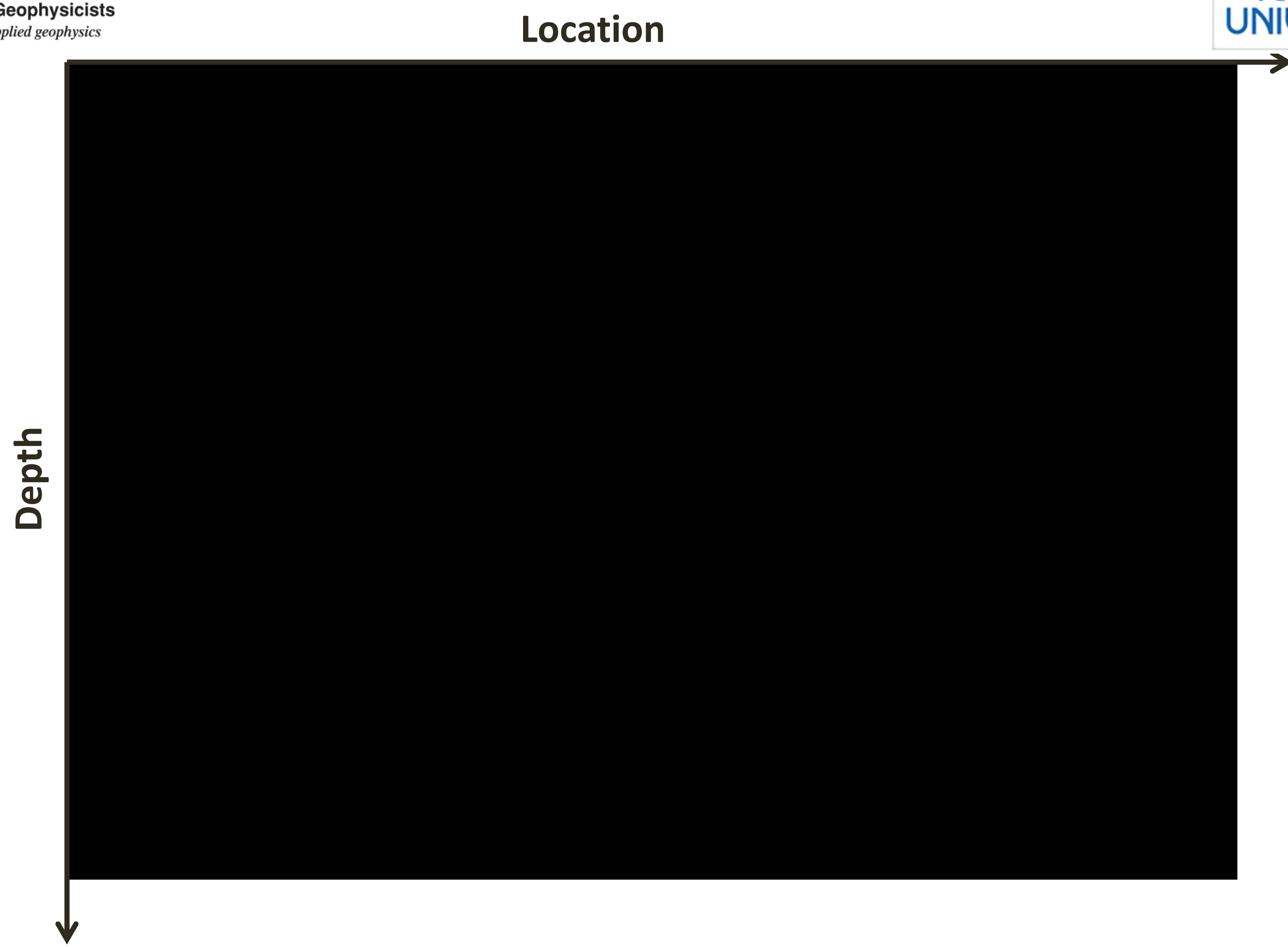


Plate 1. Models representative of the five more likely regions in the model space. The estimated a posteriori probability of each model is 0.32, 0.08, 0.05, 0.05 and 0.04.


Location
Depth

The present status of FWI

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The so-called “full-waveform inversion” or FWI is “« …technical bubble, and self-proclaimed seismic cure-all”

“… all the current approaches to so-called full-waveform inversion are:
(1) using the wrong data,
(2) using the wrong algorithms, and
(3) using the wrong Earth model, as well.»

“A timely and necessary antidote to indirect methods and so-called P-wave FWI»

A. WEGLEIN, TLE, 2013

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The present status of FWI

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"The truth is… seismic waves that propagate in the earth hardly satisfy any wave equations."

Migration Velocity Inversion with Semblance Analysis
H. Zhou, Q. Liao & F. Ortigosa, Repsol, EAGE 2009

Popper, Bayes and The Inverse Problem

ALBERT TARANTOLA, Nature, 2006

“Observations should only be used to falsify possible solutions, not to deduce any particular solution.”

Summary

- Today a large number of semi-heuristic algorithms and strategies exist, but they **do not solve** the inverse problem

- Bayesian approach **in principle** provides a framework for combining the a priori model information with the information contained in the data to arrive at the a posteriori model distribution

- ***In fact, the validation of our assumptions regarding the real model is the most crucial step in inversion.***

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What do we need to achieve alternative subsurface image?

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- ***Develop a fundamentally new procedure that can construct the image without precise velocity information***

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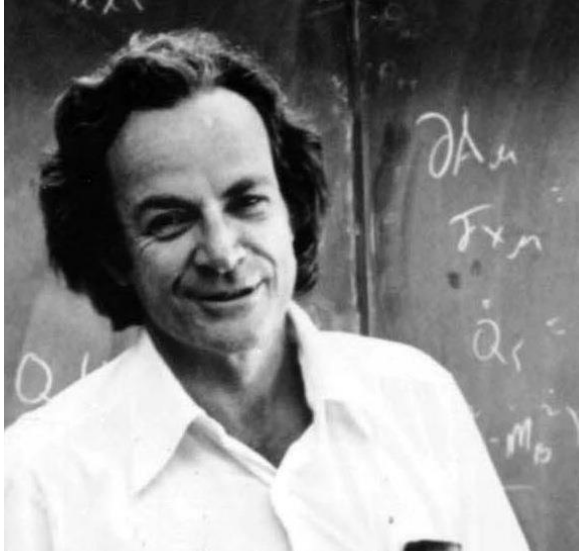


"An idea which looks completely paradoxical at first, if analyzed to completion in all its details and in experimental situations, may in fact not be paradoxical"

Richard Feynman

Quantum seismic imaging: is it possible?

Feynman‘s « path-integral » picture of the world:

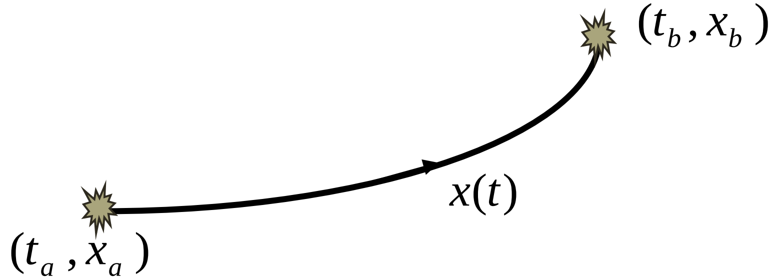
- The world is kind of tapestry in which all kind of things can gone

- To predict the future you start with a known state in the past, allow everything to happen in the intermediate time and simply add up the contributions from all the histories

- Each history contributes certain probability amplitude. The amplitude is just an integral over time and space volume between past and the future

Classical mechanics

In classical Newton's theory a particle have just a single trajectory



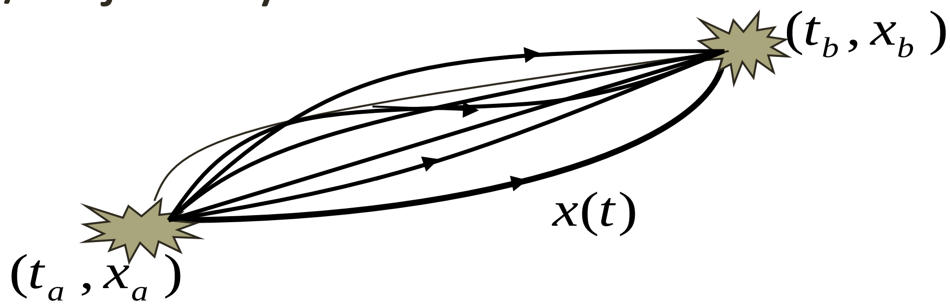
The classical path $\widehat{x}(t)$ is singled out of all possible paths as the one having the least action S

$$S = \int_{t_a}^{t_b} dt\, L(x, \dot{x}, t)$$

where L – is the Lagrangian

Quantum mechanics

In Feynman's path-integral approach, a particle does not have just a single history/trajectory as it would have in classical theory.



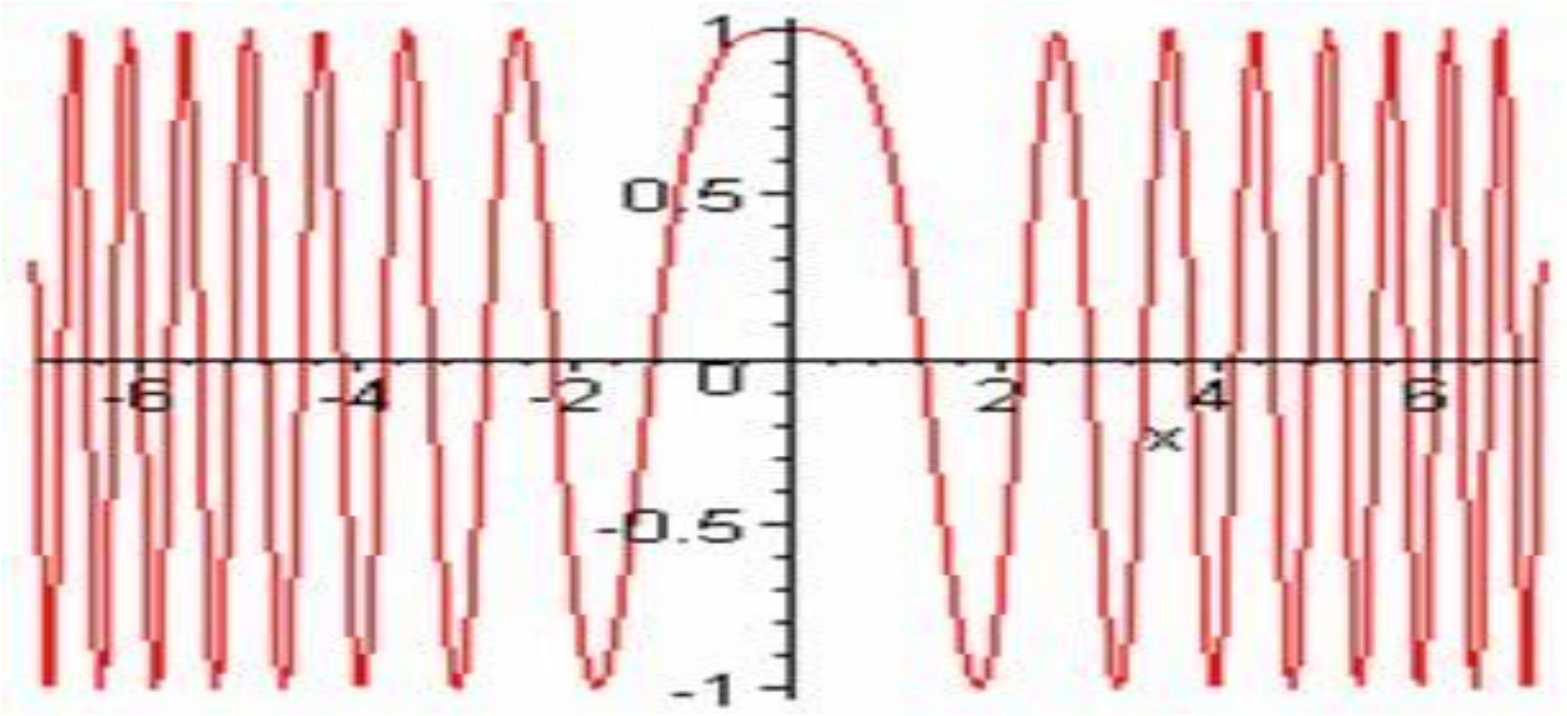
Instead of only considering the classical trajectory, consider every possible path between *a* and *b*. Each path contributes to the total amplitude. This amplitude is

$$K(b,a) = \sum_{\substack{all\ paths \\ from\, a\, to\, b}} A_i(x(t))$$

where *A* is the contribution of each individual path

$$A_i(x(t)) = const * \underbrace{\exp[iS_i(x(t))/\hbar]}_{w}$$

Quantum mechanics and Newton's physics



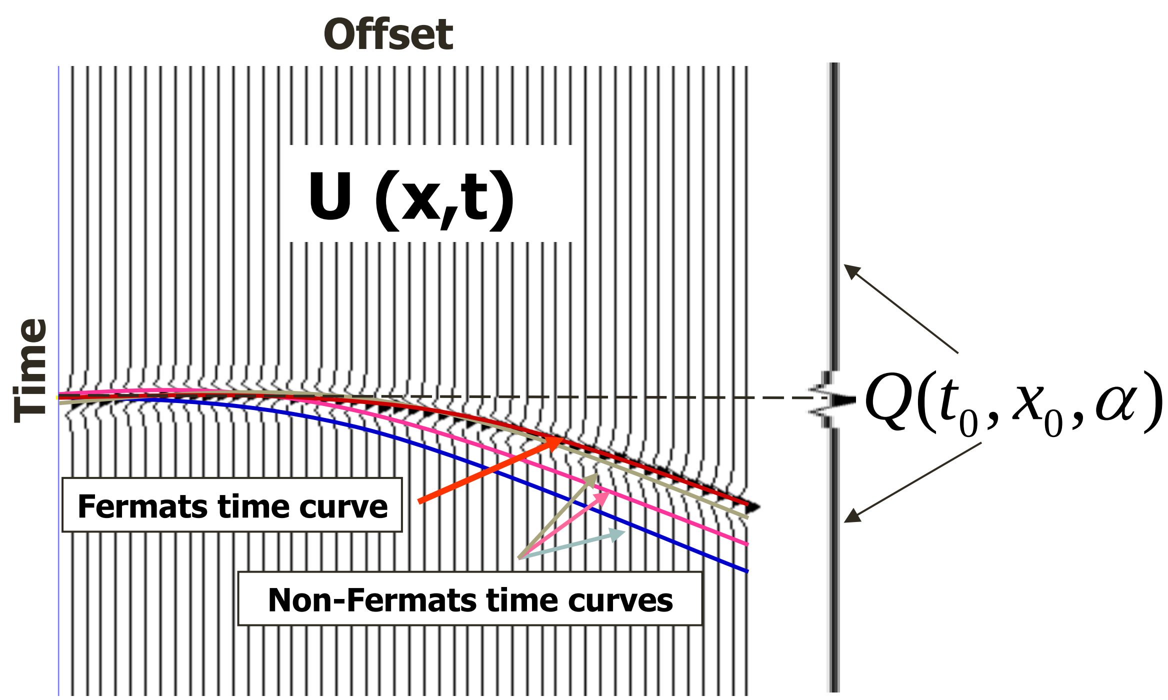
$$w(x(t)) = \exp[iS(x(t))/\hbar]$$

STACK

$$Q(t_0, x_0; \alpha) = \int dh \int dt U(t,h) \delta(t - \tau(x_0, t_0, h; \alpha))$$

where $U(t, h)$ is the recorded CDP gather for location x_0, h - is the offset to be summed over the measurement aperture. The quantity $\tau = \tau(x_0, t_0, h; \alpha)$

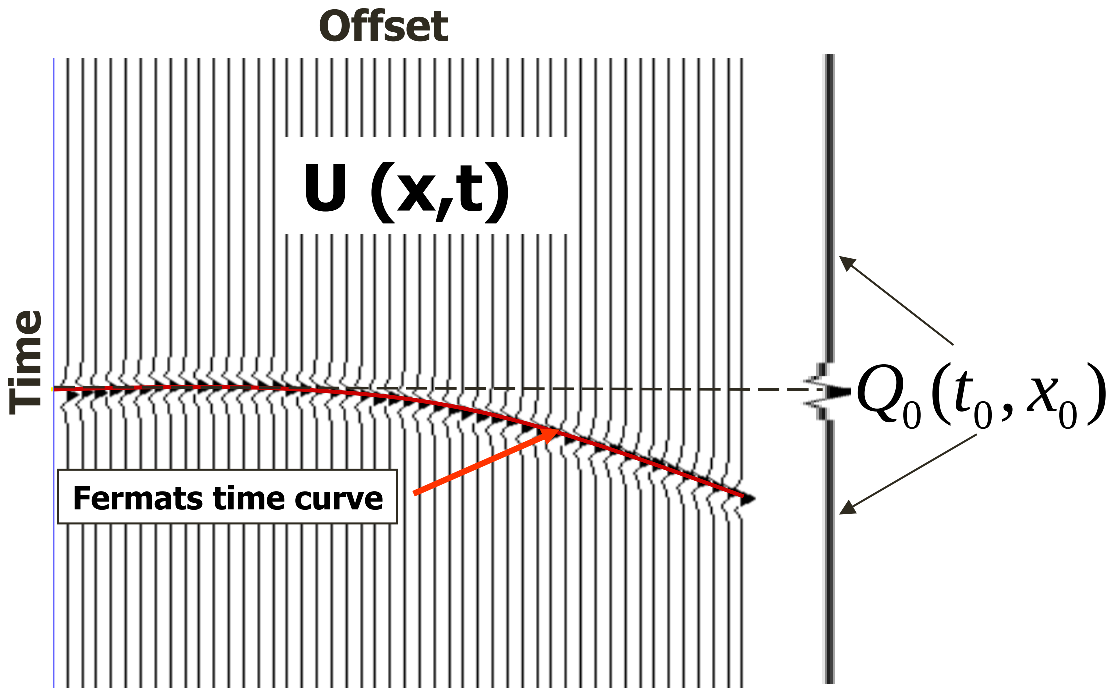
represents the time-integration path/trajectory, which is parametrized by a parameter α



STACK

The conventional zero-offset stack is obtained by optimizing for α, i.e.

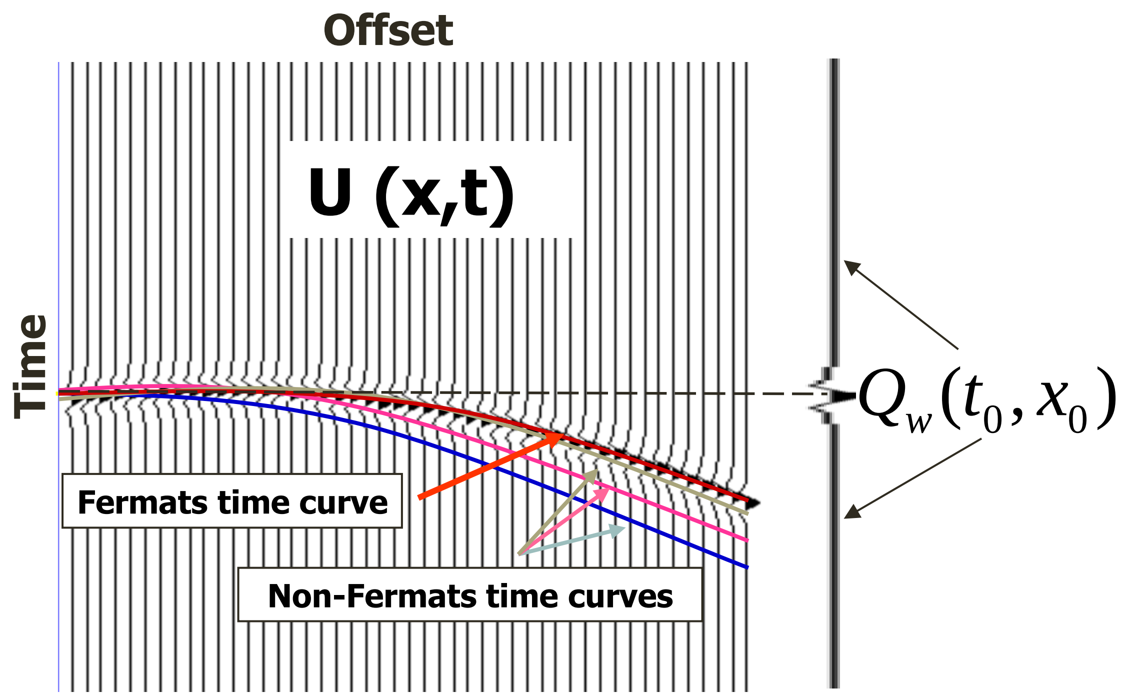
$$Q_0(t_0, x_0) = Q(t_0, x_0; \alpha_0)$$



Path-integral stack

$$Q_w = \int d\alpha w(\alpha) Q(\alpha)$$

Instead of stacking seismic data along only one time trajectory corresponding to the Fermat path our construction involves summation over all possible time trajectories.



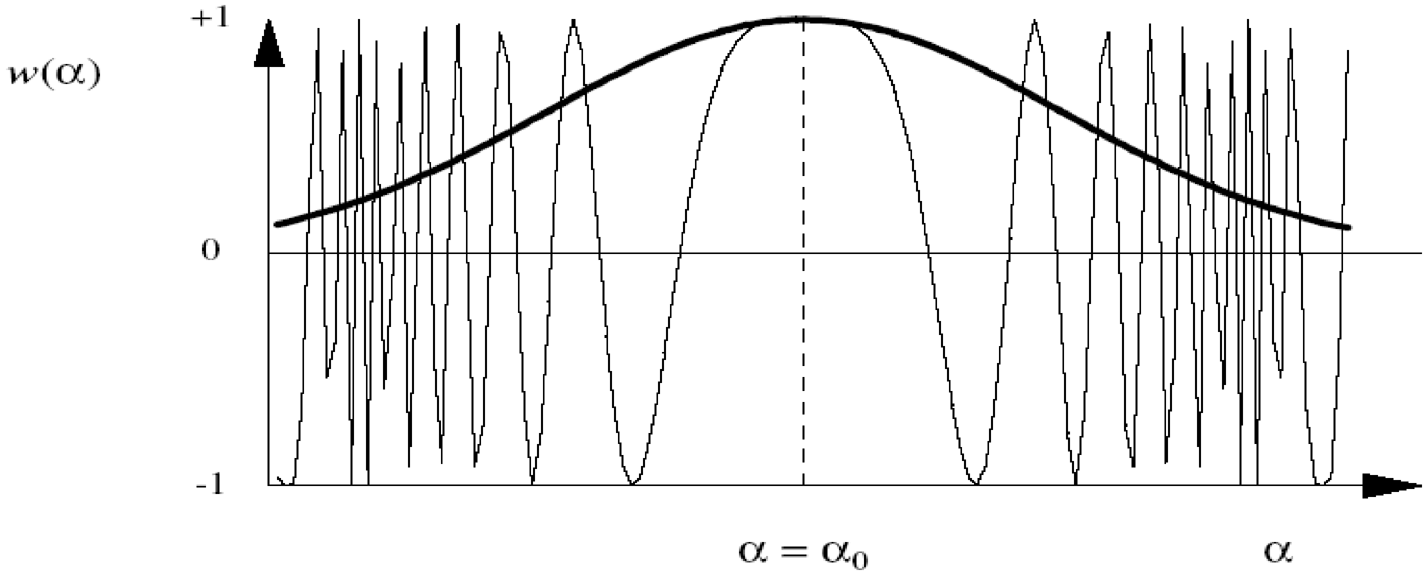
Path-integral stack

The path-integral stack Q_F approach the classical limit Q_O for $\beta \rightarrow \infty$.

This can be shown by a stationary-phase approximation under the assumptions

$$Q(\alpha) \rightarrow 0,\ \ when\ |\alpha - \alpha_0| \rightarrow \infty, S'(\alpha_0) = 0, S''(\alpha_0) \neq 0$$

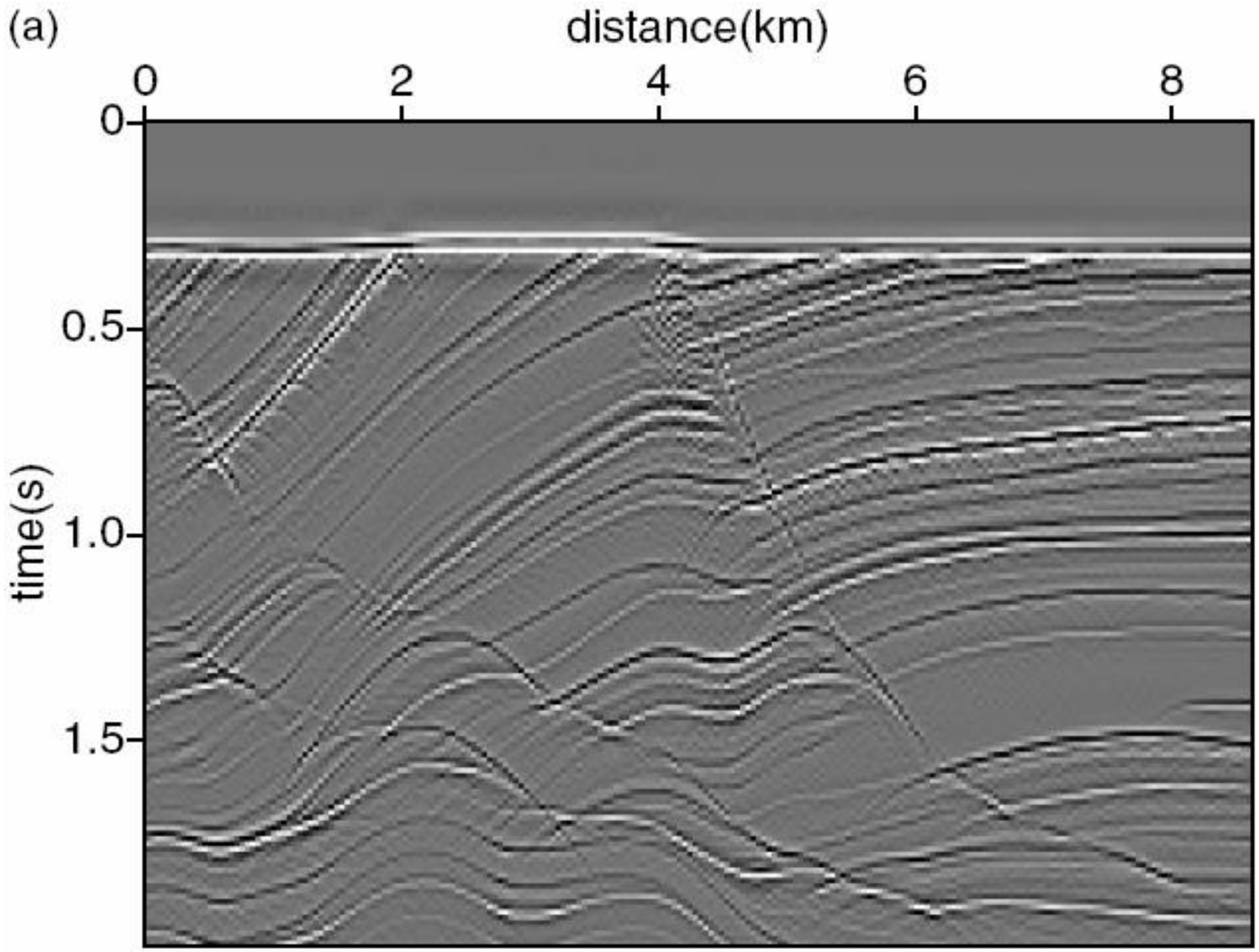
$$Q_F \approx \exp[i\beta S(\alpha_0) + i\mu\pi/4]\sqrt{\frac{2\pi}{\beta|S''(\alpha_0)|}}Q_0$$



Path-integral imaging

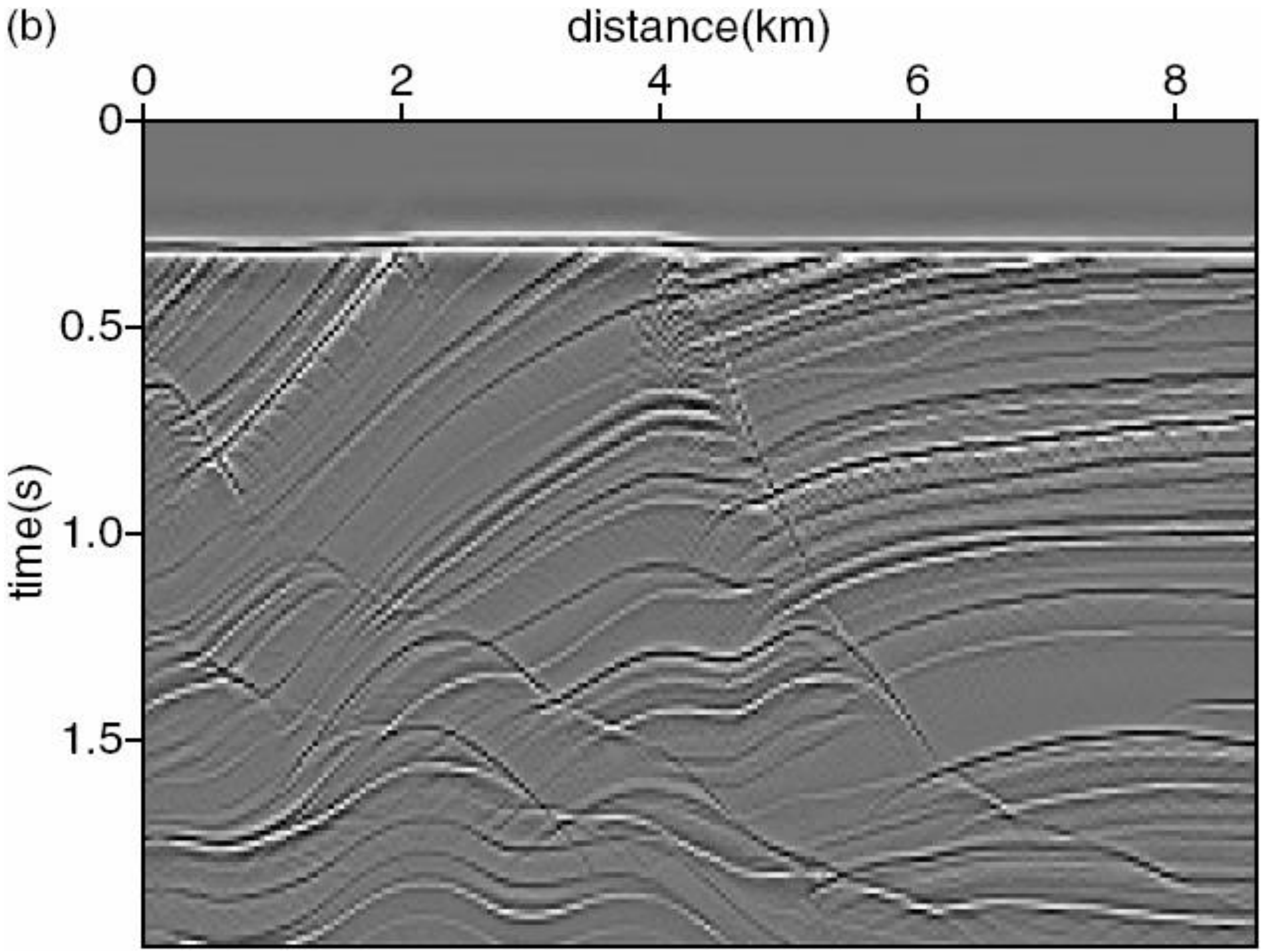
The imaging consists of weighted summation along a representative sample of all possible travel time curves (paths) between the source and observation points

Path-integral imaging



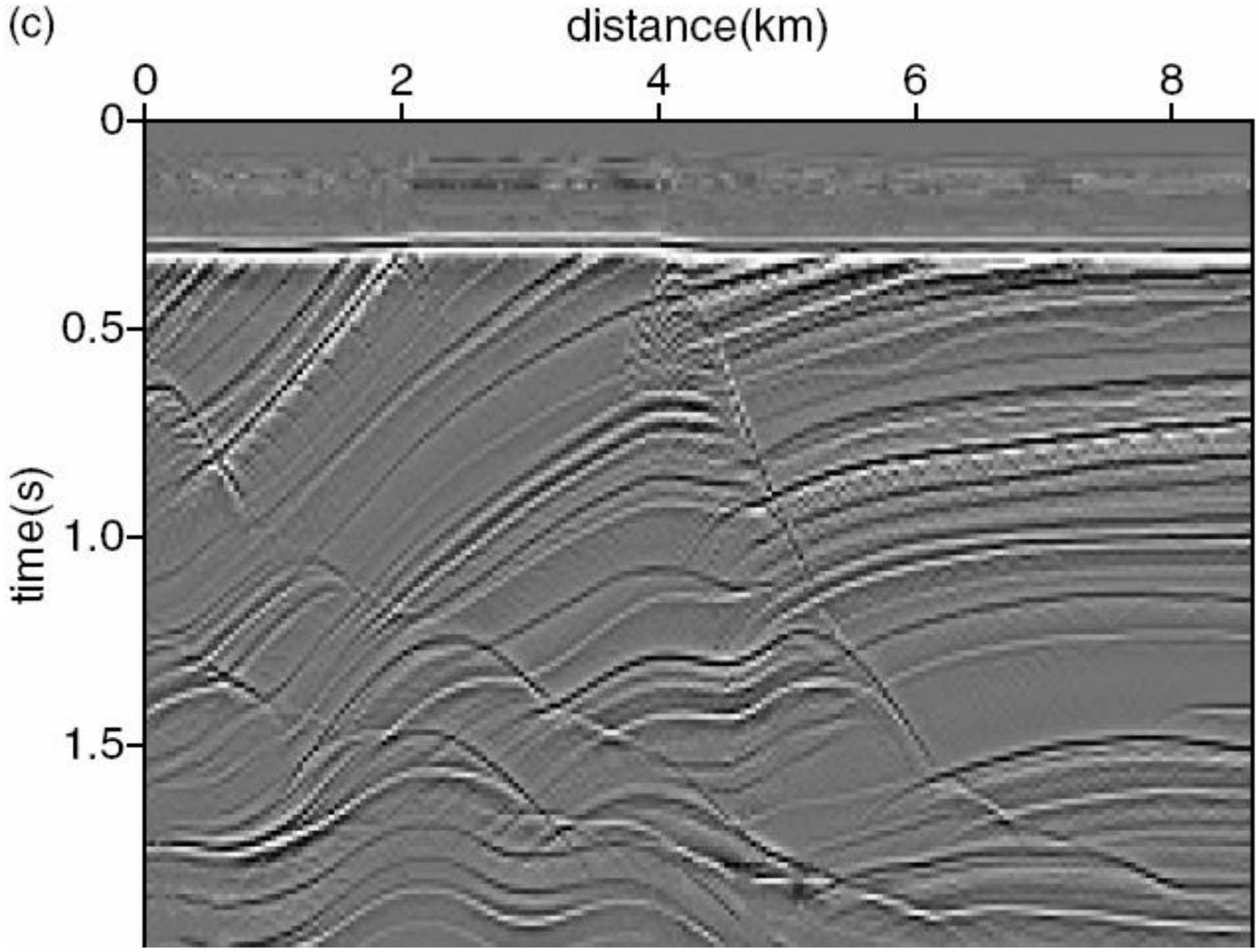
Zero-offset section

Path-integral imaging



Stacked section

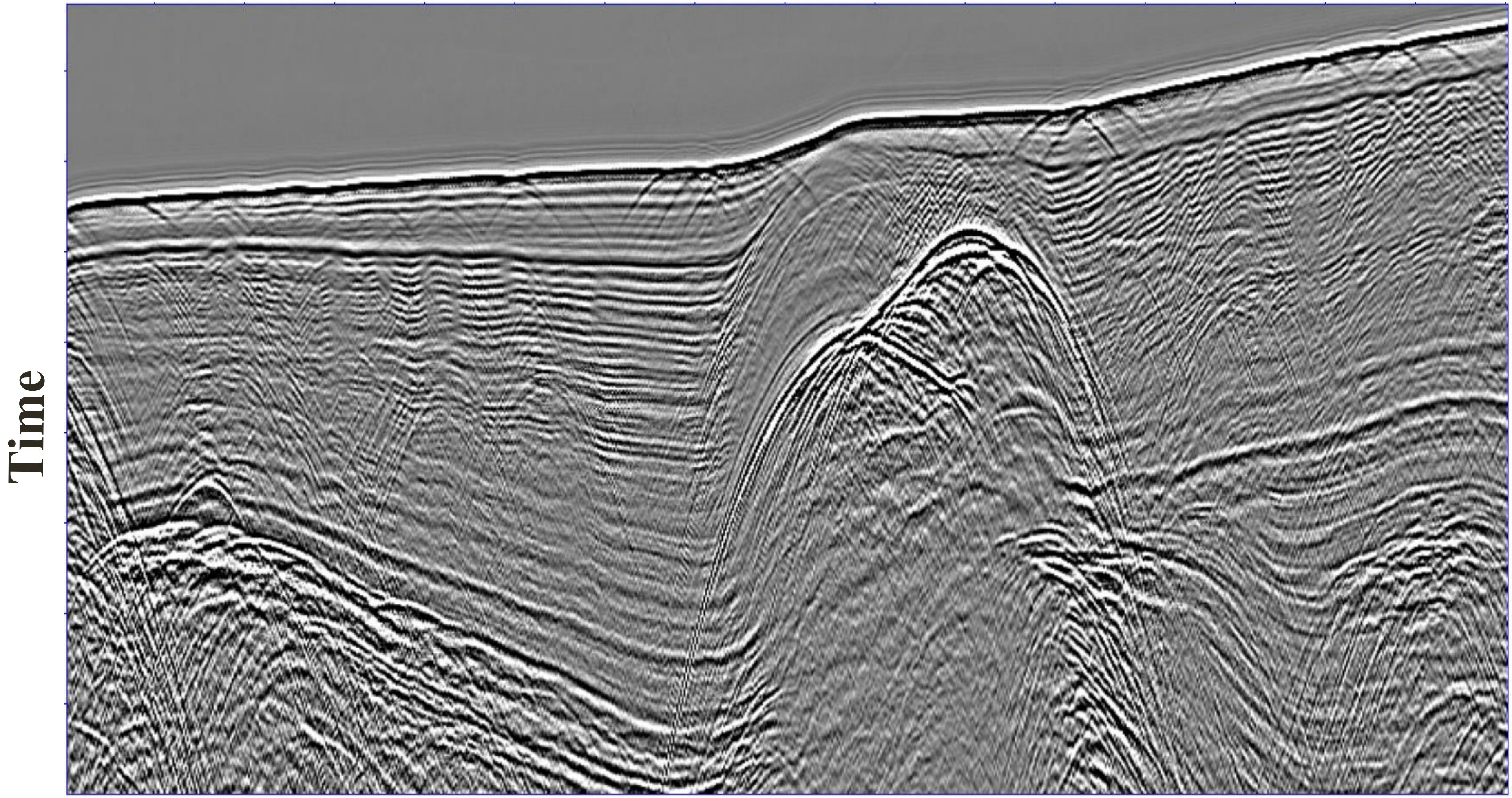
Path-integral imaging



Path-summation section

Path-integral imaging

CMP



Near offset section

Path-integral imaging

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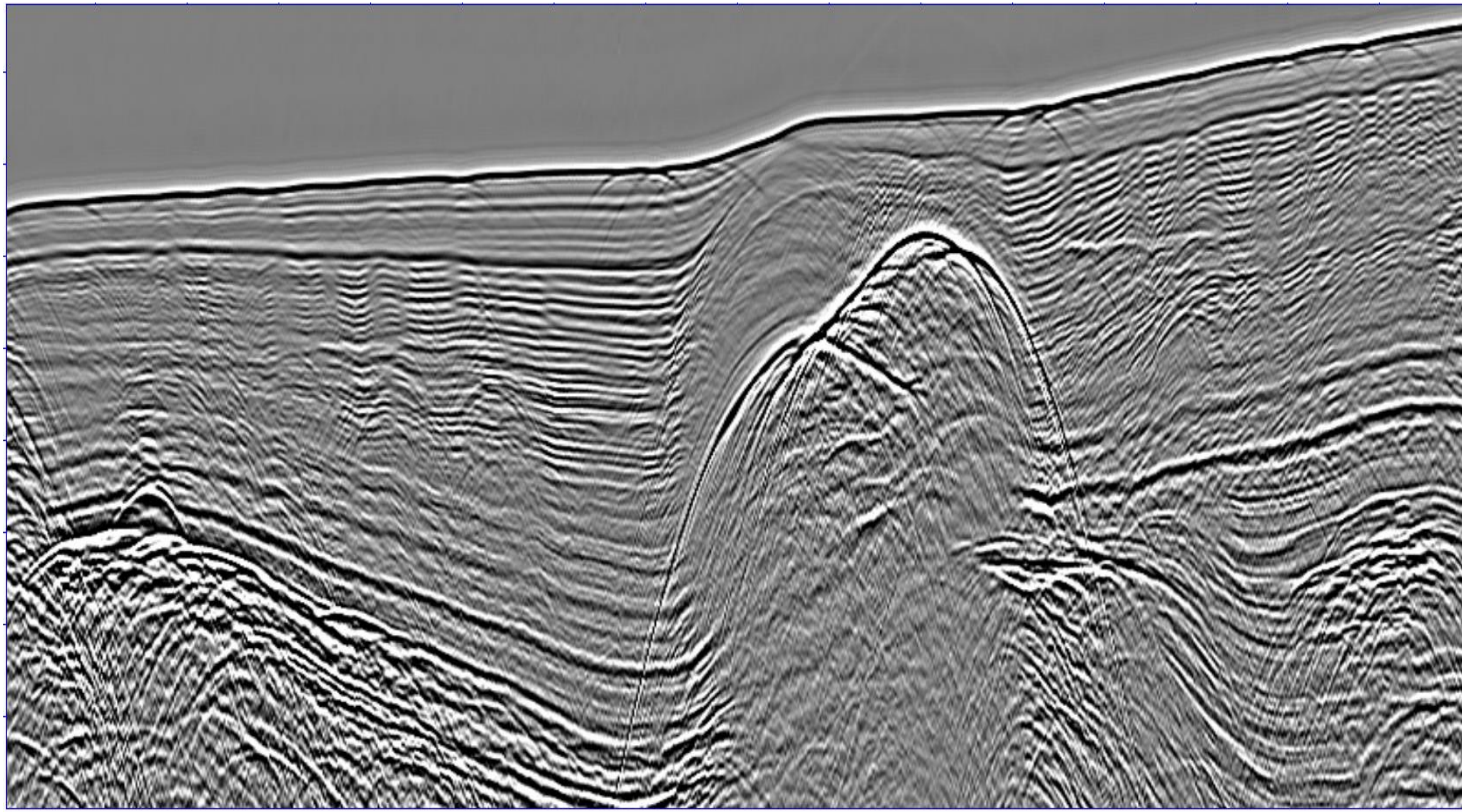
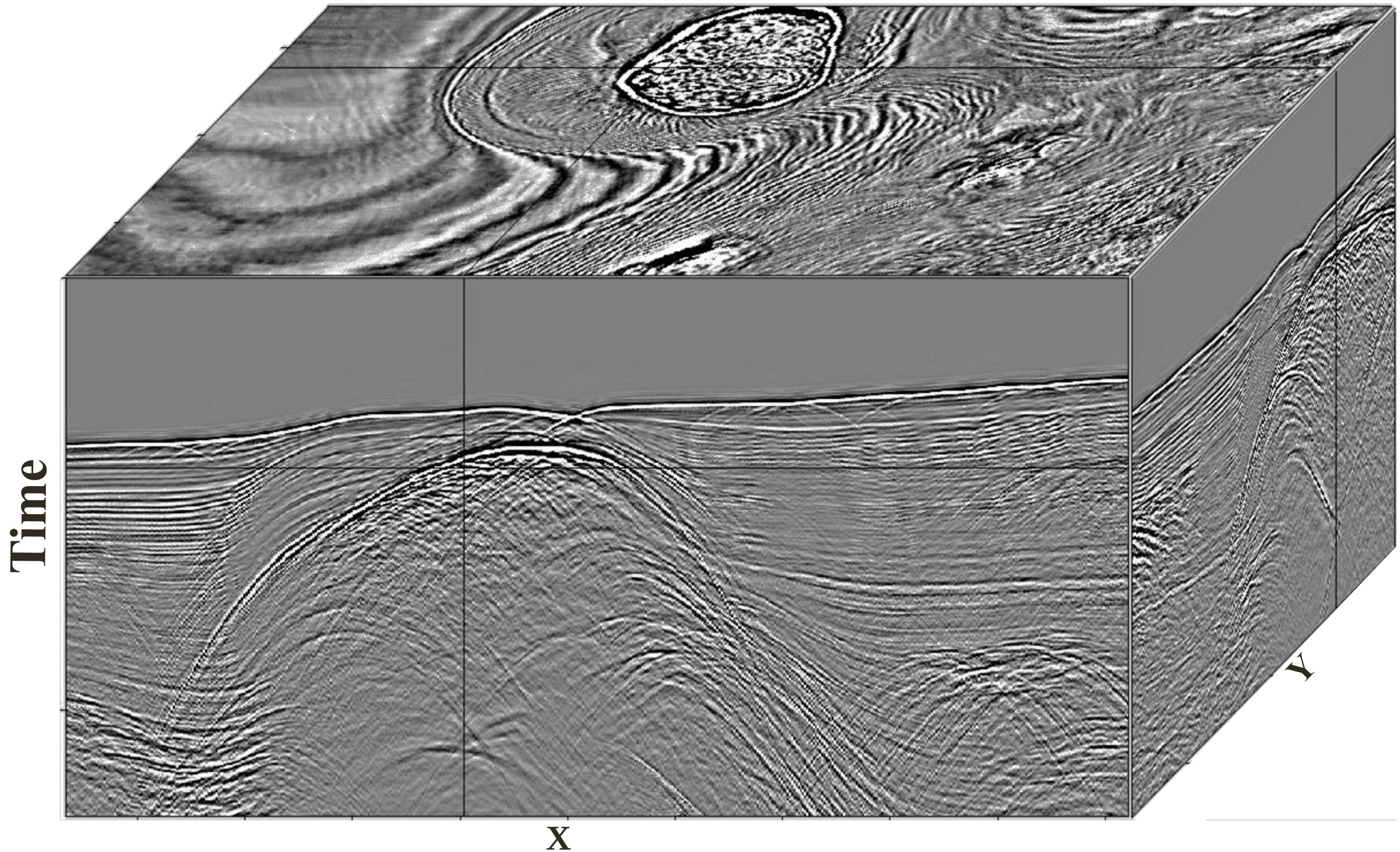


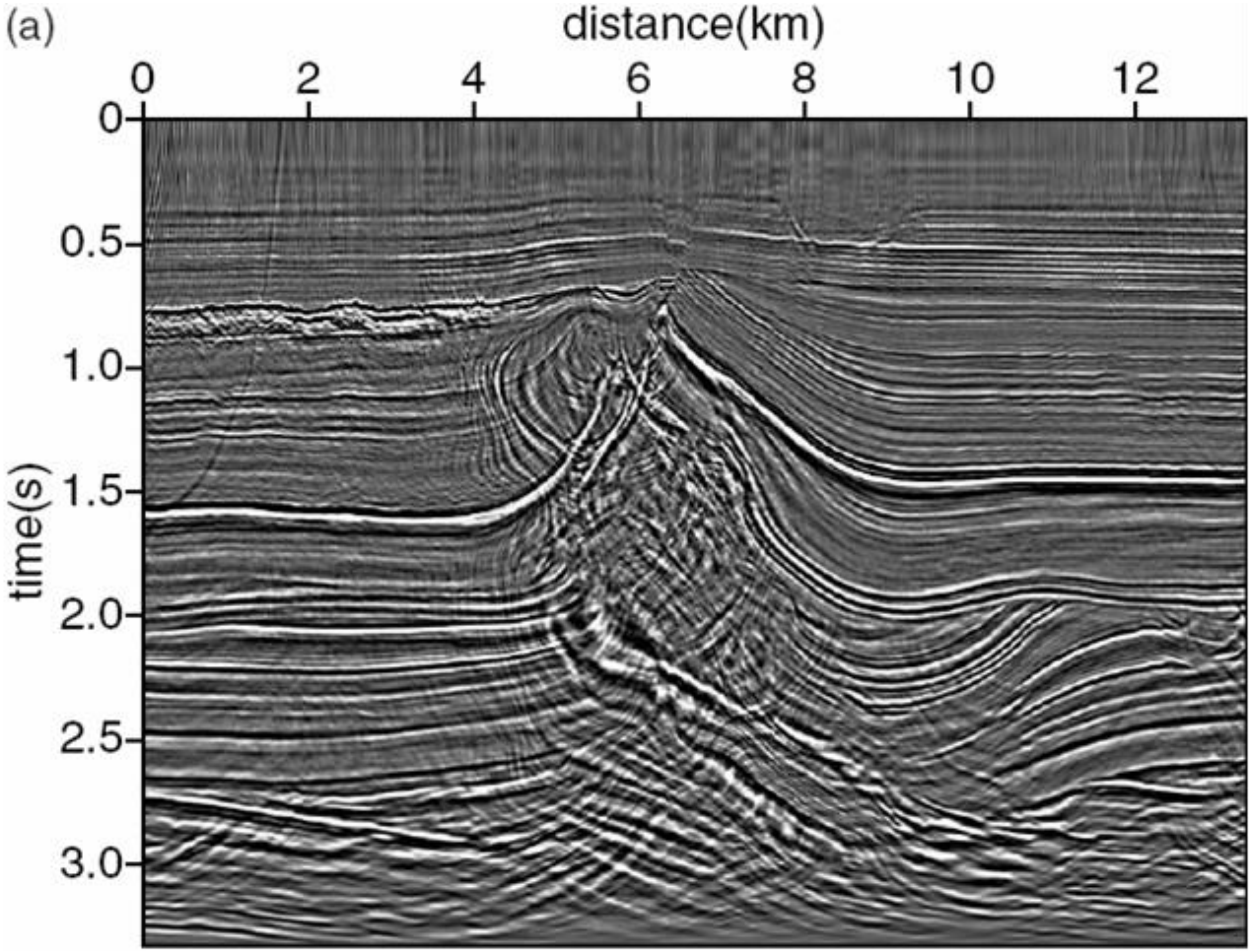
Path Summation stack

Path-integral imaging



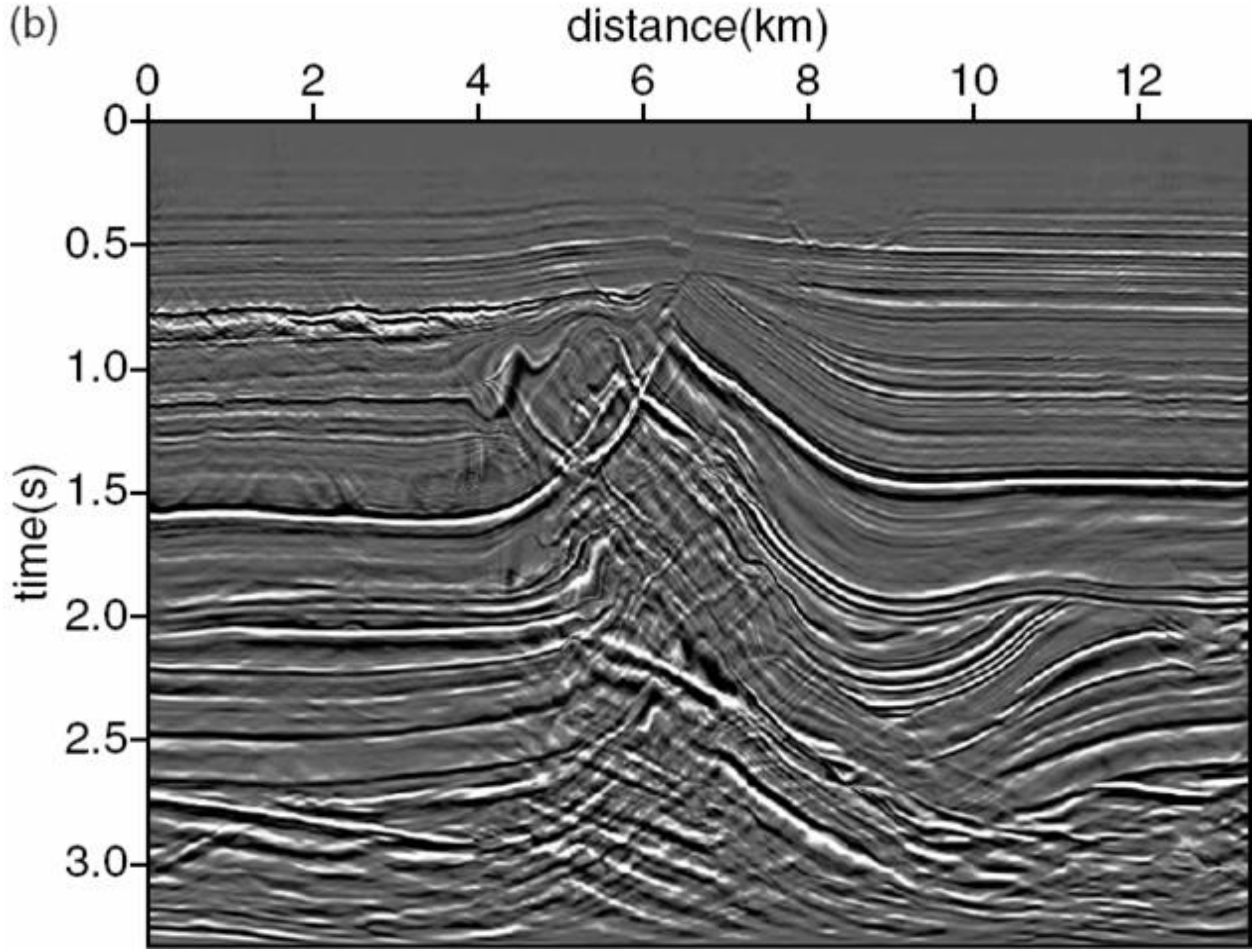
Path Summation cube

Path-integral imaging



PSTM

Path-integral imaging



Path-summation time migration

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Summary

- Quantum seismic imaging method provides a new framework for subsurface imaging without precise knowledge or selection of a velocity model
- Quantum seismic imaging can be considered as a model-independent technique, since it does not involve any velocity or parameter estimation in a common sense
- The image is constructed by summation over many possible trajectories
- The quantum imaging converges to a standard imaging procedure only in trivial situations of a deterministic and known velocity model

“Today many quantum physicists believe that quantum principles in fact apply on all scales. By combining the (quantum) approach with other (e.g. mechanical) systems, or by applying its basic ideas in different contexts, it may be possible to bring quantum effects ever closer to our everyday experience.”

Lvovsky, Ghobadi, Simon, Chandra and Prasad
“Observation of micro-macro entanglement of light.”
Nature, Physics, 2013

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Path integrals have been introduced in seismic ***wave modeling*** (Lomax, 1999; Schlottmann, 1999).

Bayesian approach, Monte Carlo and simulated annealing methods can also be formulated and interpreted in terms of the Feynman path integral (Lemm et al., 2005, Lee et al., 2000).

Interferometry can be considered in the path integral framework.

"… physics … has been reduced to calculating only the probability of an event, and not predicting exactly what happens… Yes. That's the way it is: **Nature permits us to calculate only probabilities…**" (Feynman, 1988).

"Good continuous reflectors are for kids; unconformities are for men."
Nigel Anstey

Diffraction imaging

- Reflection seismology is a method to estimate the properties of the Earth's subsurface from *reflected* seismic waves

- Specular *reflections* are the ones being used conventionally

- Specular *reflections* are generated by smooth interfaces

Diffractions are direct indicators of small scale heterogeneities in the subsurface…

There are many evidences that diffractive component of the wavefield is a key ingredient in establishing resolution…

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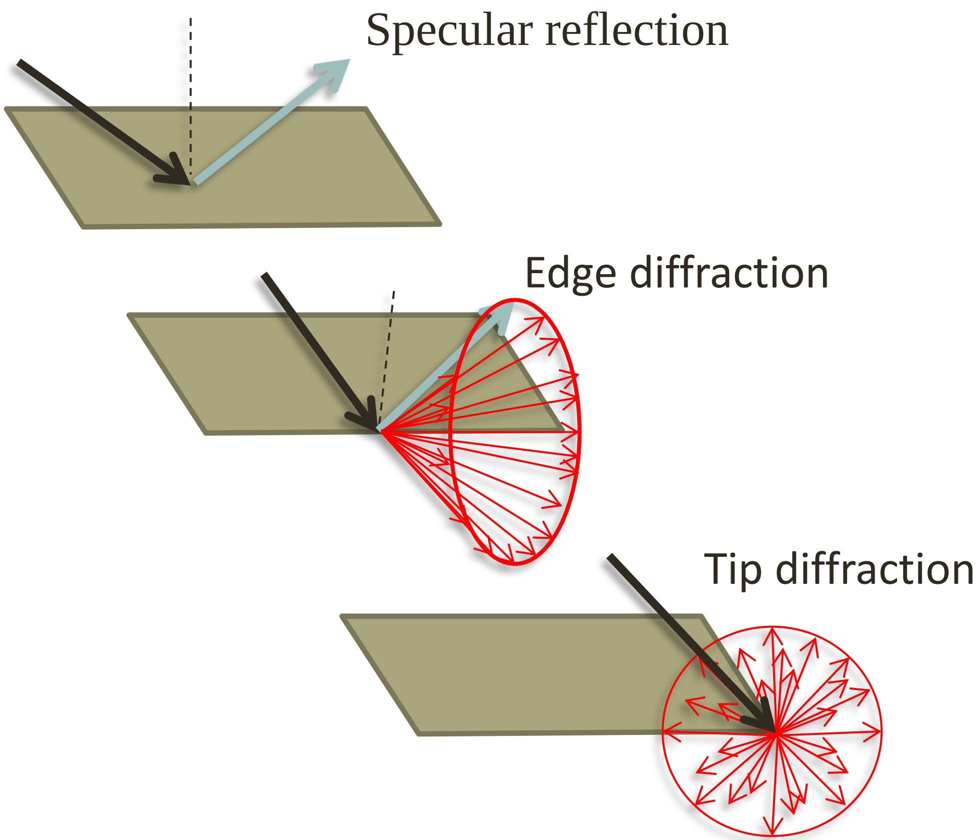
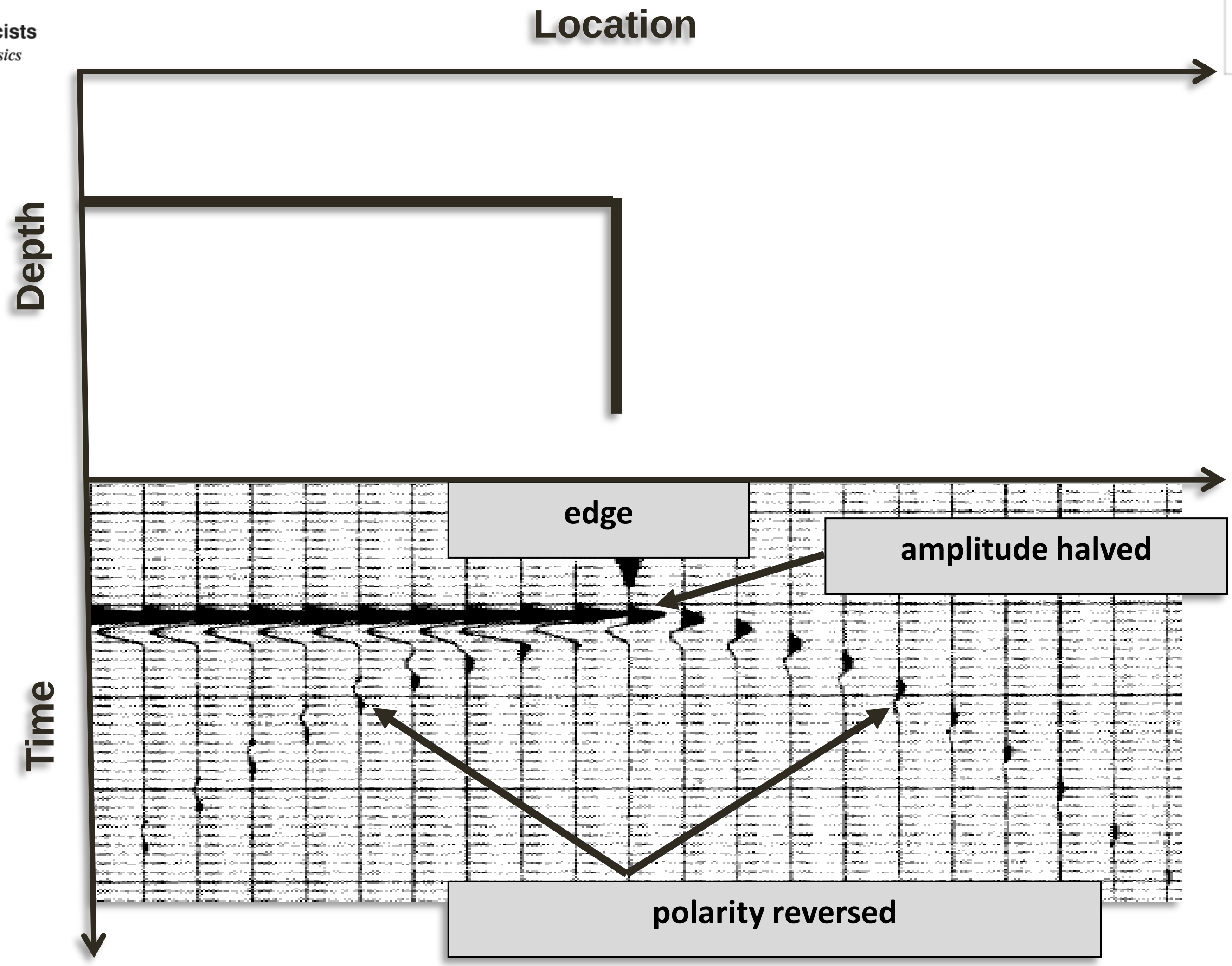


Seismic diffraction



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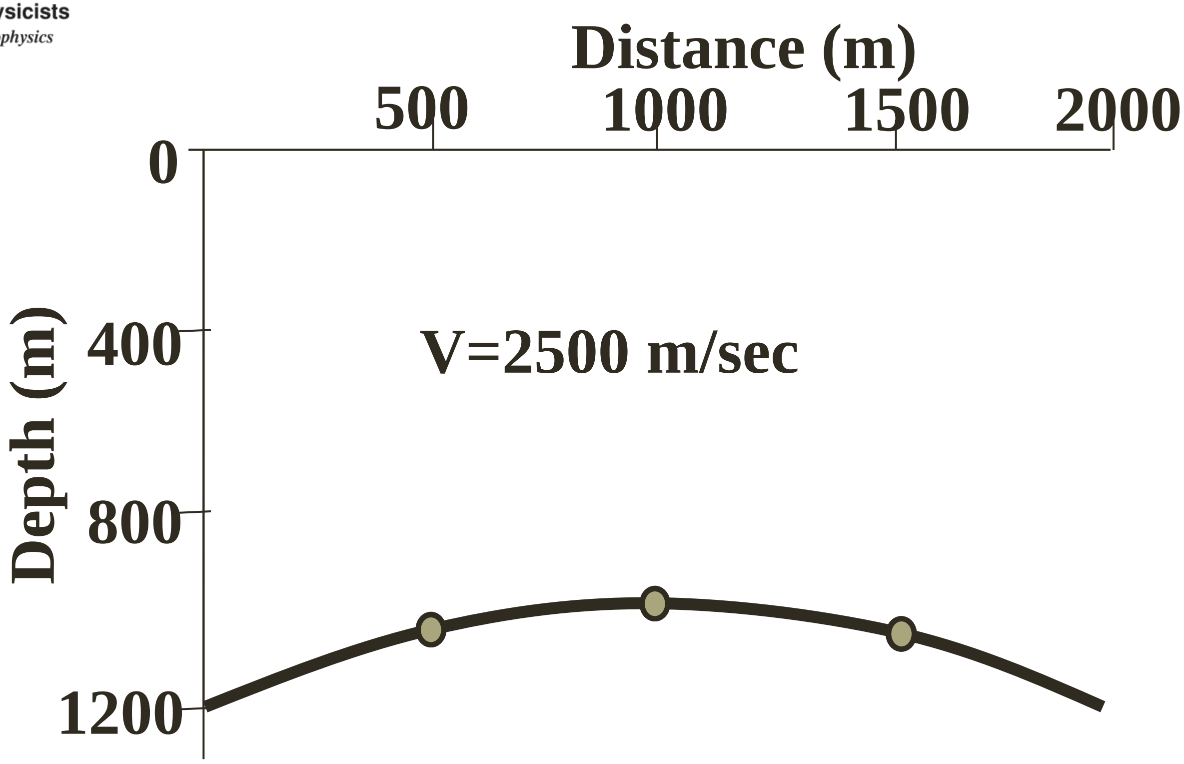
Ways to separate diffractive and reflective components:

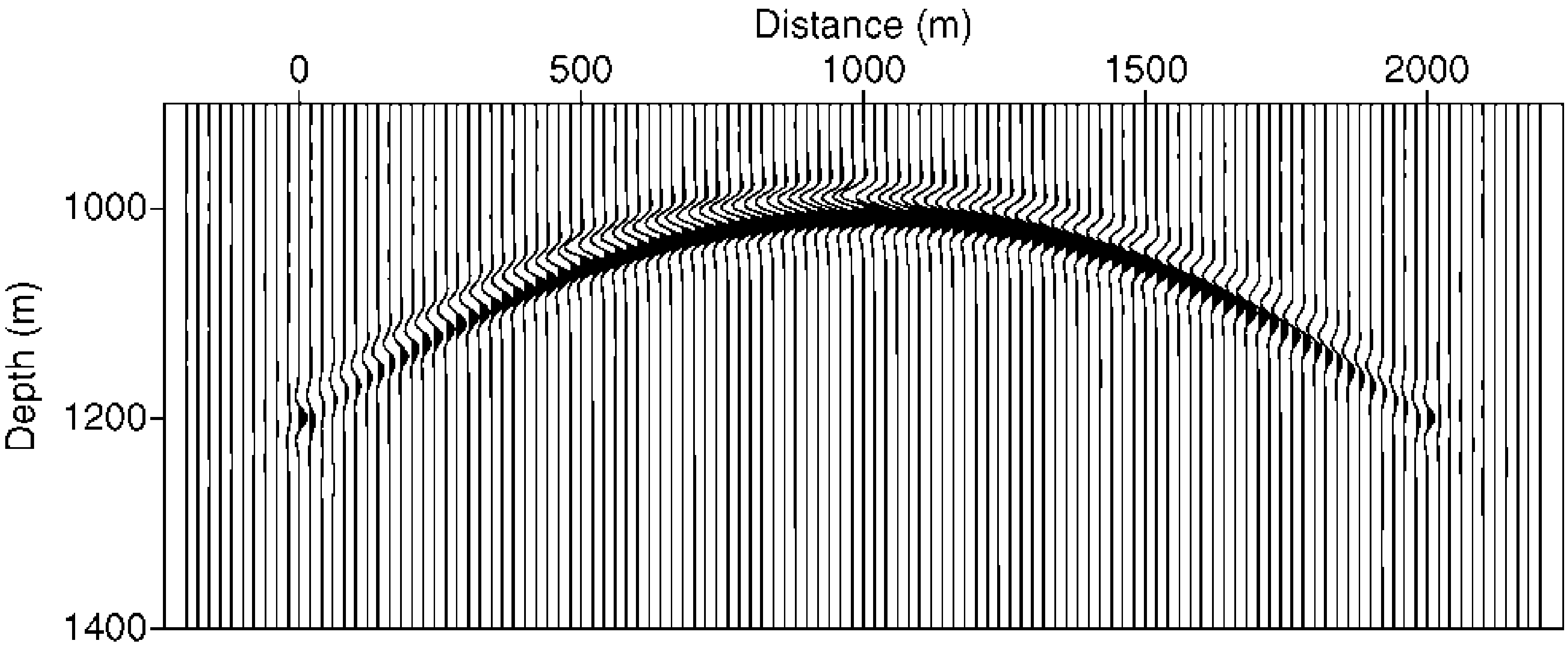
1. Weighted summation
2. Modified Kirchhoff migration
3. Plane wave destructor
4. Radon transform in the dip angle domain
5. Local Angle (LAD) domain

Model



Model with one reflector and a constant velocity. Three point scatterers are located directly on the reflector

PSDM



Prestack depth migration image of the full wavefield. Scatterers are almost invisible due to their weak amplitudes

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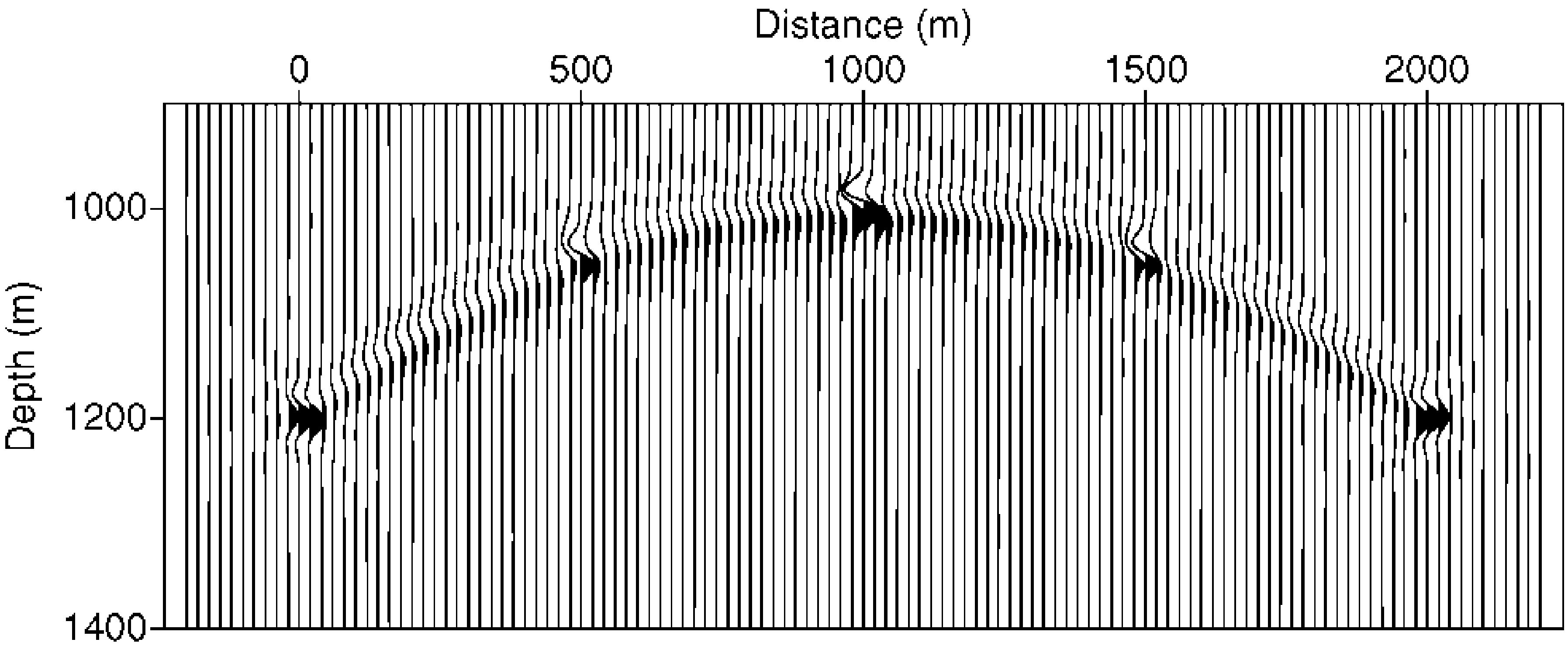
Diffraction image



Depth migration of diffraction shot gathers. Three point scatterers are well imaged and can be reliably detected from interpreting the image. Two additional diffractors located at the left and right sides of the figure are caused by the edges of the interface

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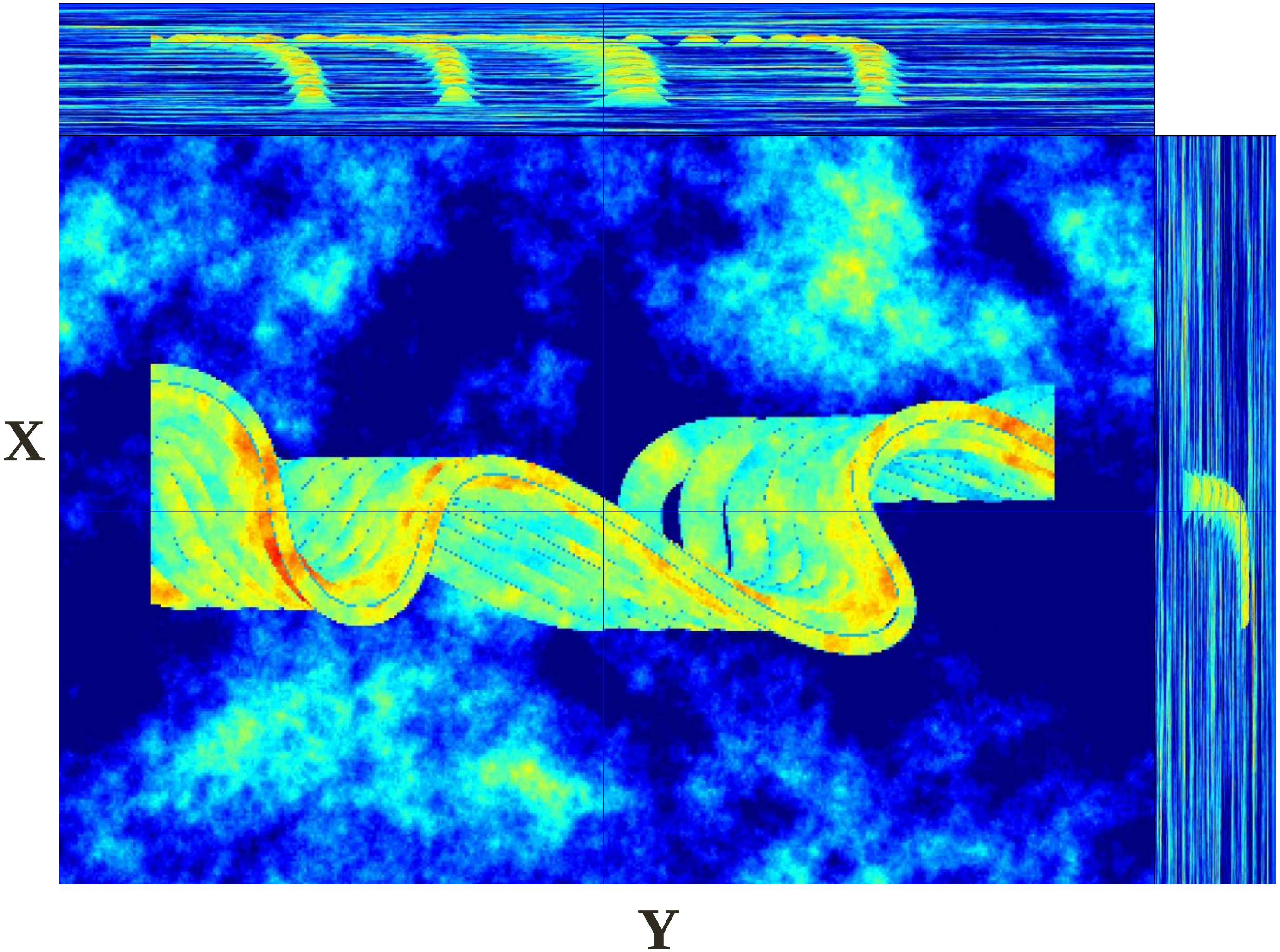
Synthetic velocity model for a channelized reservoir



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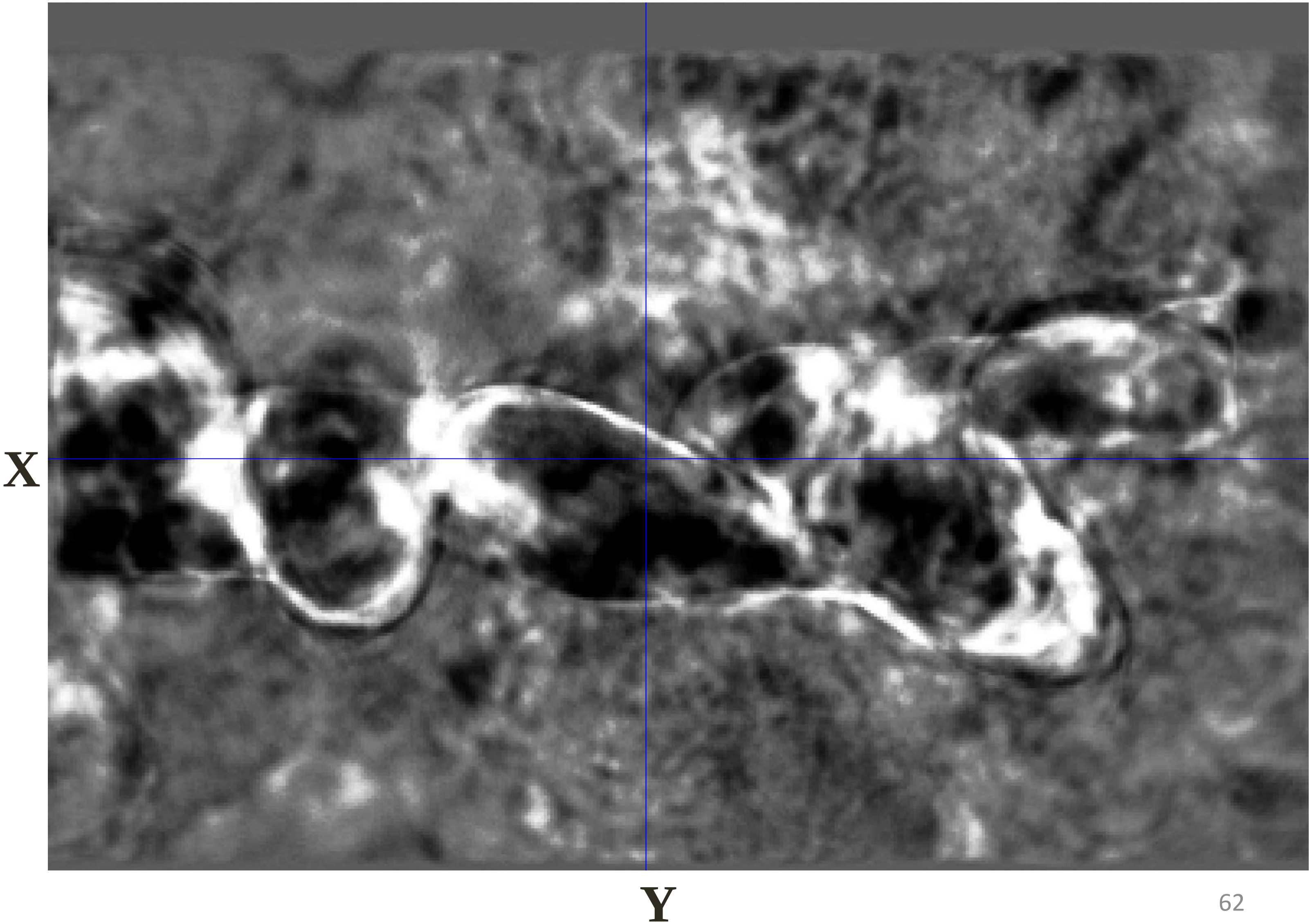
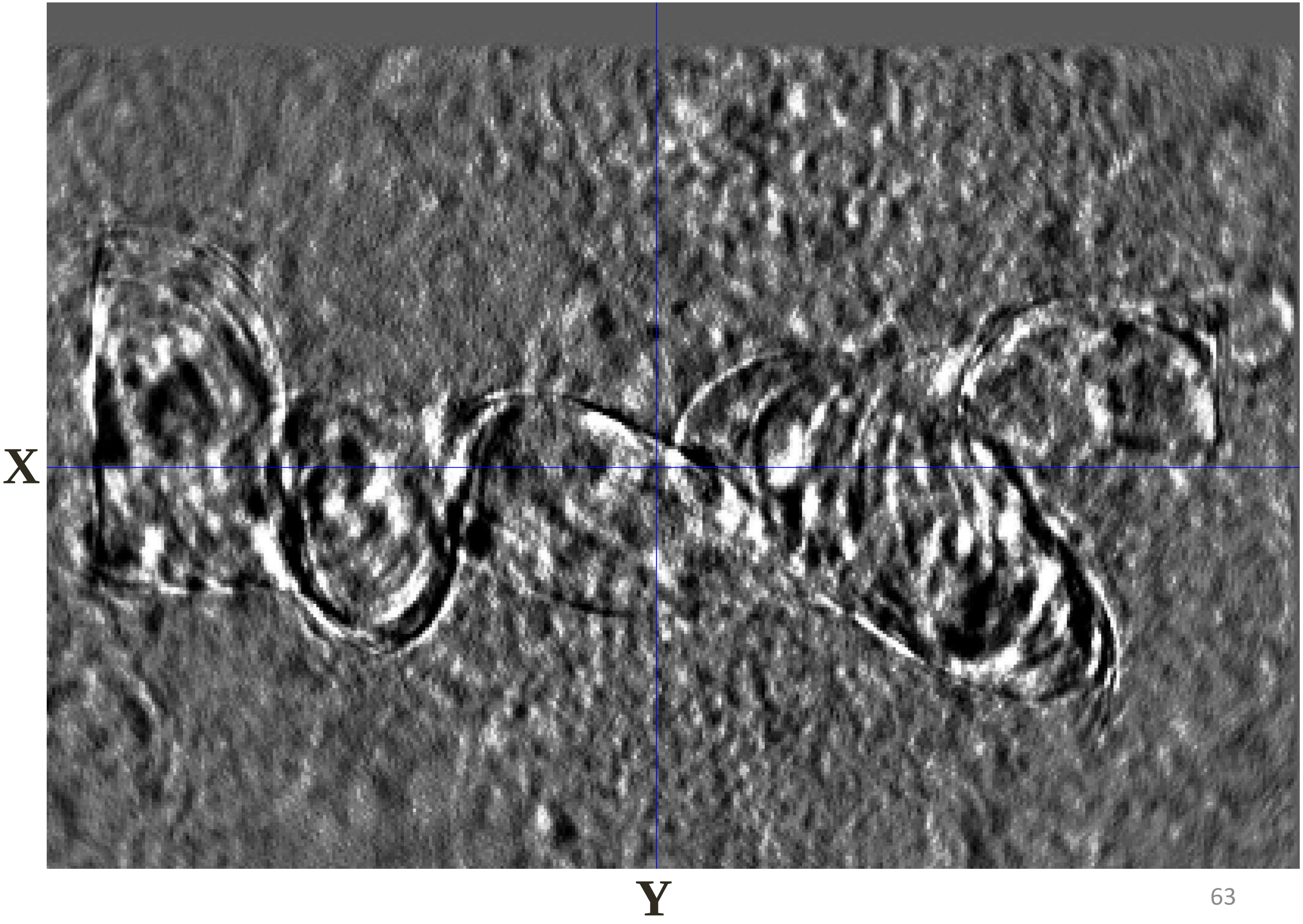
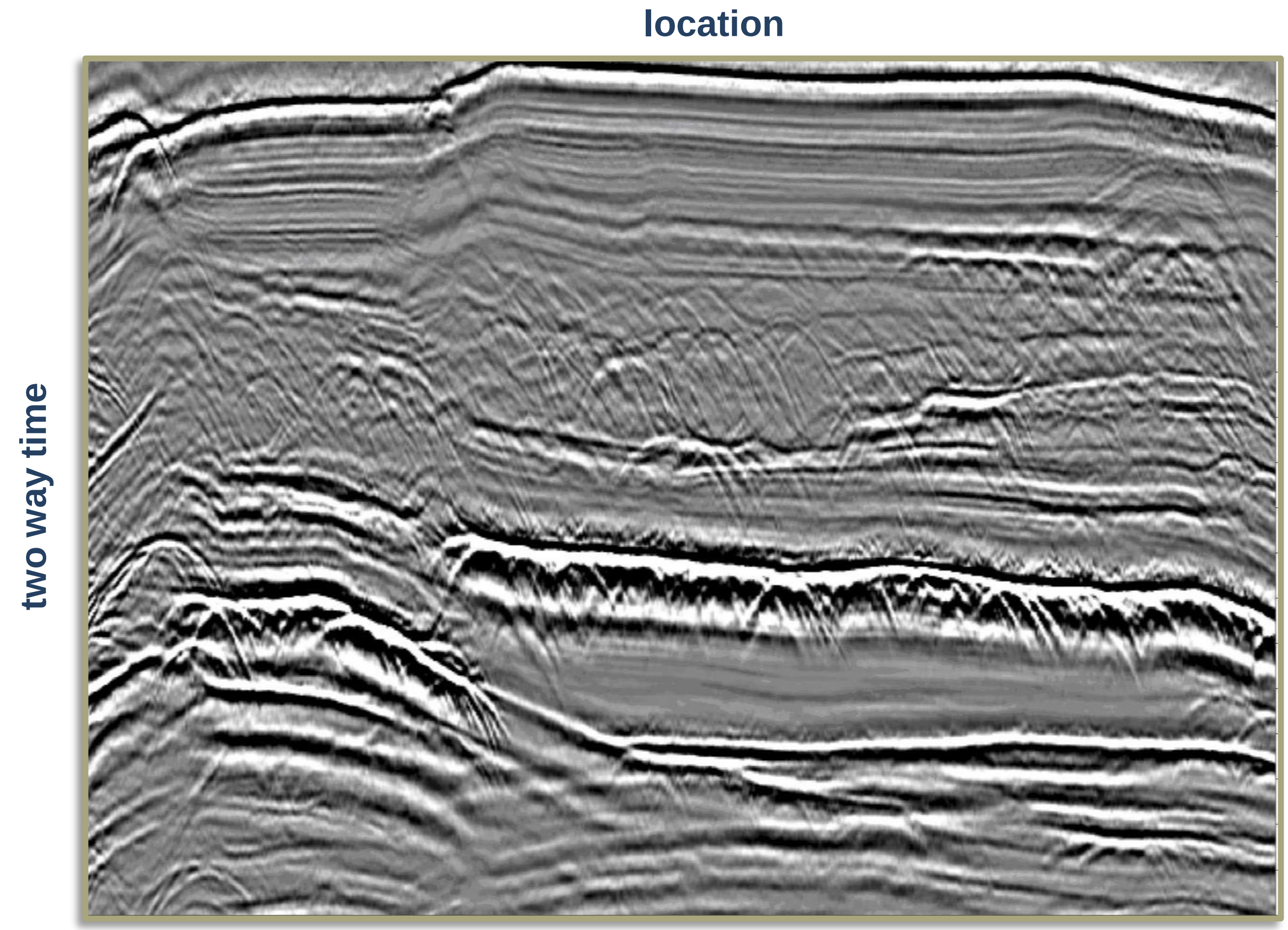
Migrated time slice of the full wavefield



Migrated time slice of the diffractive component



Stacked section

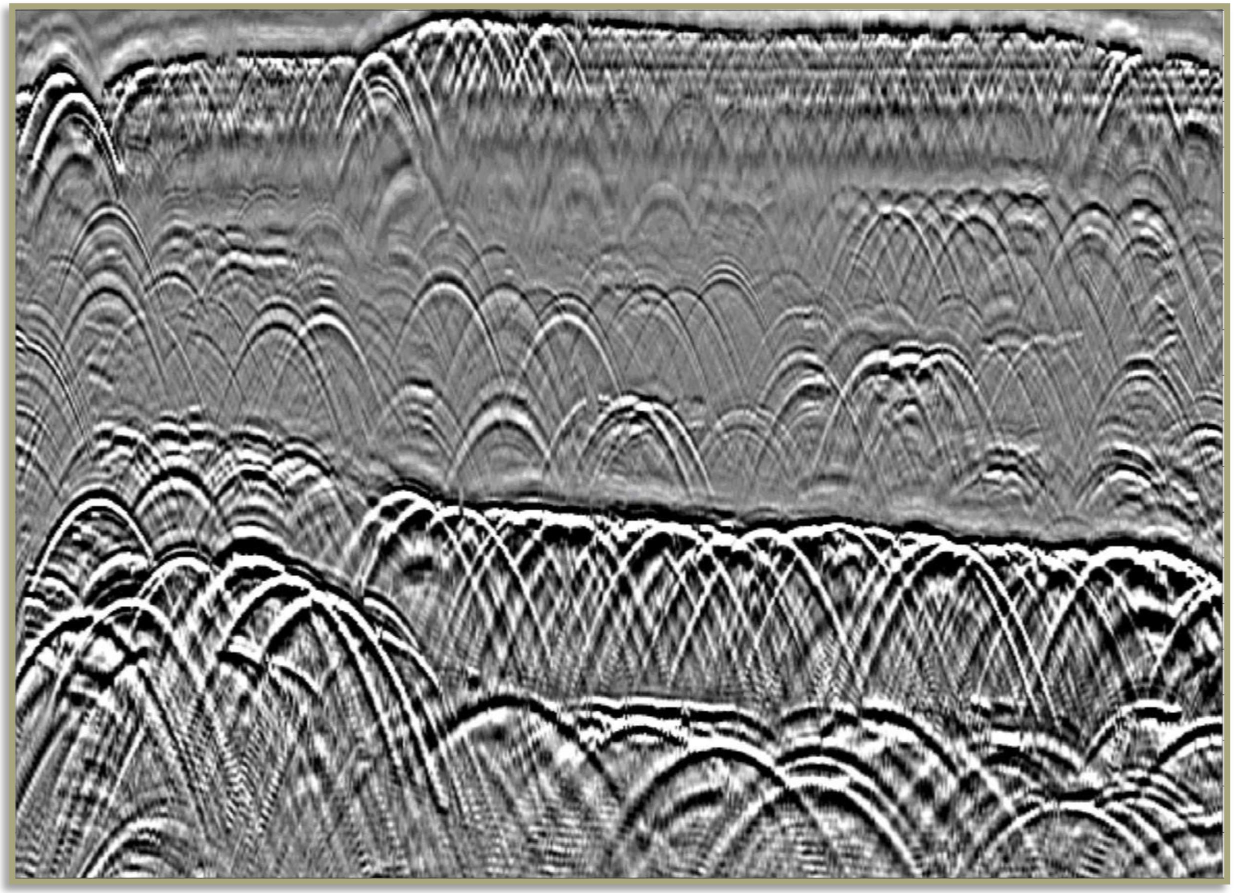


After Berkovich et al., 2009

Diffraction stacked section

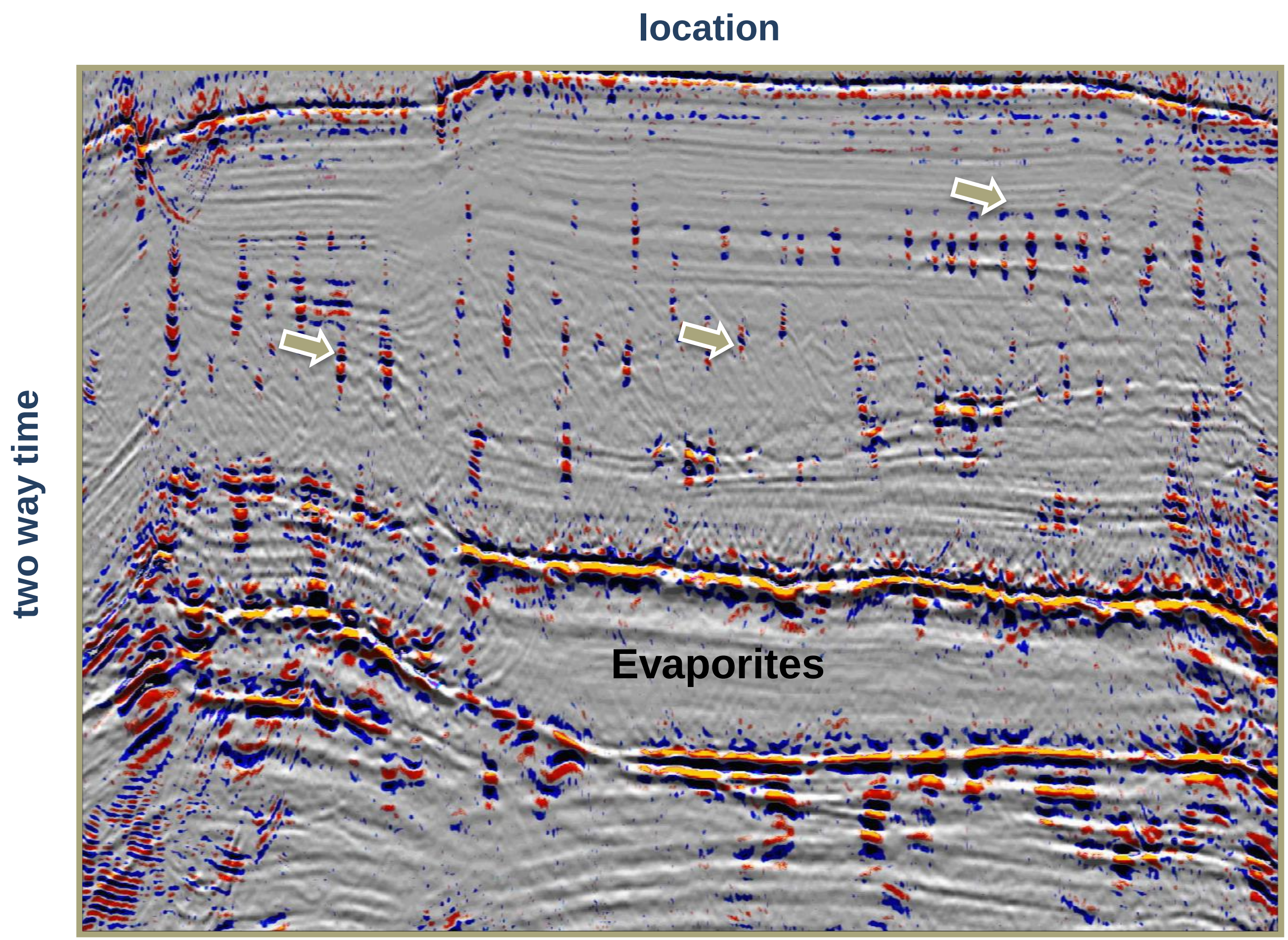
location

two way time



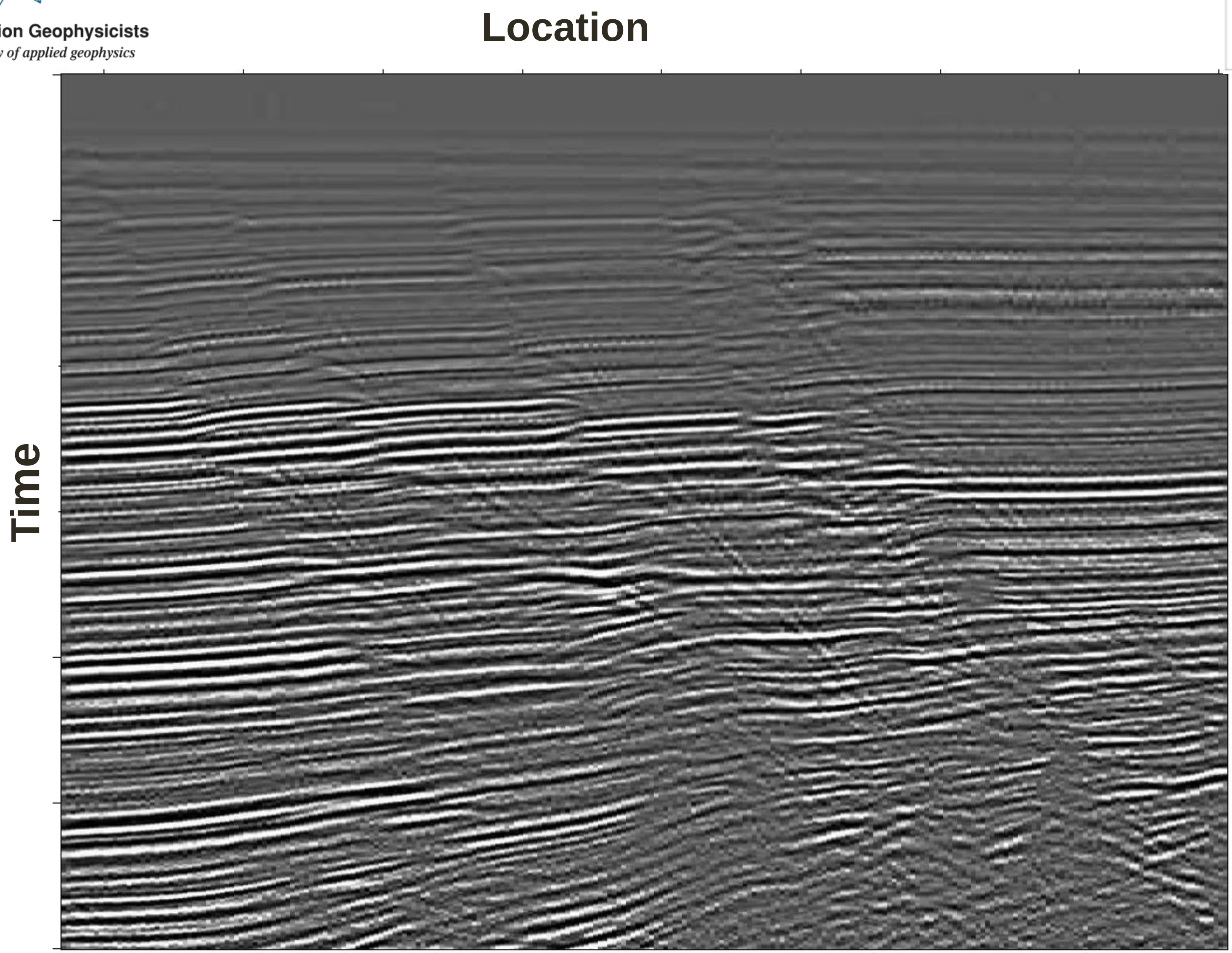
After Berkovich et al., 2009

Migrated diffraction image



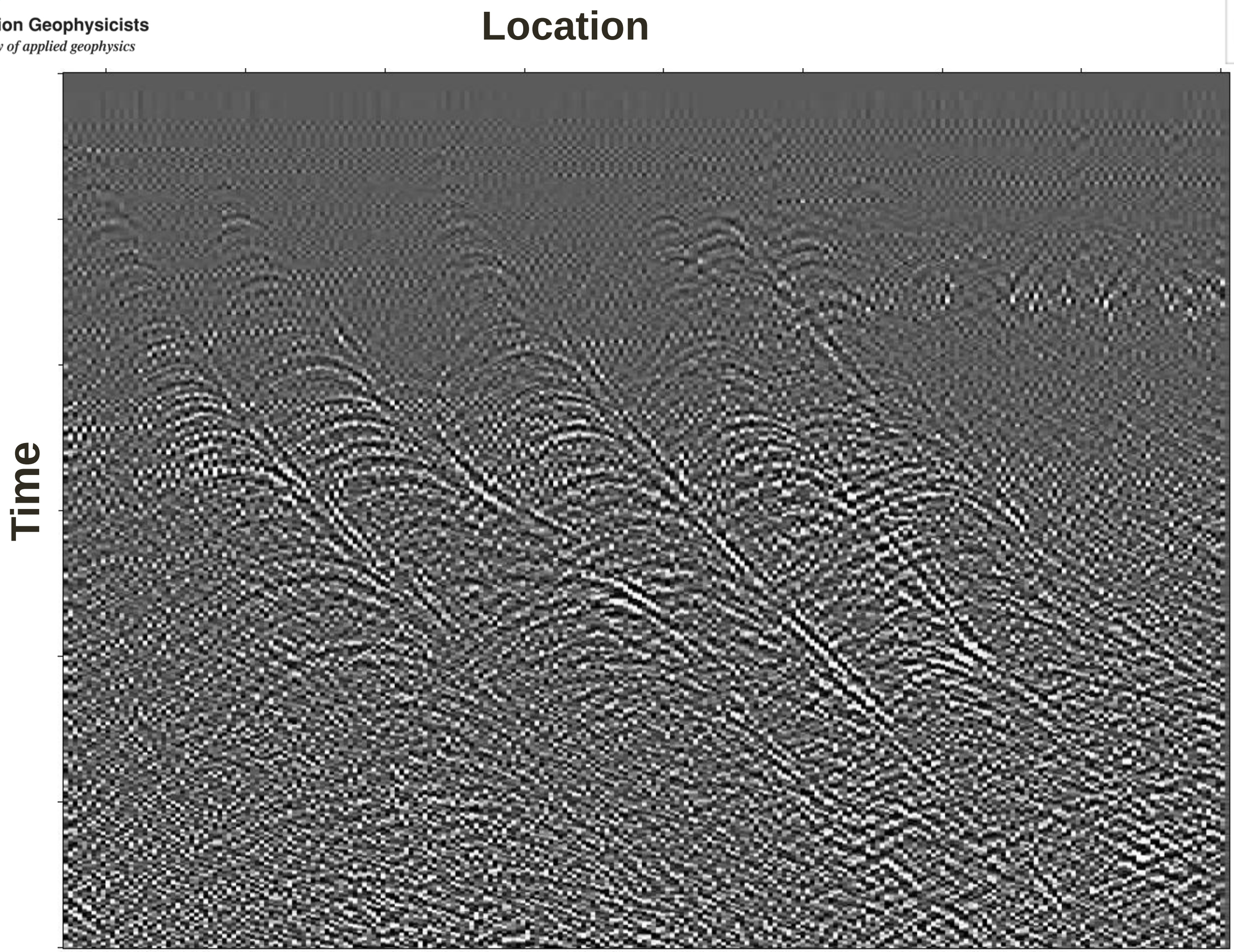
After Berkovich et al., 2009

Stacked section



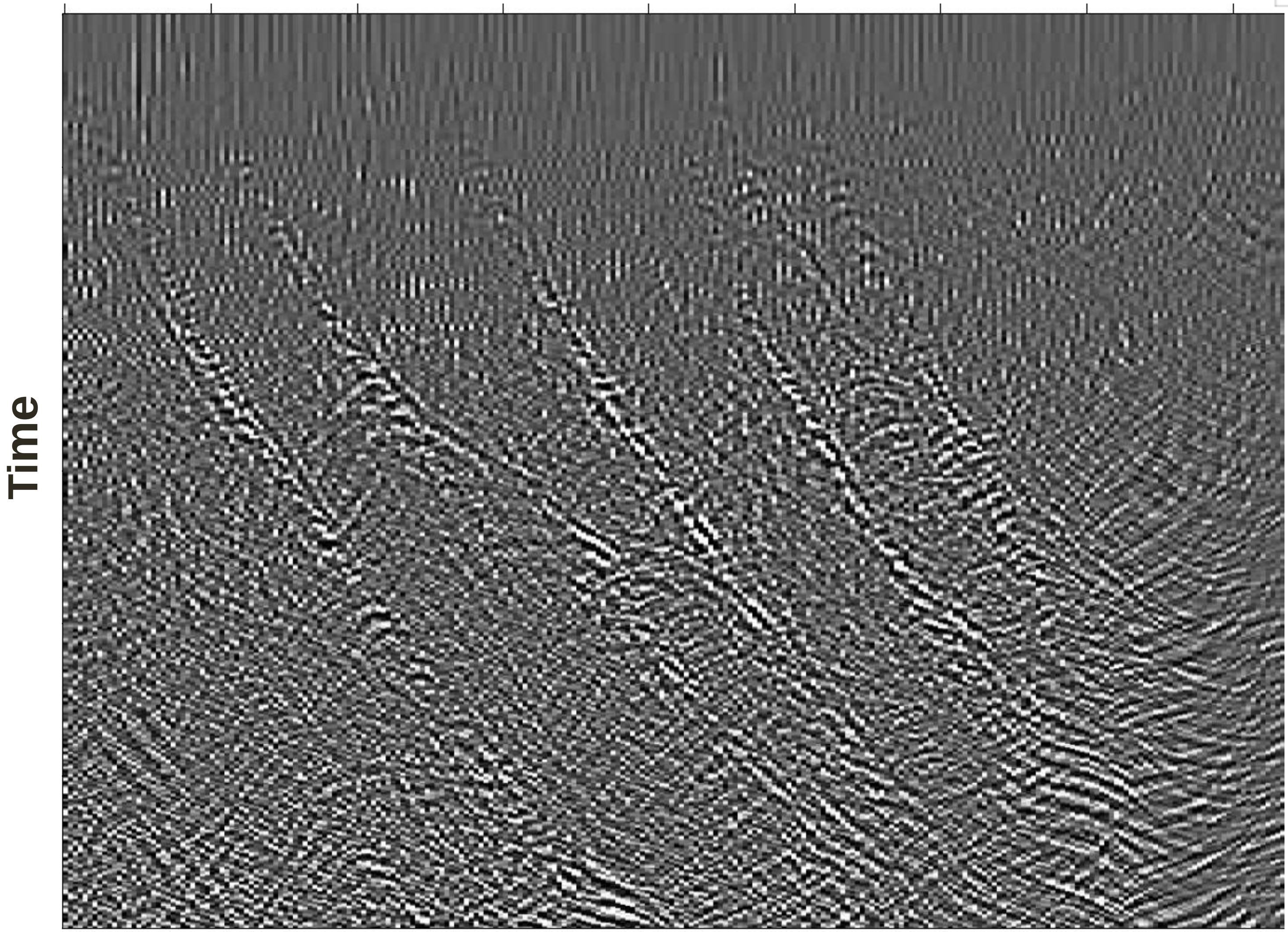
After Fomel, Landa and Taner, 2007

Diffraction stack



After Fomel, Landa and Taner, 2007

Migrated diffraction stack

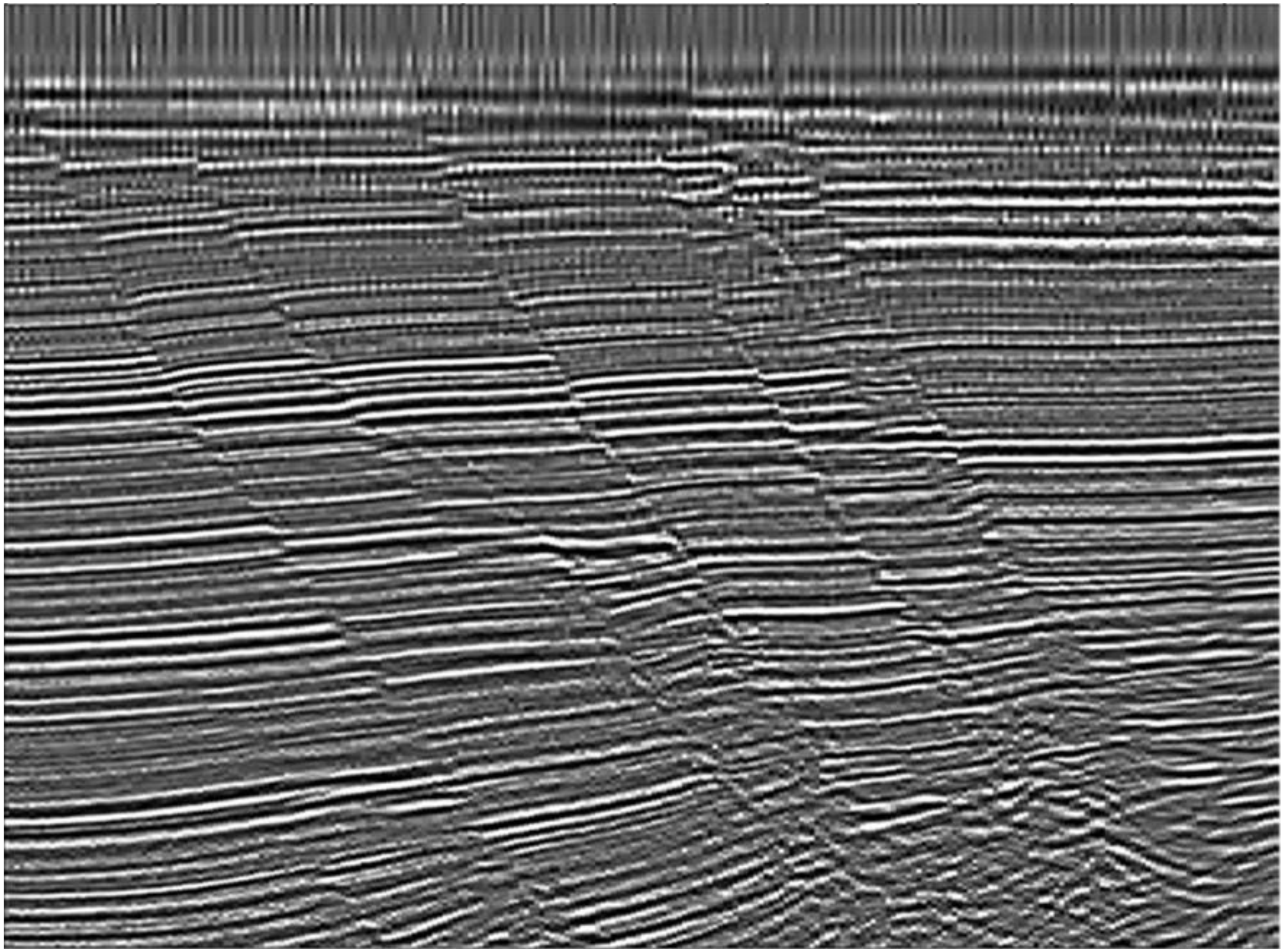


After Fomel, Landa and Taner, 2007

Migrated full stack

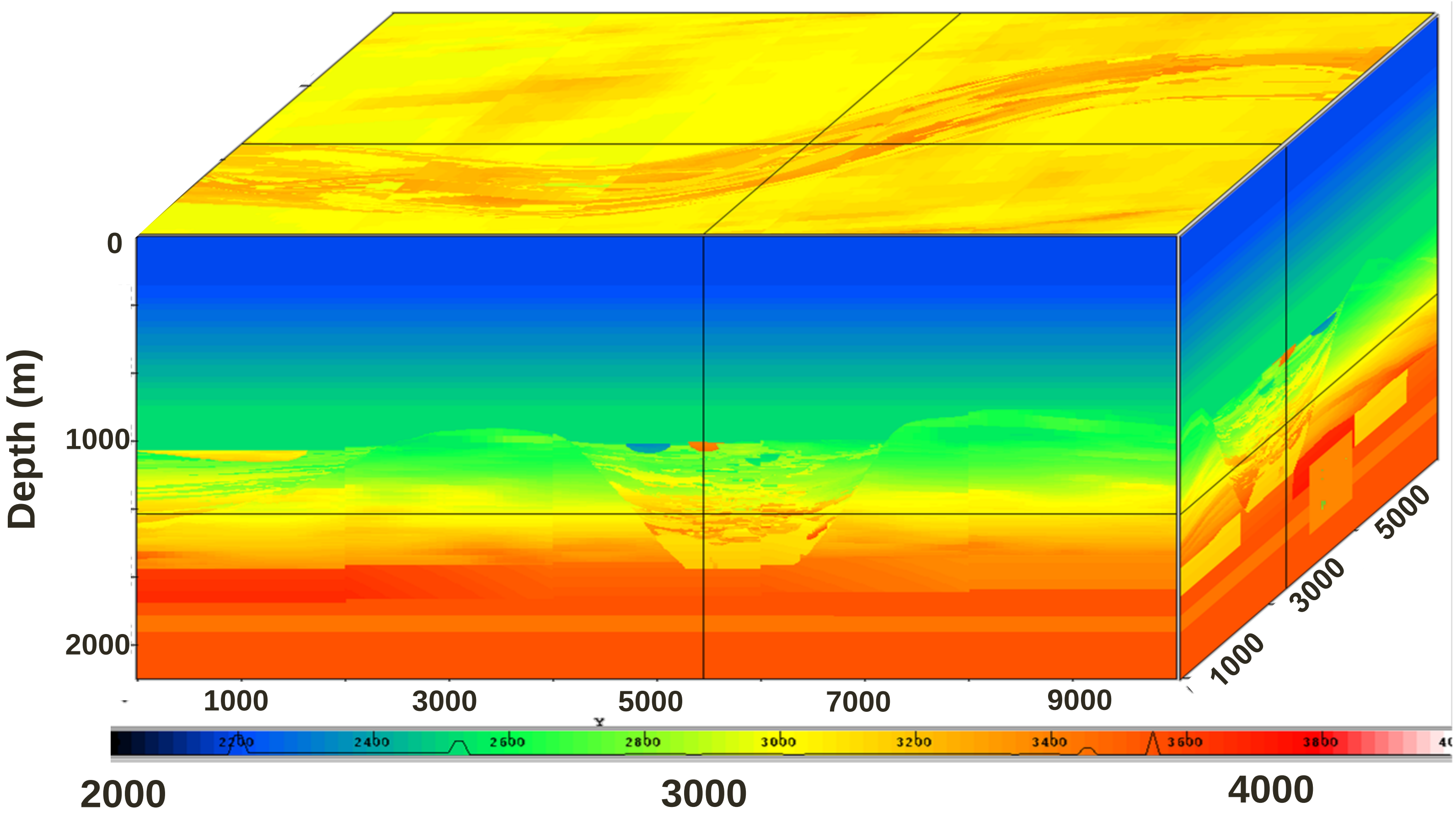
Location

Time



After Fomel, Landa and Taner, 2007

Velocity model



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Velocity model

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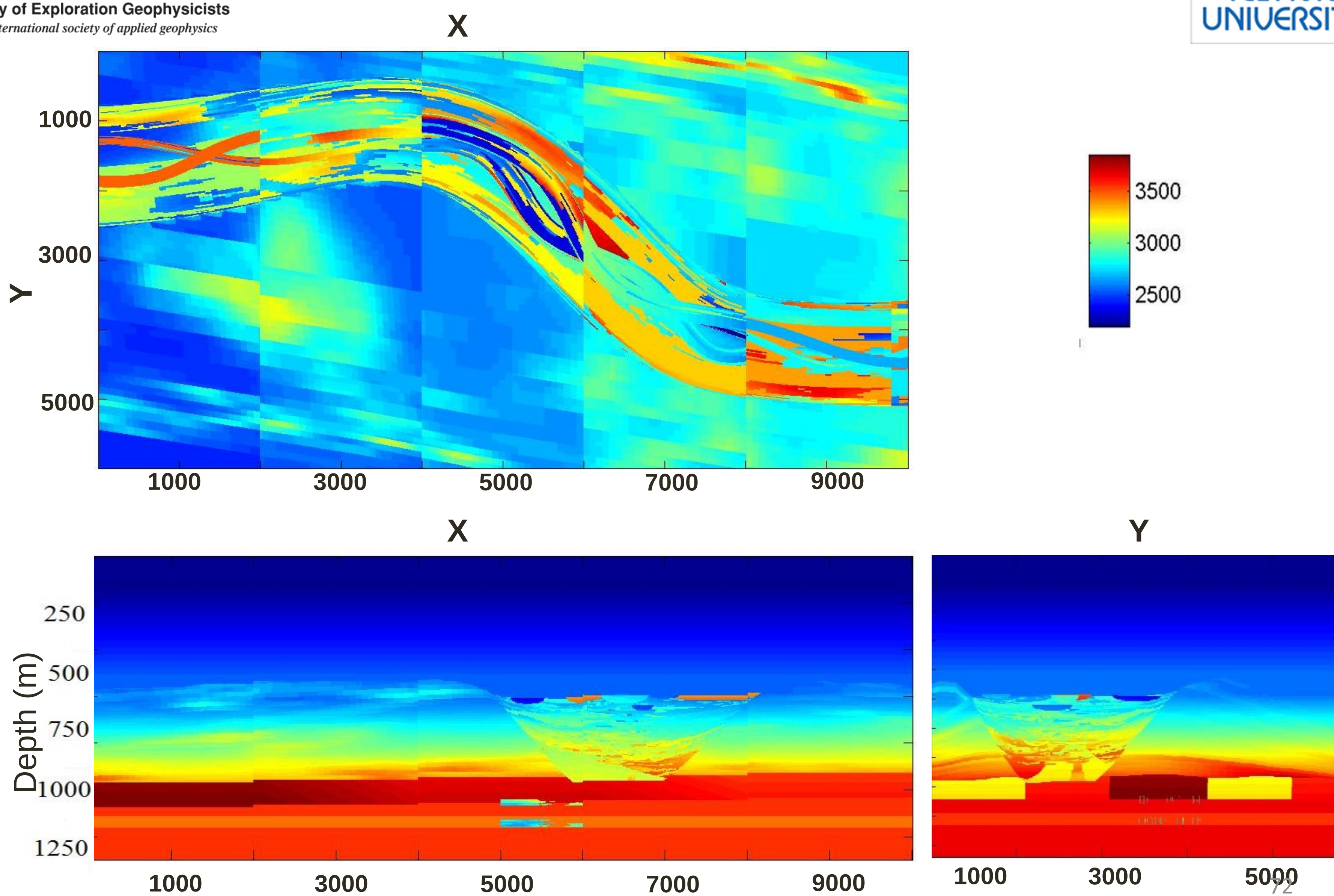

Zero-offset cube (channel time slice)

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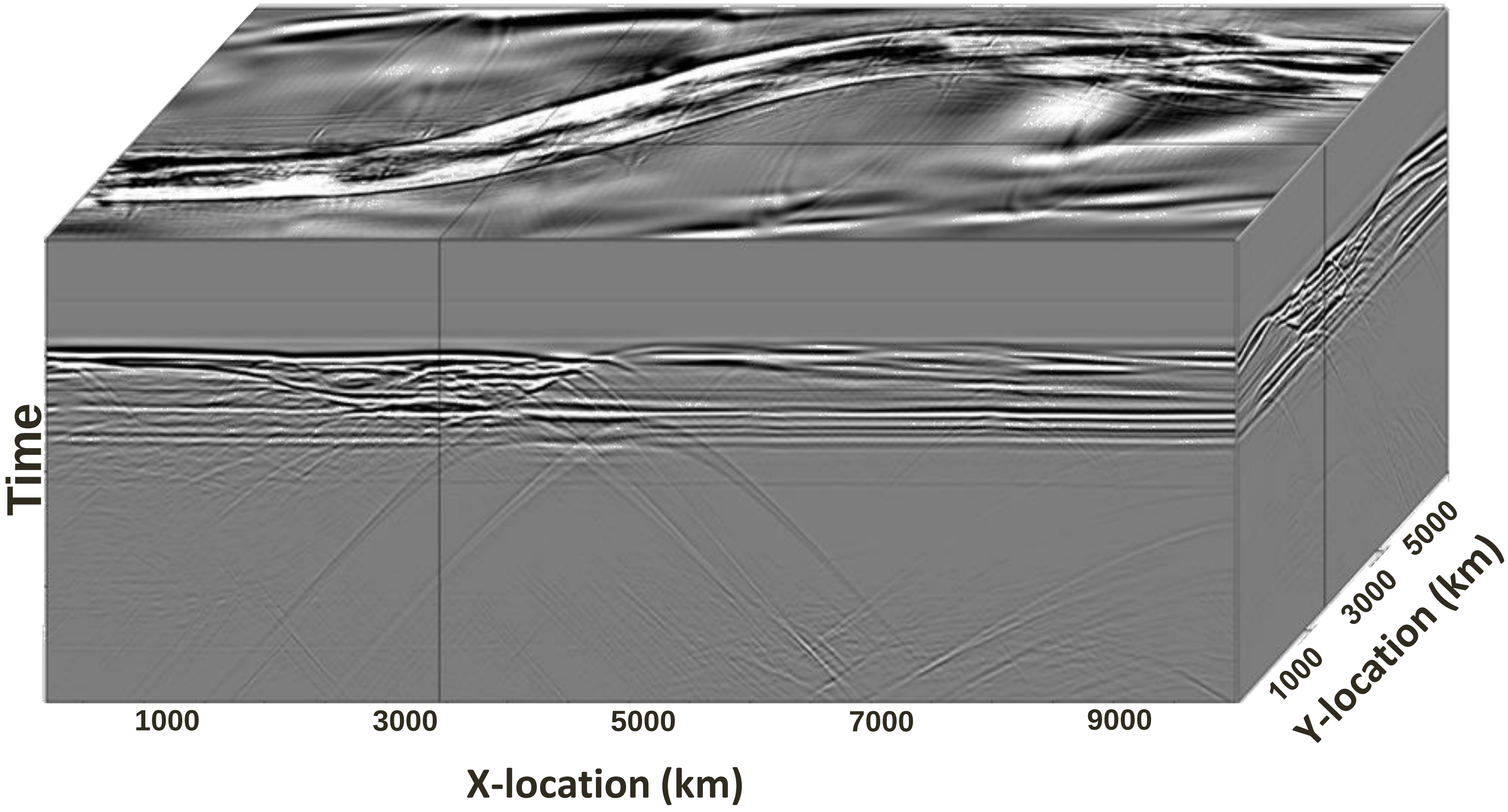
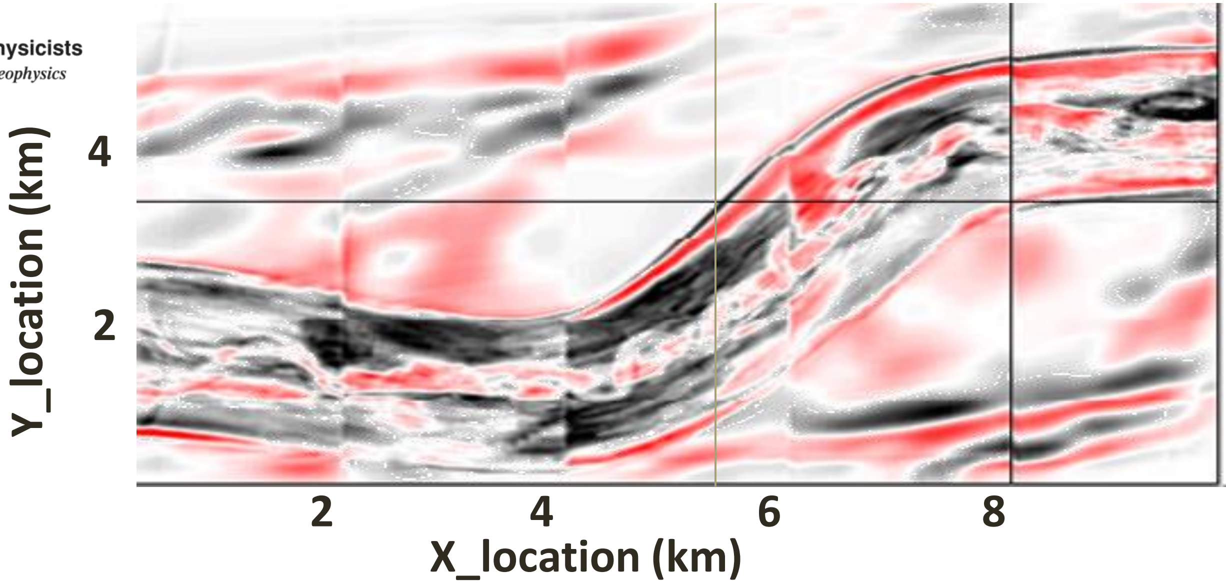
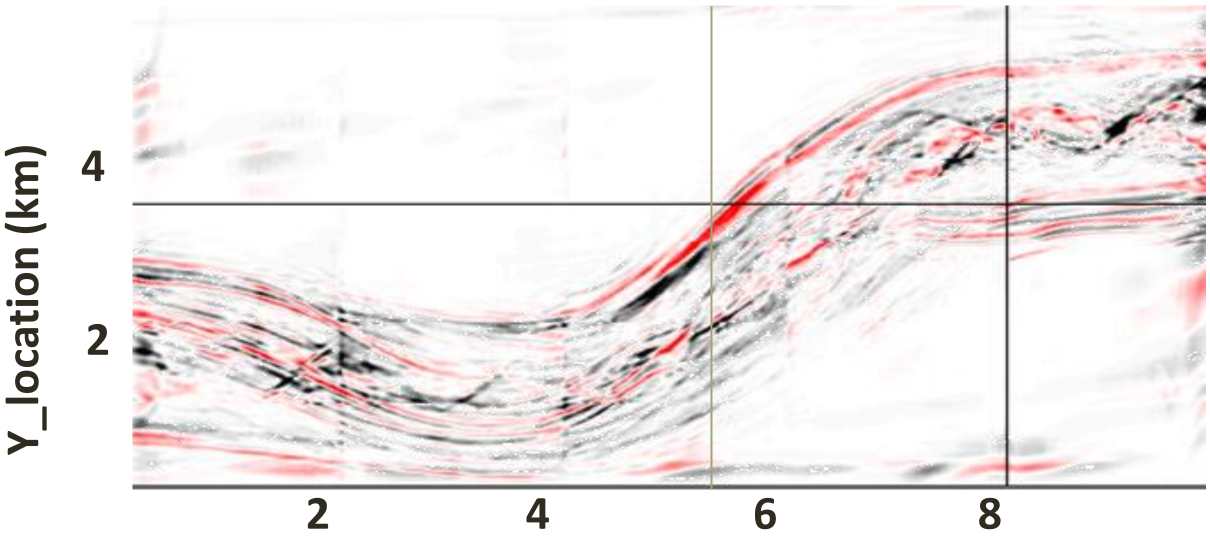


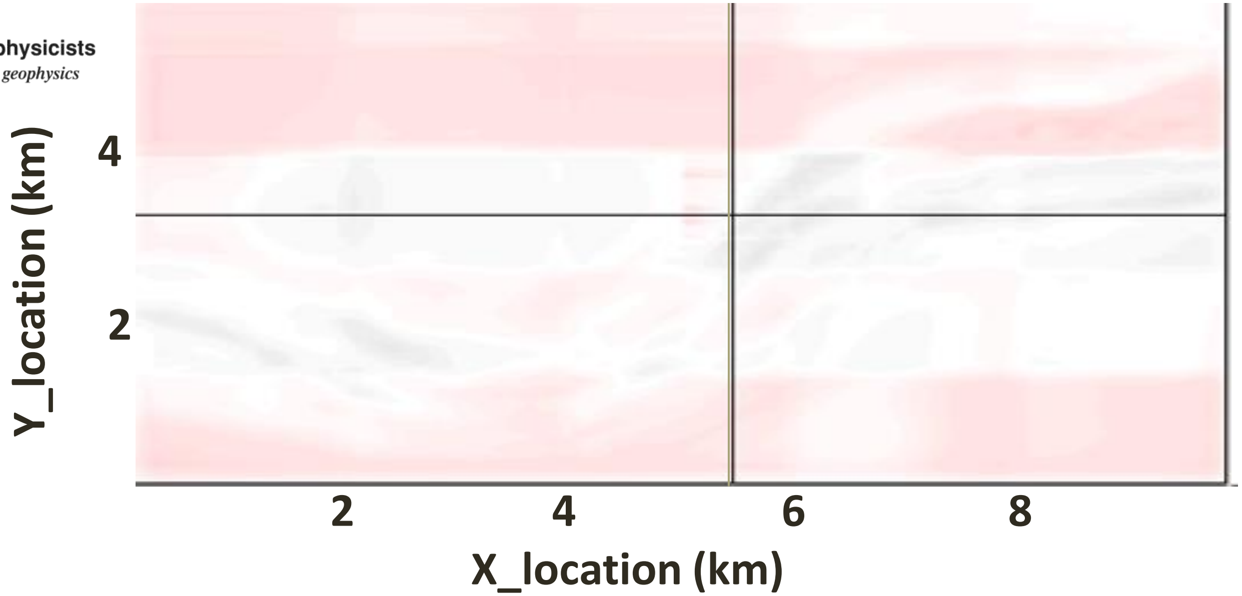
Migrated full field

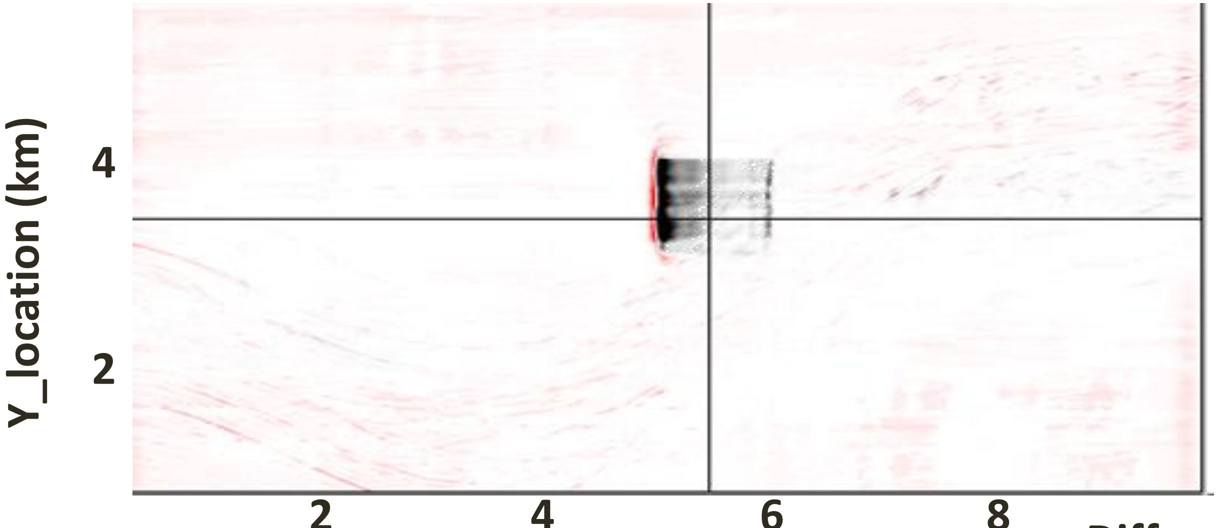


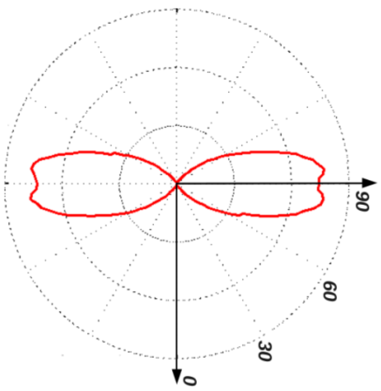


Migrated diffractive component

Migrated full field







Diffractivity diagram

Migrated diffractive component

Summary

• Traditional seismic processing and imaging tends to highlight reflectors and obscure non-reflecting elements, such as small faults, edges, fractures and small scattering objects

• Diffraction is direct indicator of small and medium scale subsurface elements

• Diffraction imaging method allows us finding objects less than seismic wavelength

Is the future bright?...

Can we overcome the non-uniqueness, instability and uncertainty in our solutions?

- How far the geophysical inverse problem can be formalized?
- The gap between a solution as obtained for very refined mathematical assumptions and reality can be very large
- Does the level of mathematics really define the maturity of a science?
- Overcoming uncertainties…

"I can live with doubt and uncertainty and not knowing. I think it is much more interesting to live not knowing than to have answers that might be wrong" *Richard Feynman*