

# Networked Distributed Energy Resources

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Electrical Engineering  
Computing + Math Sciences



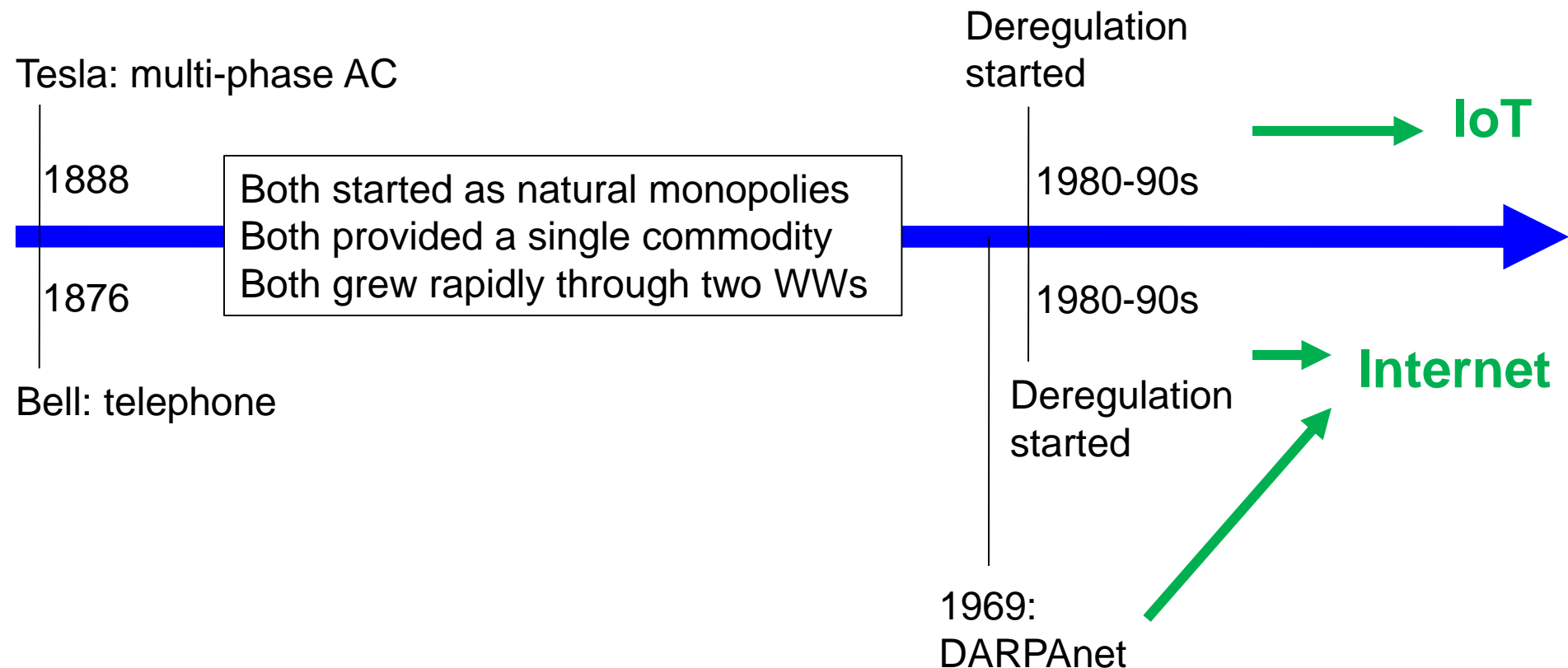
Caltech

Sept 2015  
Energy Colloquium, Skoltech, Moscow

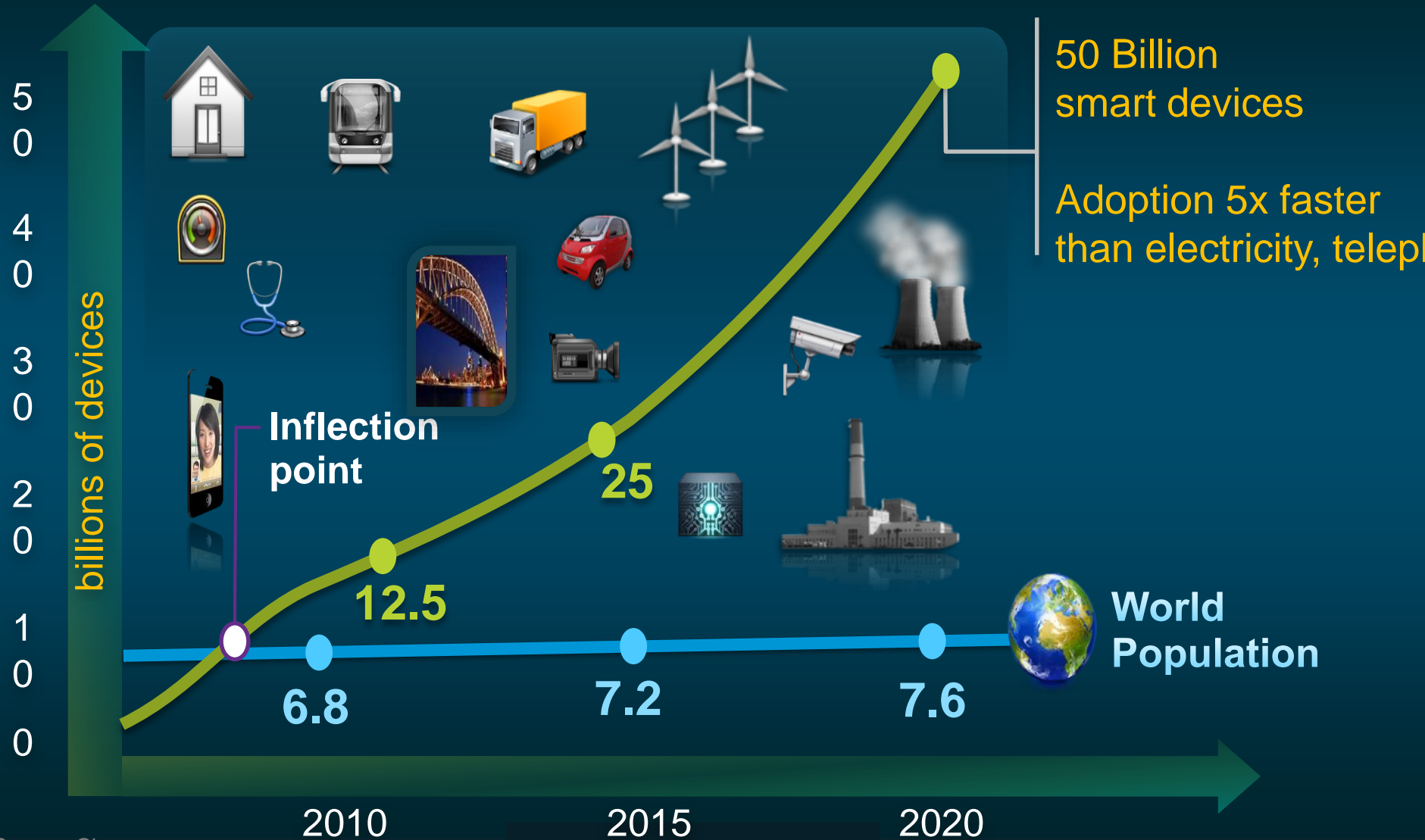


# Watershed moment

Power network will undergo similar **architectural transformation** that phone network went through in the last two decades



# Internet of Things (IoT)





# Challenges & opportunities

Industries will be destroyed and created

AT&T, MCI, McCaw Cellular, **Qualcom**

**Google, Facebook, Twitter, Amazon, eBay, Netflix**

Infrastructure will be reshaped

Centralized intelligence, vertically optimized

**Distributed intelligence, layered architecture**

**What will drive power network transformation ?**



# Four drivers

Proliferation of renewables

Electrification of transportation

} challenges

Advances in power electronics

Deployment of sensing, control, comm

} enablers



# Outline

## Big picture

- Energy stats and trends

## Challenges and opportunities

- Implications on smart grid

## Sample Caltech research

- Control and optimization
- Power flow and dynamics





# World energy stats (2011)

Consumption	519 quad BTU
petroleum	34%
coal	29%
gas	23%
renewable (elec)	8%
nuclear	5%

top 5  
countries

Consumption	519 (quad BTU)	per capita (mil BTU)
China	20%	78
US	19%	313
Russia	6%	209
India	5%	20
Japan	4%	164
<b>total</b>	<b>54%</b>	



# World energy stats (2011)

Consumption	519 quad BTU
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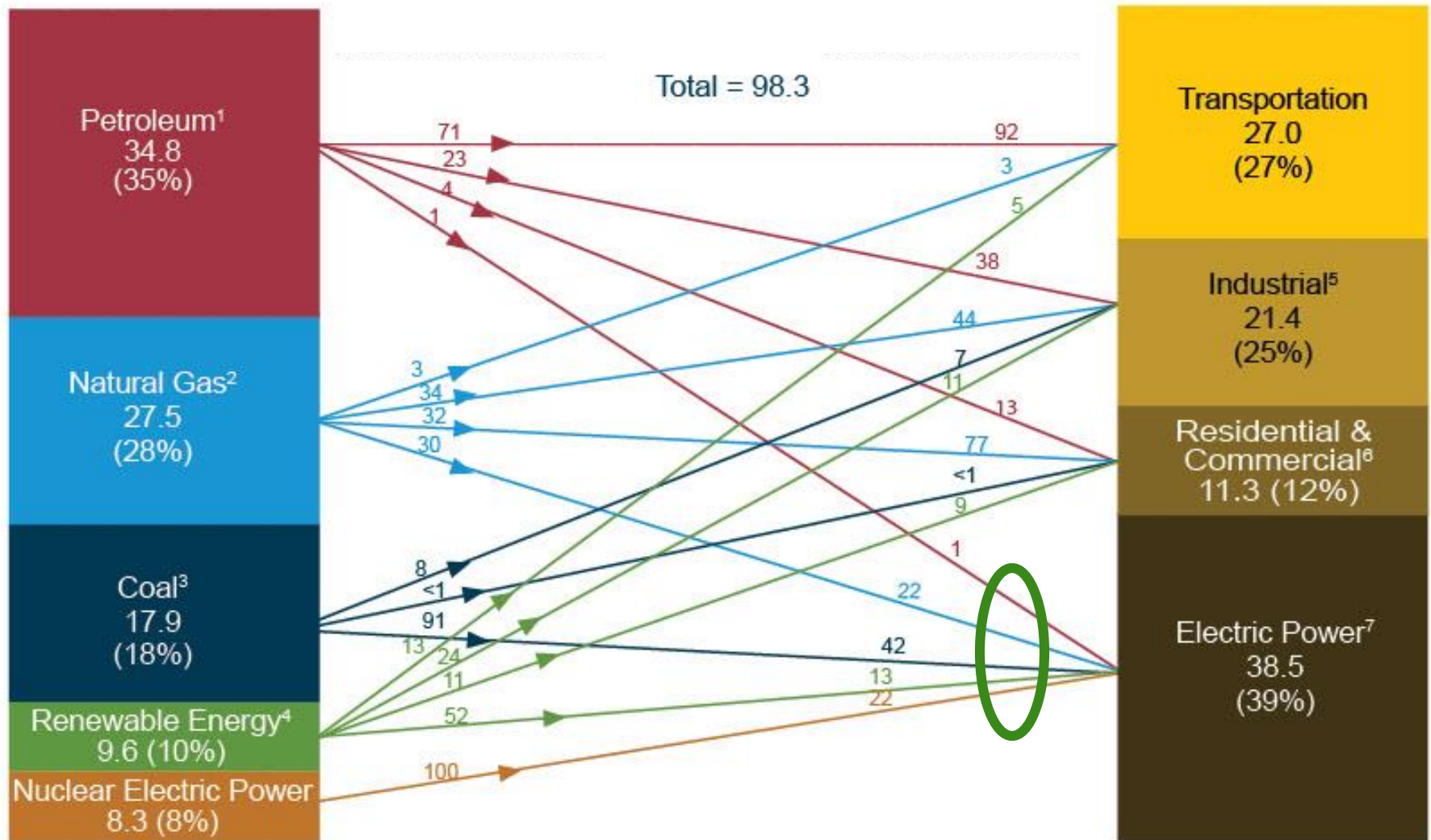
top 5  
countries

Consumption	519 (quad BTU)	CO2 emission
China	20%	27%
US	19%	17%
Russia	6%	5%
India	5%	5%
Japan	4%	4%
<b>total</b>	<b>54%</b>	<b>58%</b>



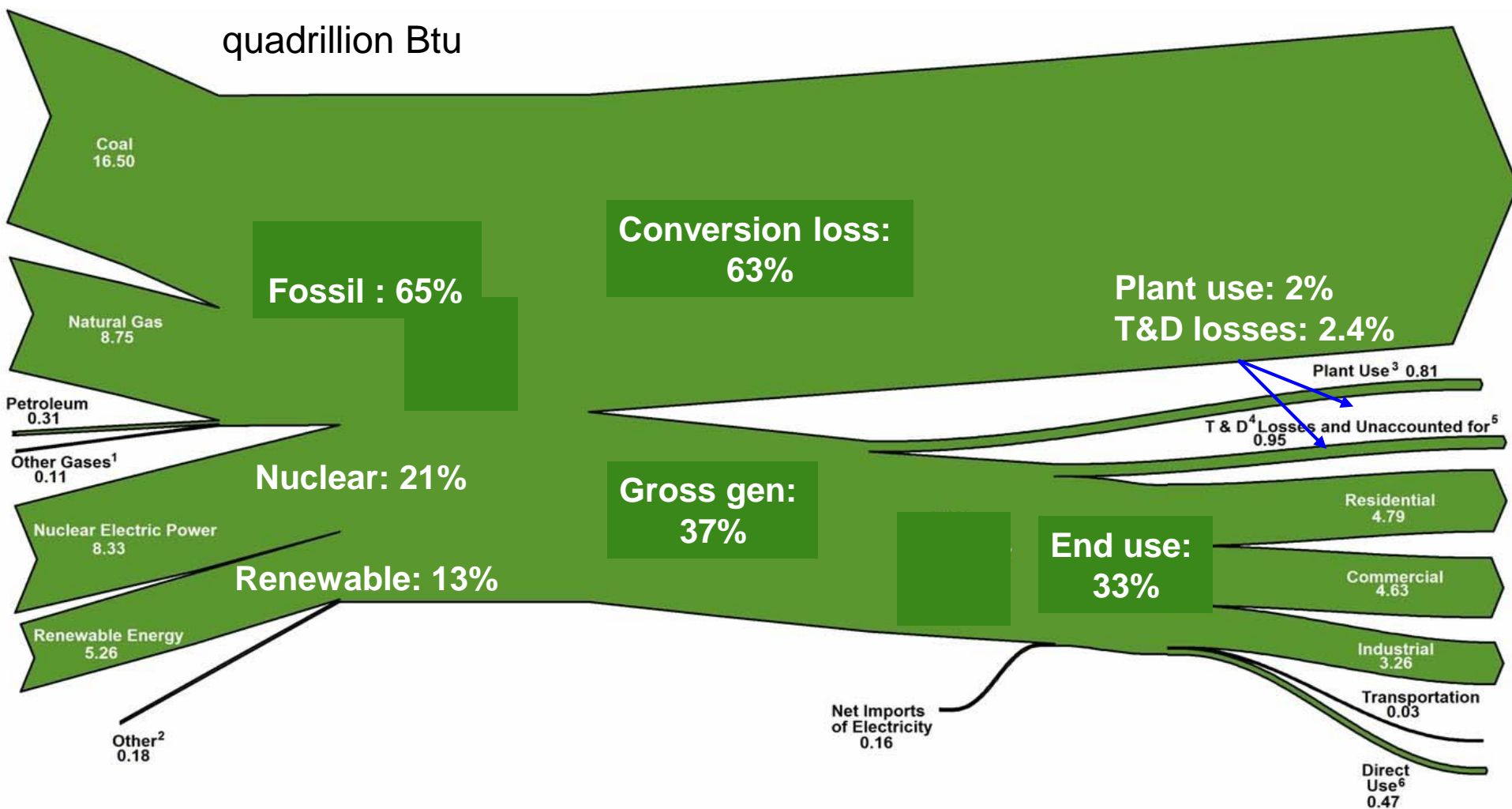
# US Primary Energy Flow 2014

(Quadrillion Btu)





# US electricity flow 2014



US total energy use: 98.3 quads  
For electricity gen: 39%

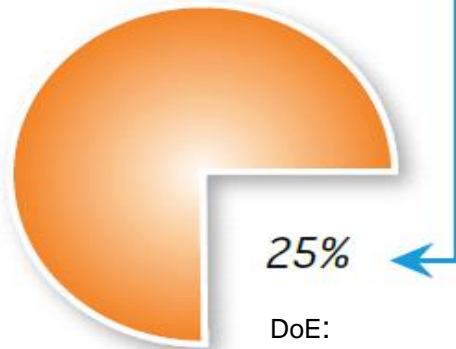
Source: EIA March 2015  
Monthly Energy Review



# Sustainability challenge



The U.S. accounts for 4% of the world's population while contributing 25% of its greenhouse gases.

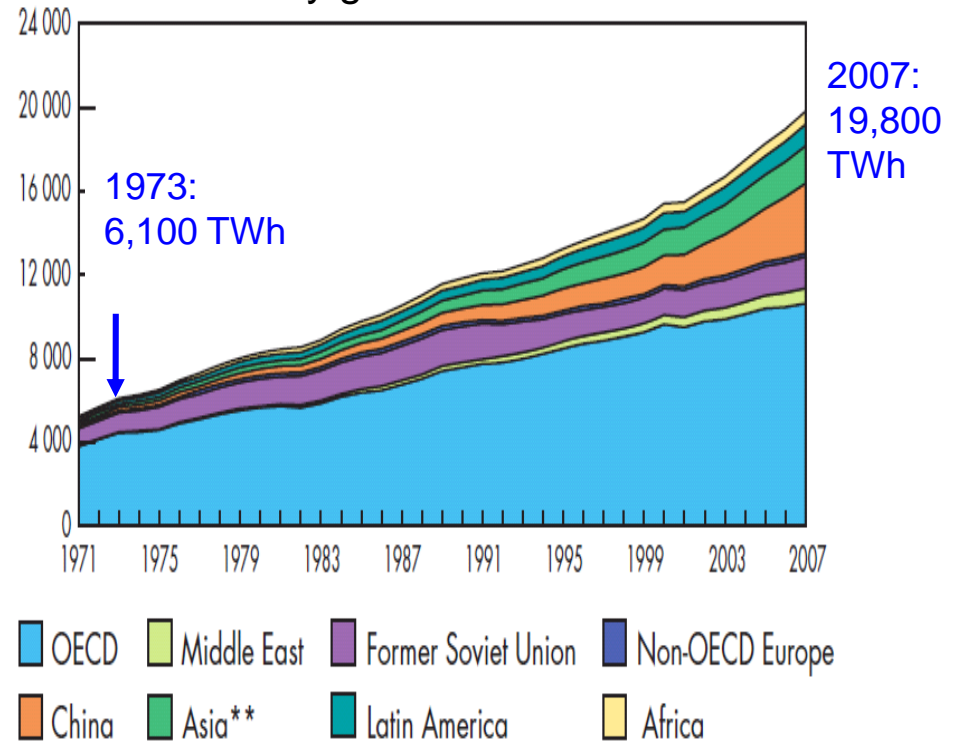


DoE:  
Smart Grid Intro 2008

US CO<sub>2</sub> emission

- Elect generation: 40%
- Transportation: 20%

Electricity generation 1971-2007

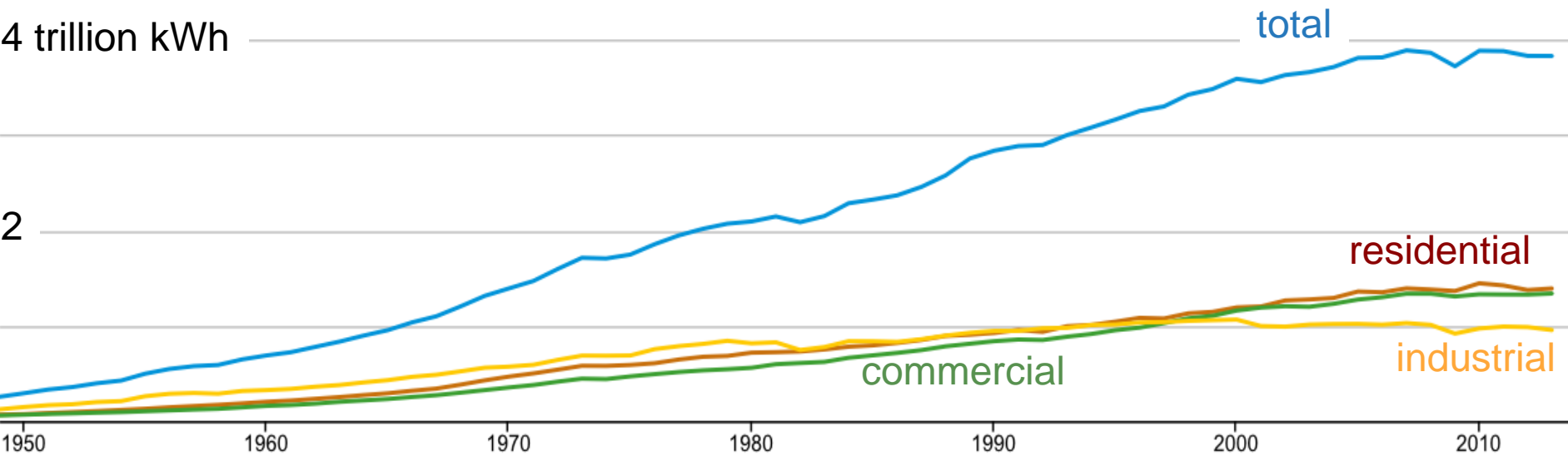


In 2009, 1.5B people  
have no electricity

Sources: International Energy Agency, 2009  
DoE, Smart Grid Intro, 2008



# US electricity use



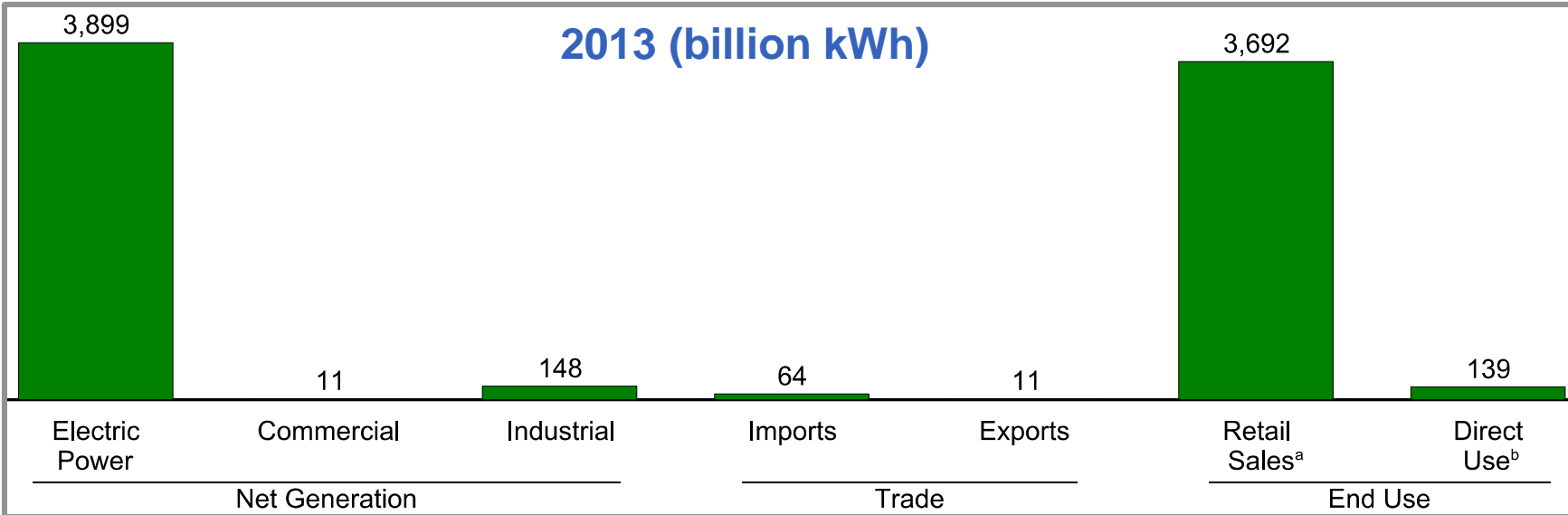
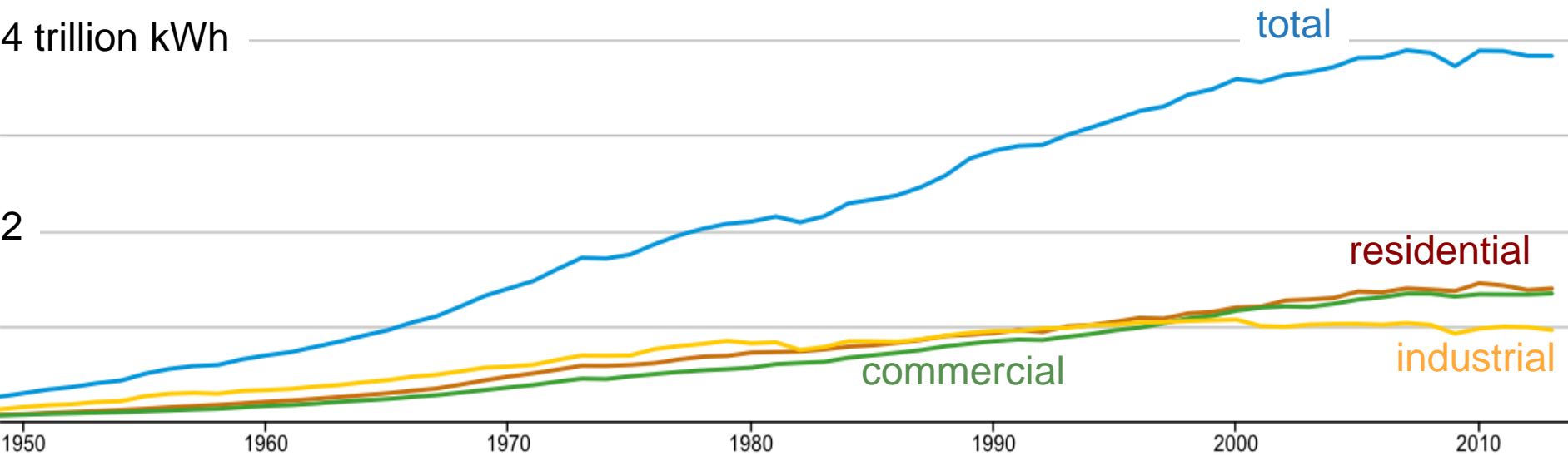
2013 (billion kWh)



Source:  
US EIA



# US electricity use

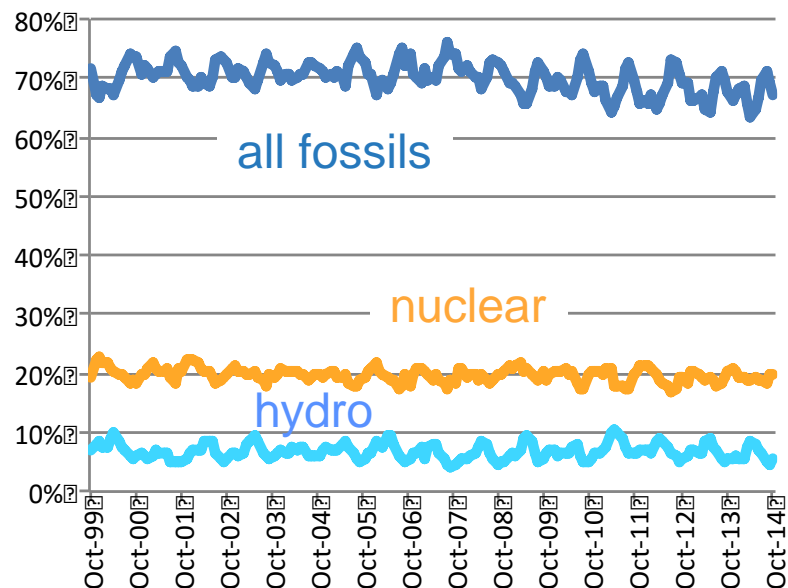
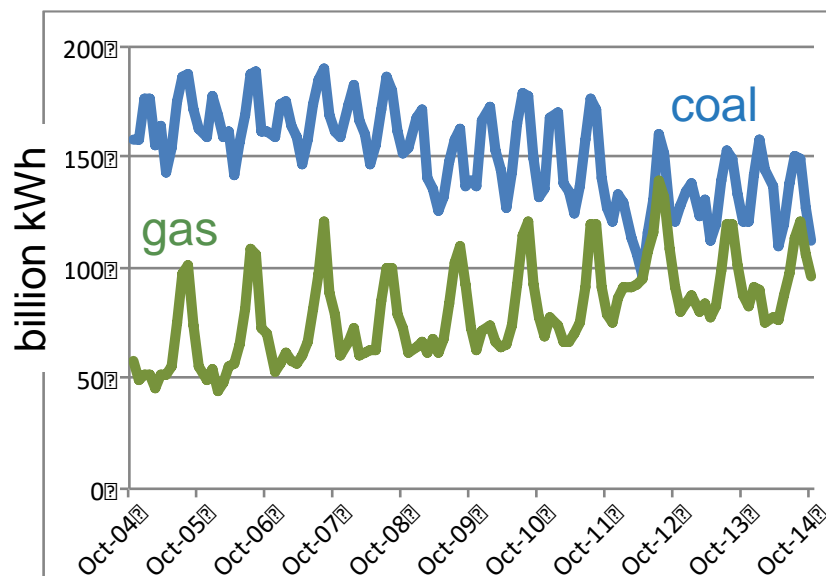
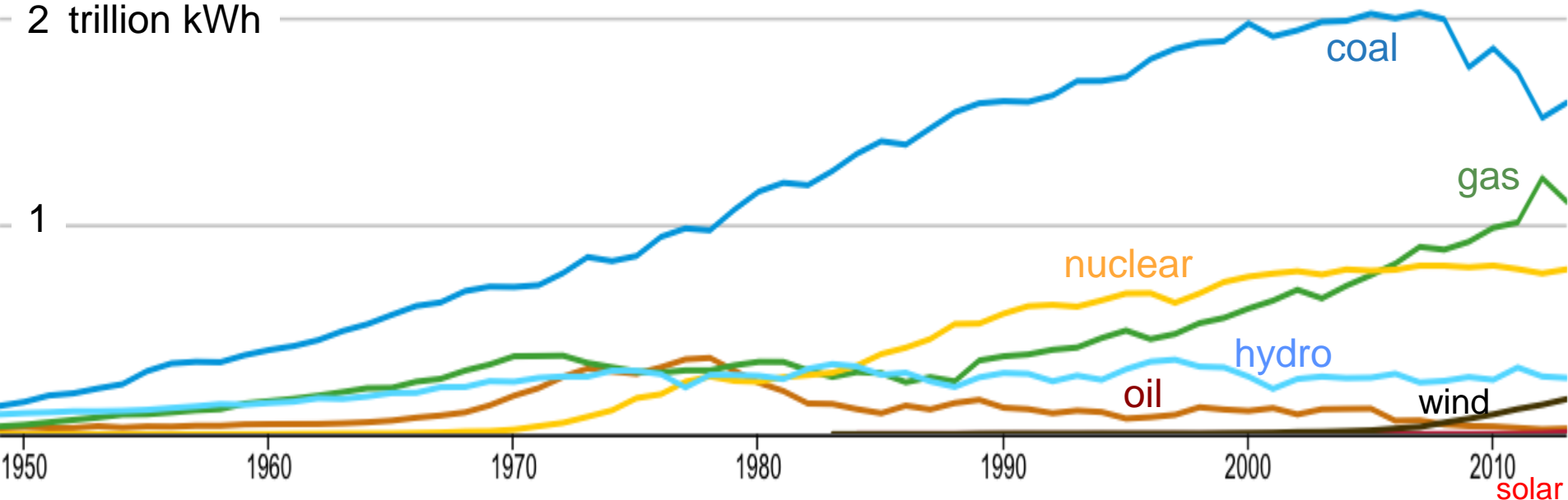




# US dirty supply

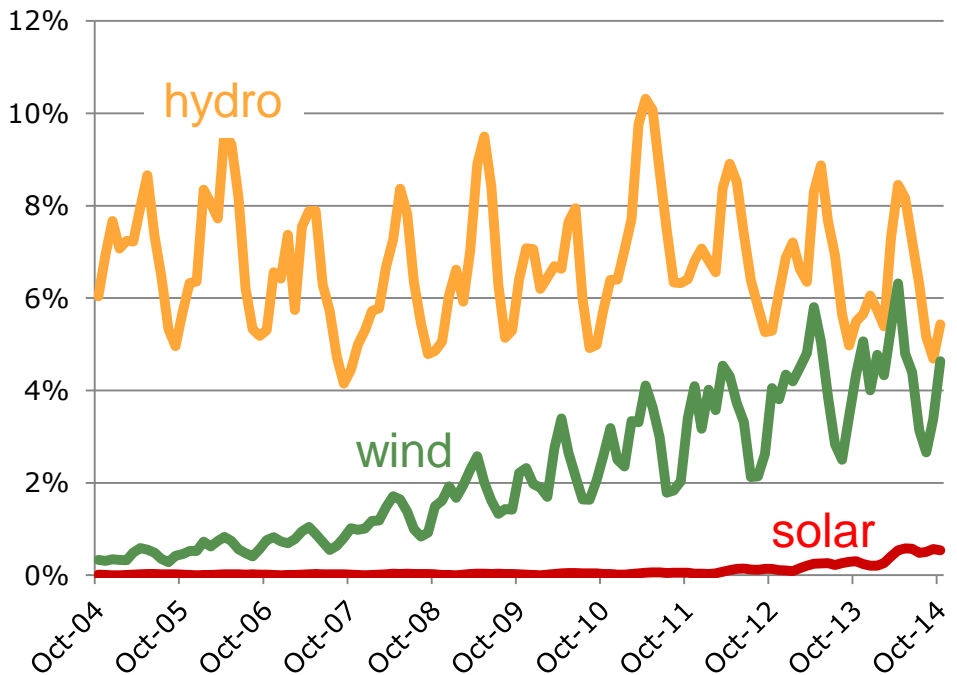
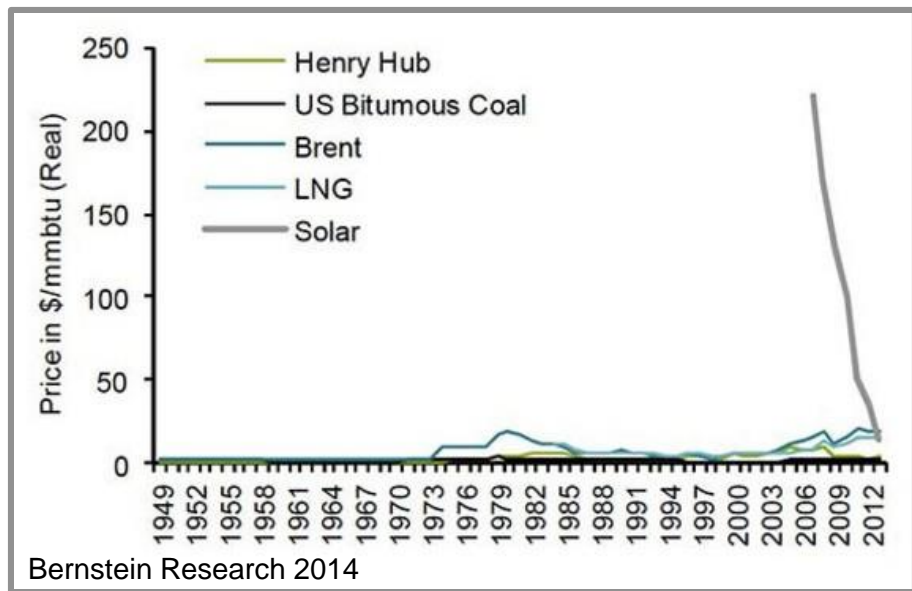
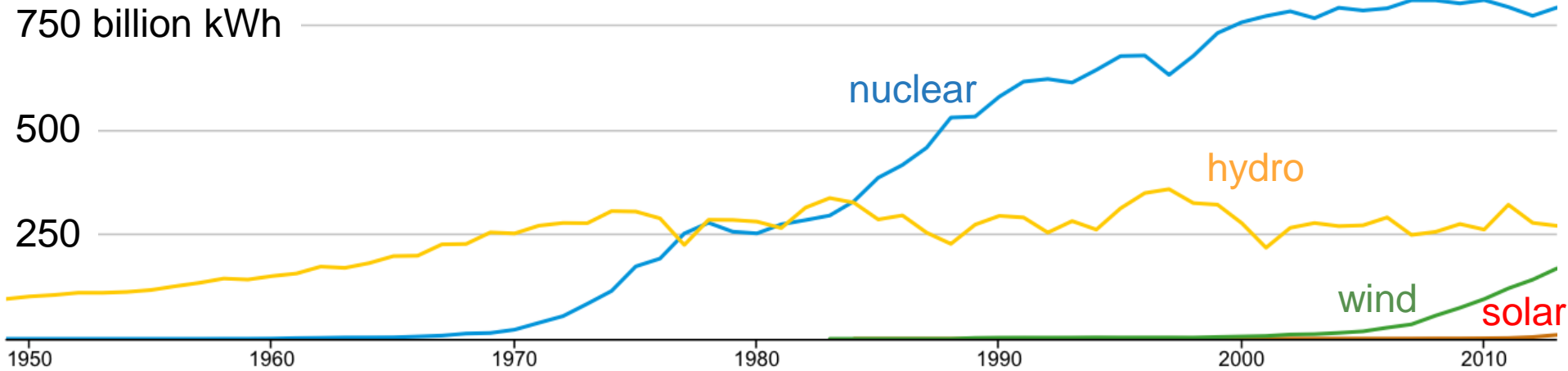
2 trillion kWh

1





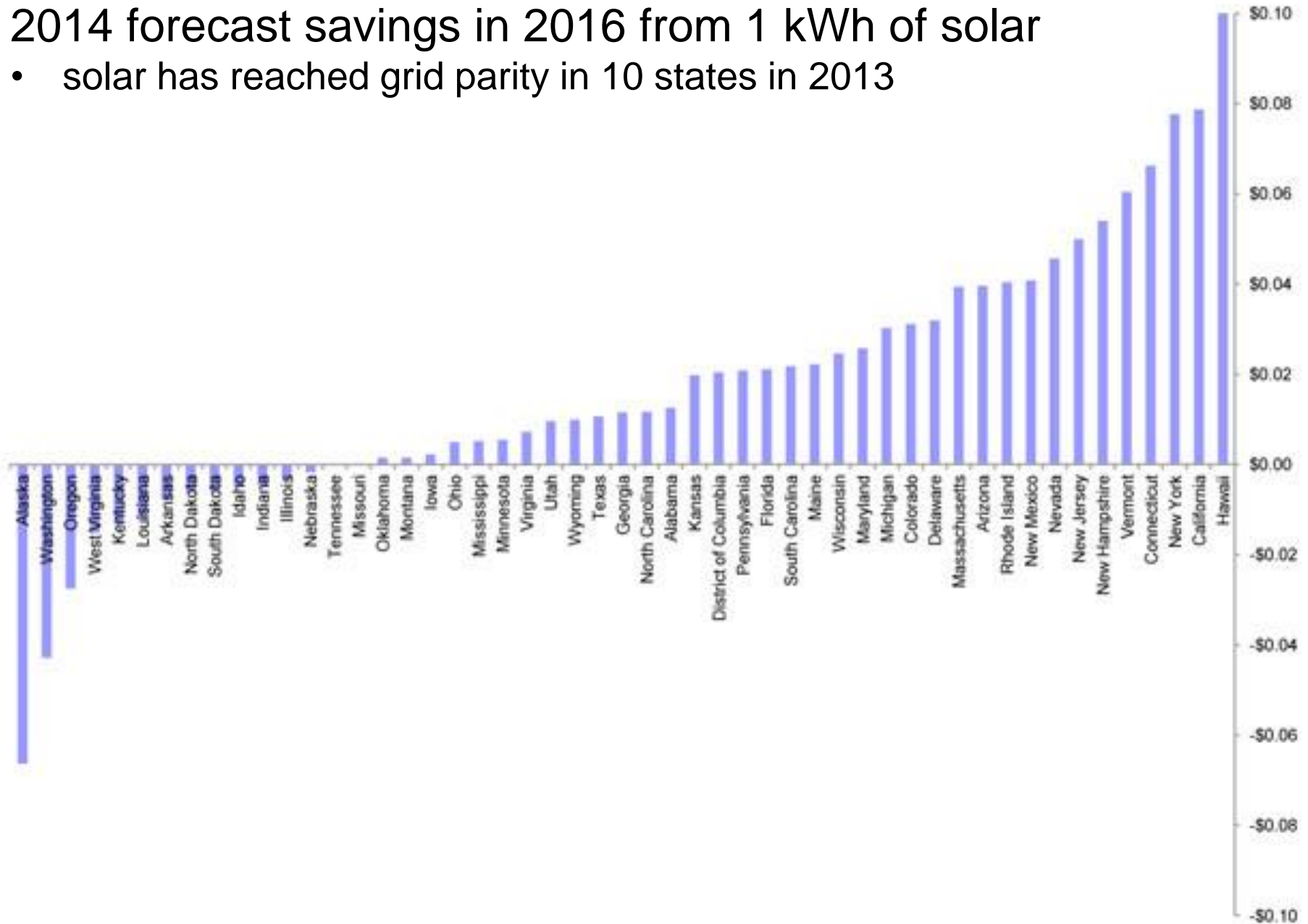
# US renewable generations





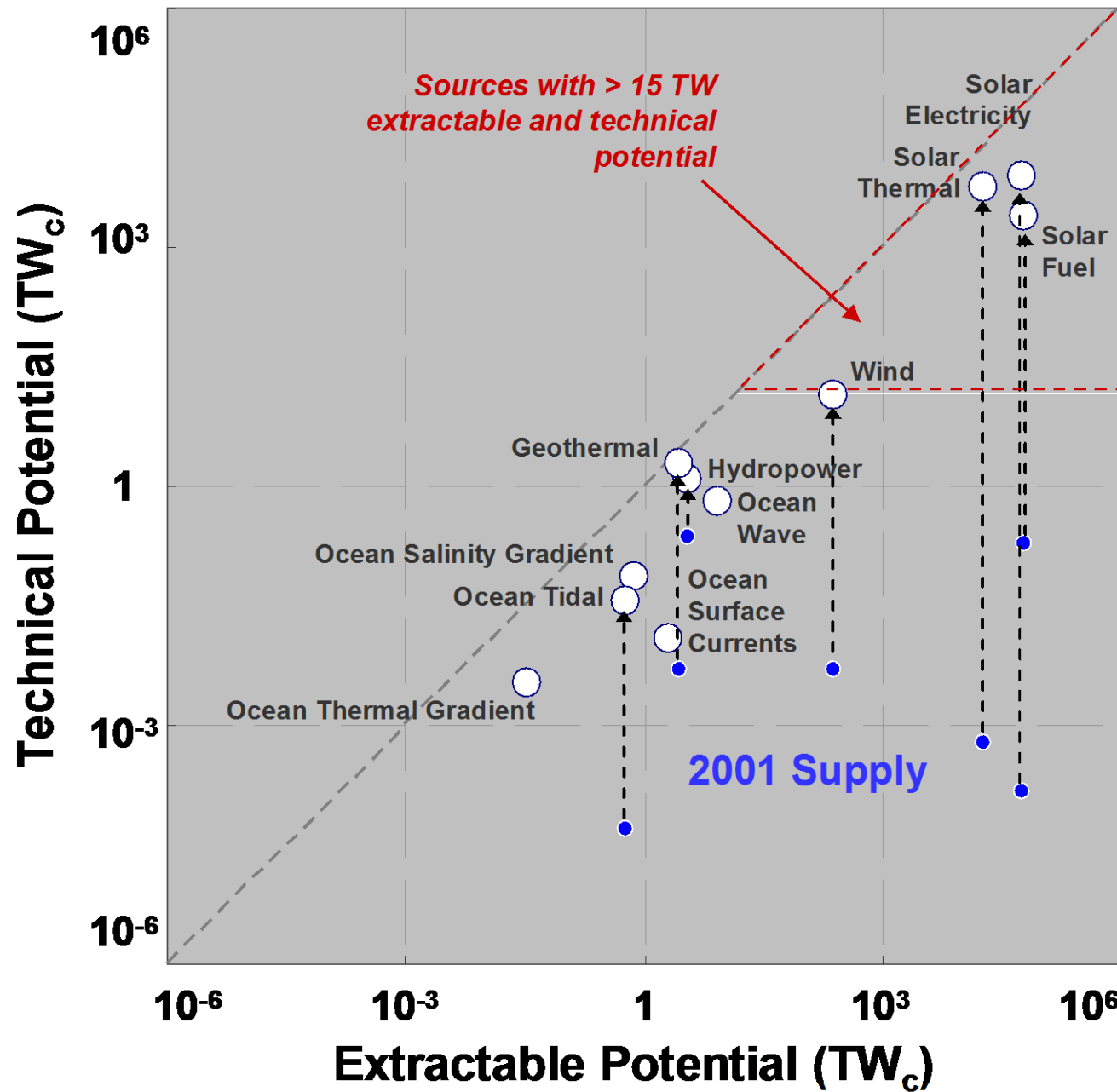
# 2014 forecast savings in 2016 from 1 kWh of solar

- solar has reached grid parity in 10 states in 2013





# Technical potential of renewable sources



solar can supply  
total world  
energy demand

# Area to power the world by solar





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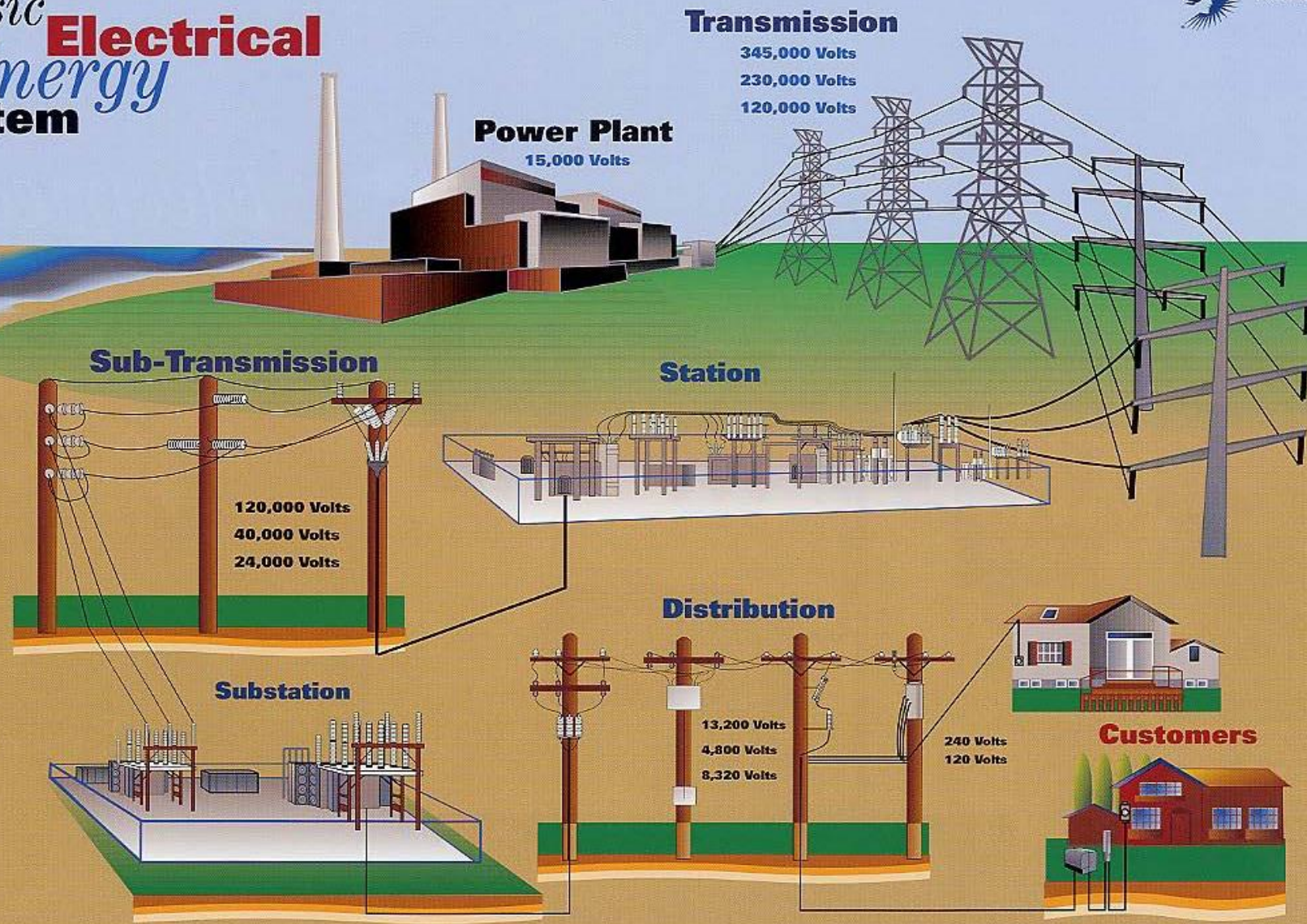
- Control and optimization
- Power flow and dynamics







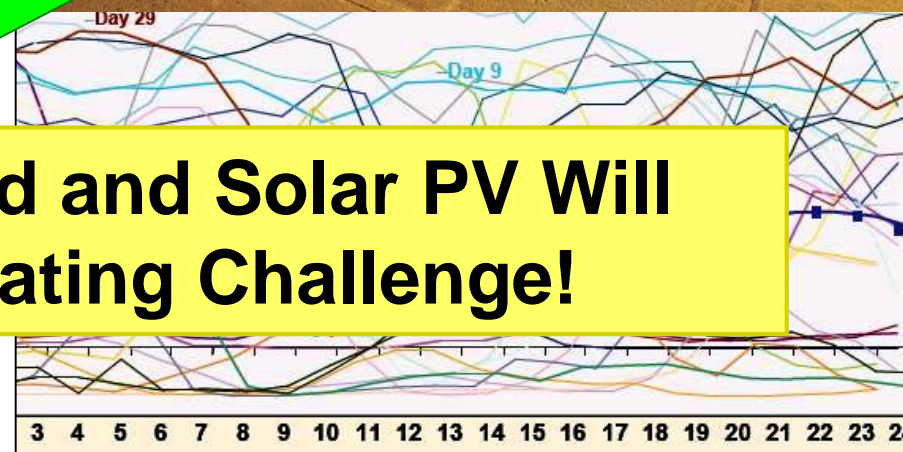
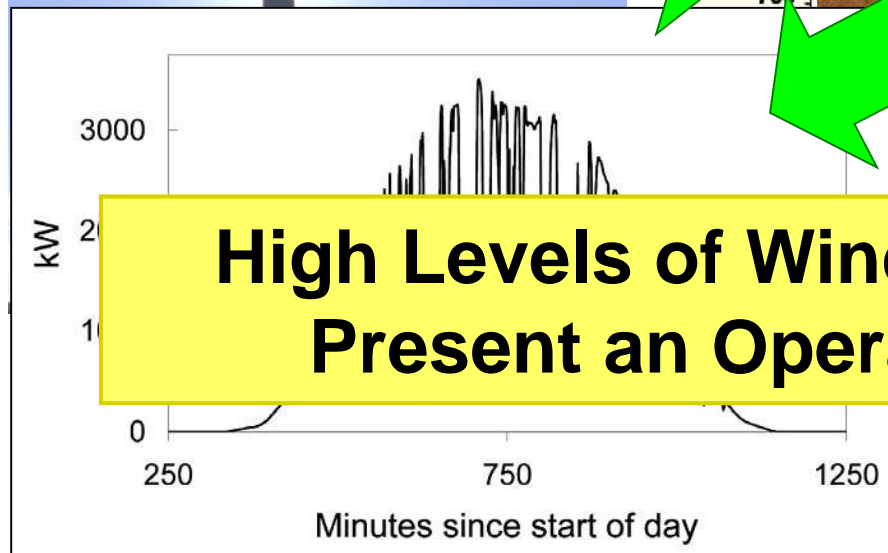
# Basic **Electrical** Energy System







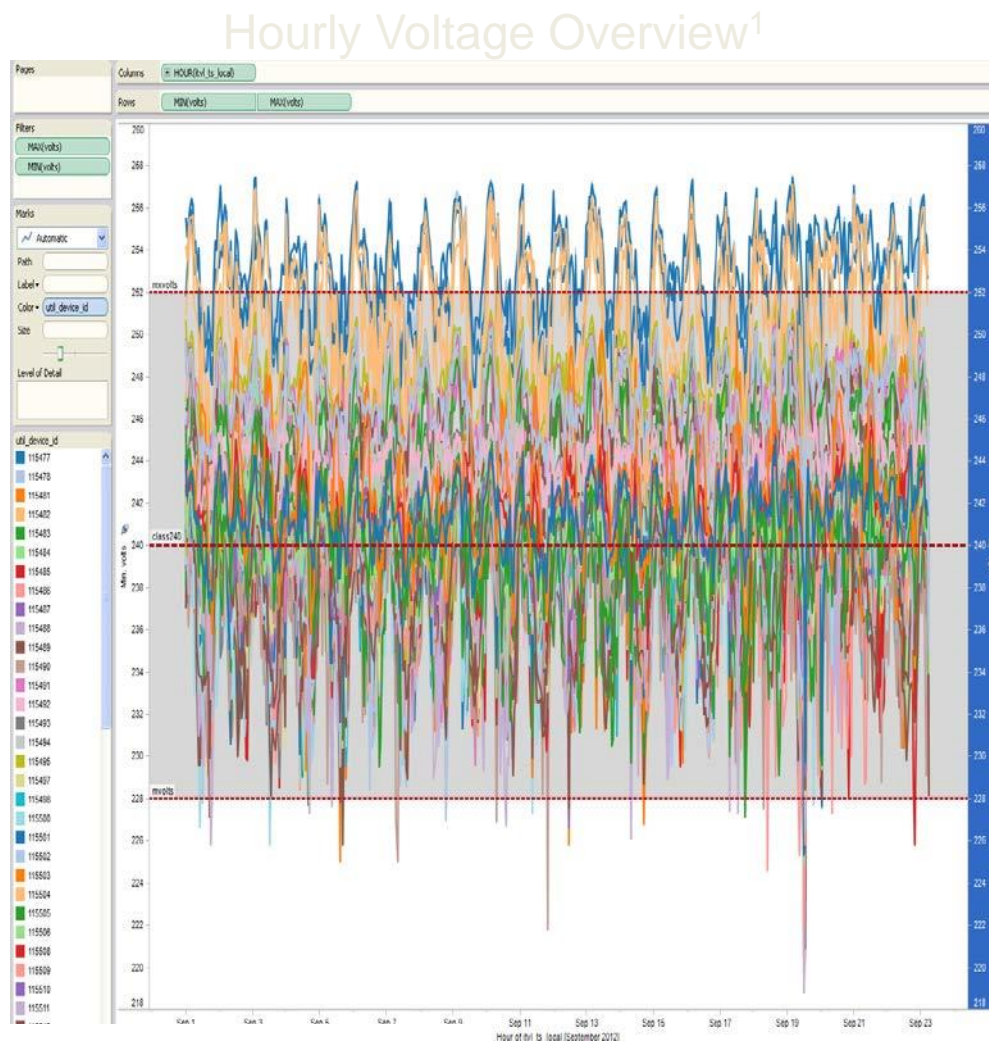
# Renewables → uncertainty



**High Levels of Wind and Solar PV Will Present an Operating Challenge!**

Source: Rosa Yang, EPRI

- 68 meters (residential)
- Sept 2012 (23 days)
- 240 volts
- $\pm 5\%$  min-228/max-252
- Hourly by meter #
- A few “high” meters
- Larger # of low meters



**Voltage violations are frequent**



High  
Penetration

**2013**  
Feb 13-14, San Diego, CA

Source: Leon Roose, University of Hawaii  
Development & demo of smart grid inverters for high-penetration PV applications

Columns

HOUR(hh:mm local)

100(volt)

200  
180  
160  
140  
120  
100  
80  
60  
40  
20  
0

Times

Power struggle: Green energy versus a grid that's not ready for the rush to renewable energy might actually

# Hawaii's solar power flare-up: Too much of a good thing?

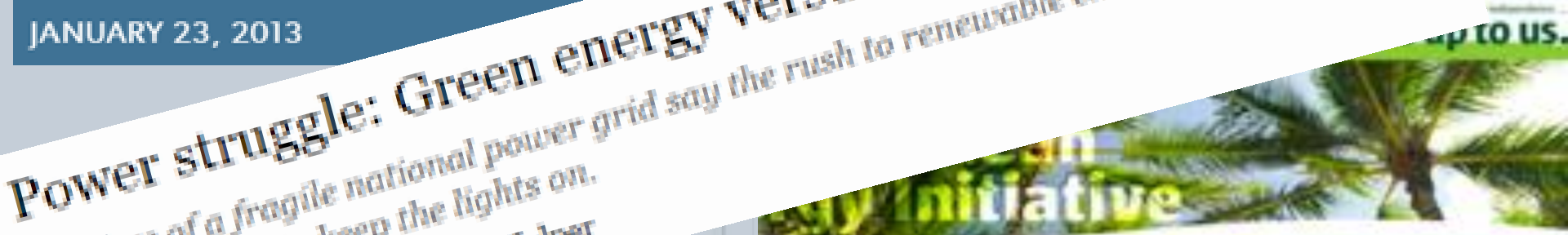
So many private solar panels are returning power to the grid that utilities' systems can't handle it all.

November 17, 2013

## Germany's Green Energy Destabilizing Electric Grids

“Energiewende”

JANUARY 23, 2013



## Power struggle: Green energy versus a grid that's not ready for the rush to renewable energy might actually make it harder to keep the lights on.

December 02, 2013 | By Evan Halper

### Renewable Revolution Hiccups: Grid Instability

By Catalina Schröder

Sudden fluctuations in Germany's power grid are causing major damage to companies. While many of them have responded by getting their own power plants, they warn that companies might be forced to deal with the issues fast.

WALL

### Power customers opt to go off grid

When Renewable Electric Co. began installing solar panels last year, it pushed energy companies, regulators and utilities to seek out other options, including a growing interest in going off the grid.

There are a lot of people on the way to being approved for rooftop solar systems as a result of a September solar rate change which allowed customers and contractors to be paid by the utility before installing photovoltaic panels. Some customers have waited as much as three months for approval and are afraid they may not get on before the credits if they don't act fast.

"We have seen a growing number of customers looking for off-grid solutions," said Chris DeBono, managing partner at Renewable and Thermal Storage Consultants.





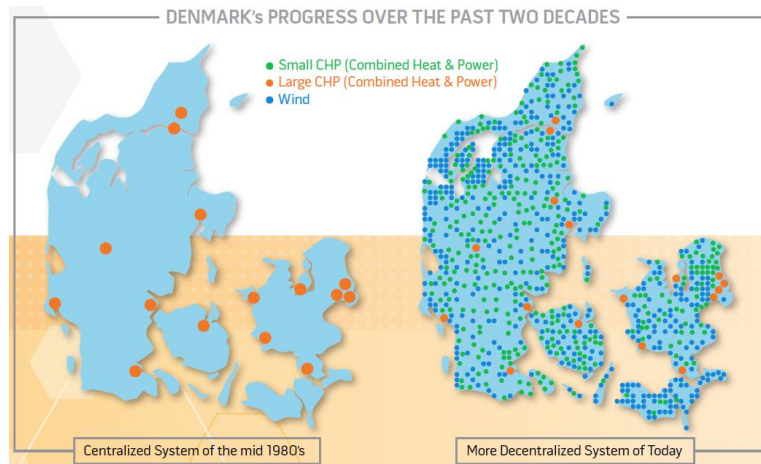
# Global trends

## 1 Proliferation of renewables

- Driven by sustainability
- Enabled by policy and investment

## 2 Migration to distributed arch

- 2-3x generation efficiency
- Relief demand on grid capacity







# Global trends

## 1 Proliferation of renewables

- Driven by sustainability
- Enabled by policy and investment

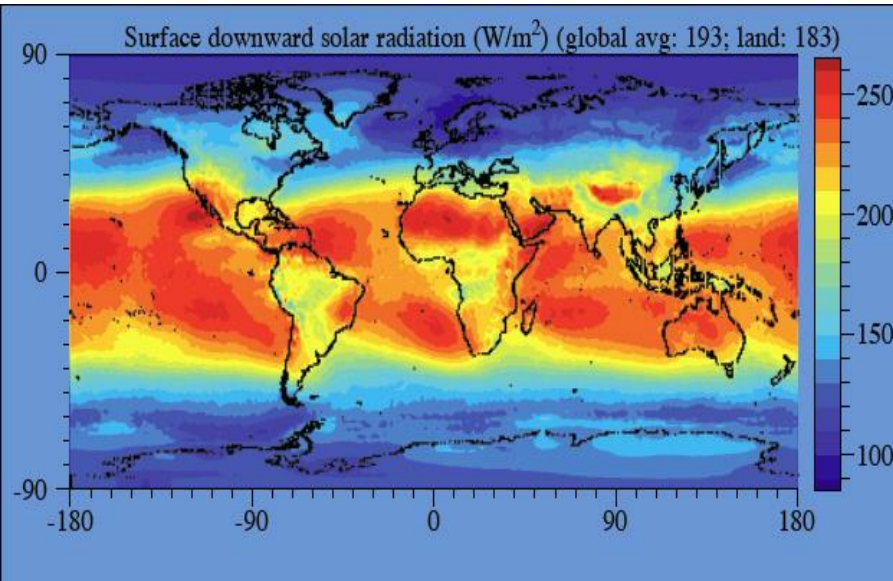
## 2 Migration to distributed arch

- 2-3x generation efficiency
- Relief demand on grid capacity

## 3 Rise of Internet of Things (IoT)

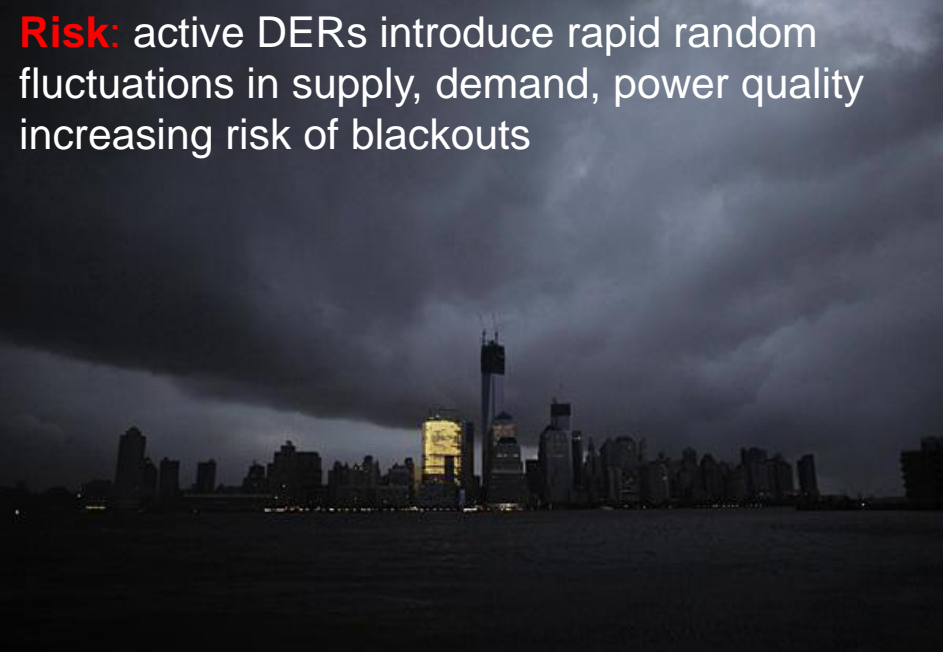
- 5x adoption rate of electricity & phone
- Impact on industry and residential

**Solar power over land:  
> 20x world energy demand**



**network of  
billions of **active**  
distributed energy  
resources (DERs)**

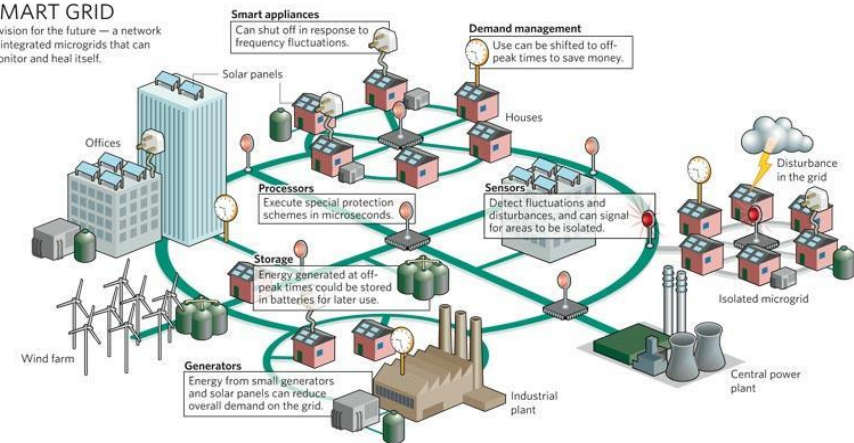
DER: PV, wind tb, EV, storage, smart bldg / appl



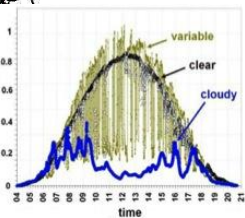
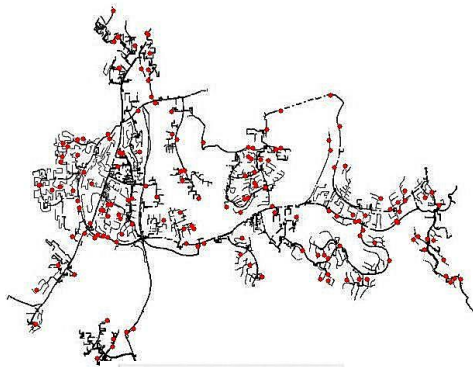
**Opportunity:** active DERs enables realtime dynamic network-wide feedback control, improving robustness, security, efficiency

#### SMART GRID

A vision for the future — a network of integrated microgrids that can monitor and heal itself.



## Caltech research: distributed control of networked DERs



- Foundational theory, practical algorithms, concrete applications
- Integrate engineering and economics
- Active collaboration with industry





# Implications

## Current control paradigm works well today

- Centralized, open-loop, human-in-loop, worst-case preventive
- Low uncertainty, few active assets to control
- Schedule supplies to match loads

## Future needs

- **Closing the loop**, e.g. real-time DR, Volt/VAR control, EV/storage mgt
- **Fast computation** to cope with rapid, random, large fluctuations in supply, demand, voltage, freq
- **Simple algorithms** to scale to large networks of active DERs



**intelligence  
everywhere  
connected**



# Recap

Global energy demand will continue to grow

Traditional supply is unsustainable

There is more **renewable** energy than the world ever needs

- Someone will figure out how to capture and store it

There will be **connected intelligence** everywhere

- Cost of computing, storage, communication and manufacturing will continue to drop

→ Power system will transform into the largest and most complex **Internet of Things**

- Generation, transmission, distribution, consumption, storage



# Recap

To develop technologies that will enable and guide the **historic transformation** of our power system

- Generation, transmission, distribution, consumption, storage
- Devices, systems, theory, algorithms
- Control, optimization, stochastics, data, economics



# Key technical challenges

## Large scale

- Distributed algorithms

## Uncertainty

- Risk-limiting approach

## Multiple timescales

- Decomposition

## Nonconvexity

- Convex relaxations





# Large scale

## Example: Southern California Edison

- 4-5 million customers

### SCE Rossi feeder circuit

- #houses: 1,407; #commercial/industrial: 131
- #transformers: 422
- #lines: 2,064 (multiphase, inc. transformers)
- peak load: 3 – 6 MW
- #optimization variables: 50,000

### SCE has 4,500 feeders

- ~100M variables

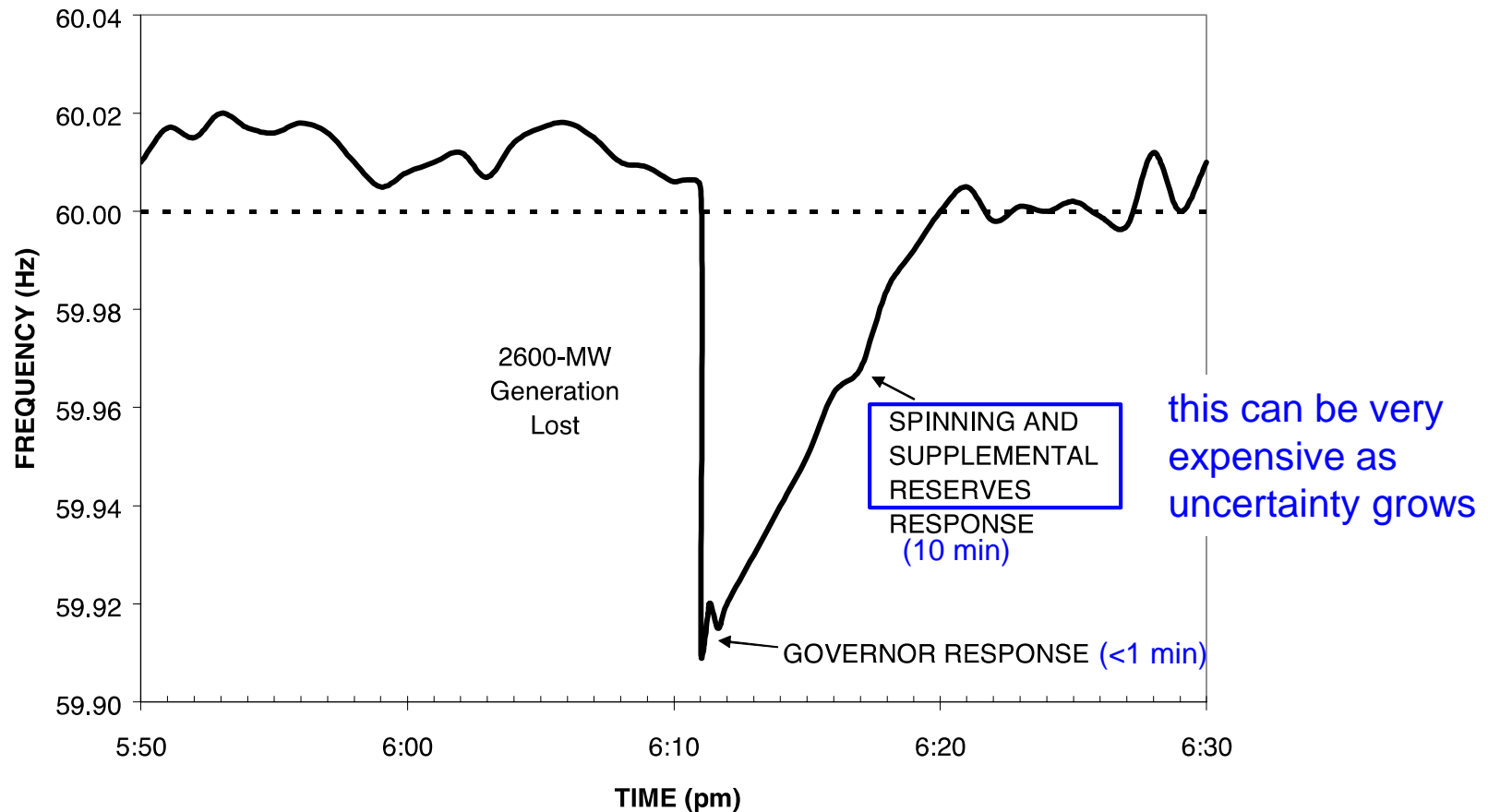
## United States

- 131M customers, 300K miles of transmission & distr lines, 3,100 utilities



# Uncertainty

Uncertainty creates difficulty in both control and markets



Loss of 2 nuclear plants in ERCOT



# Uncertainty

Real-time price can be more than 100x the average price !

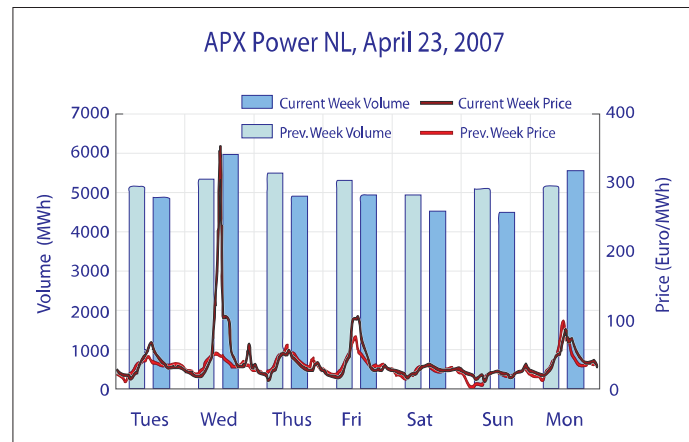
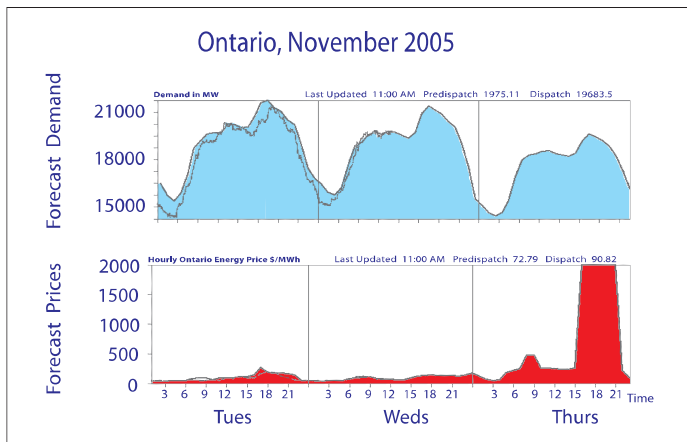
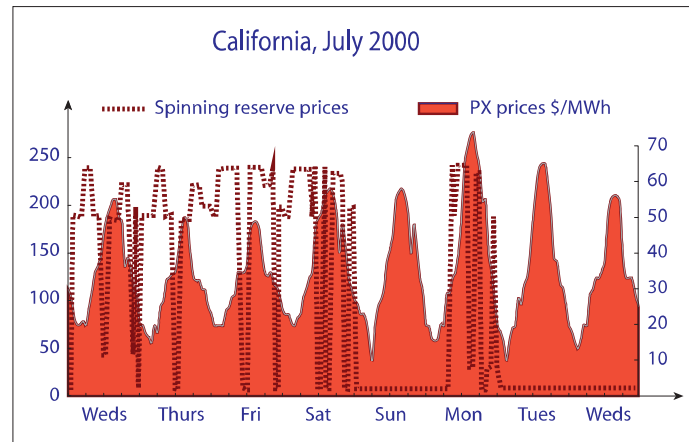
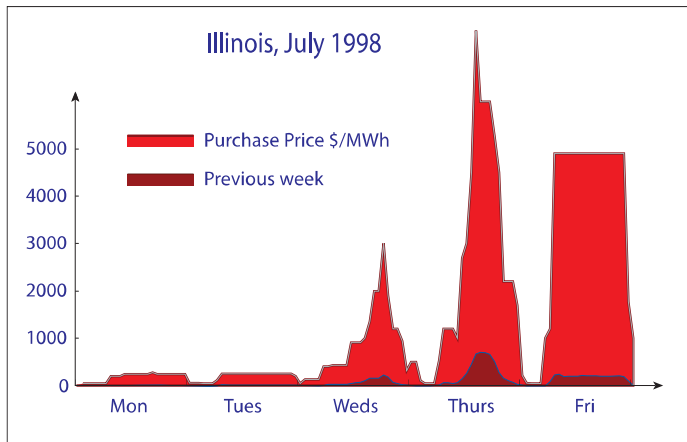


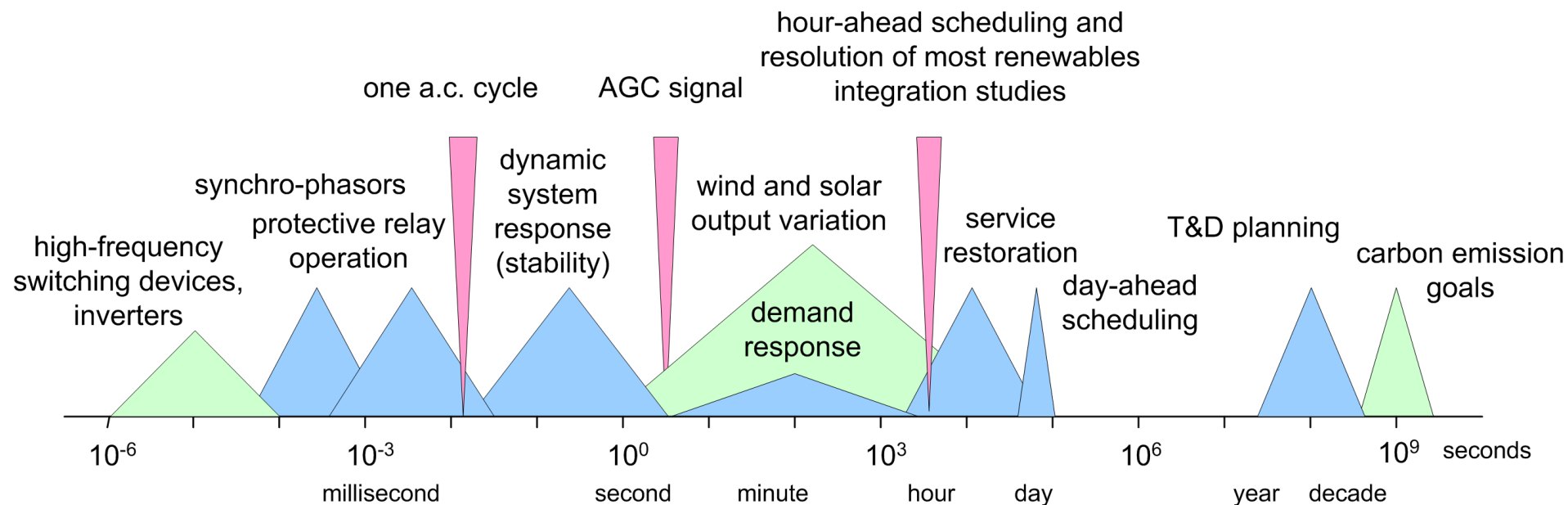
Figure: Real-world price dynamics



# Multiple timescale

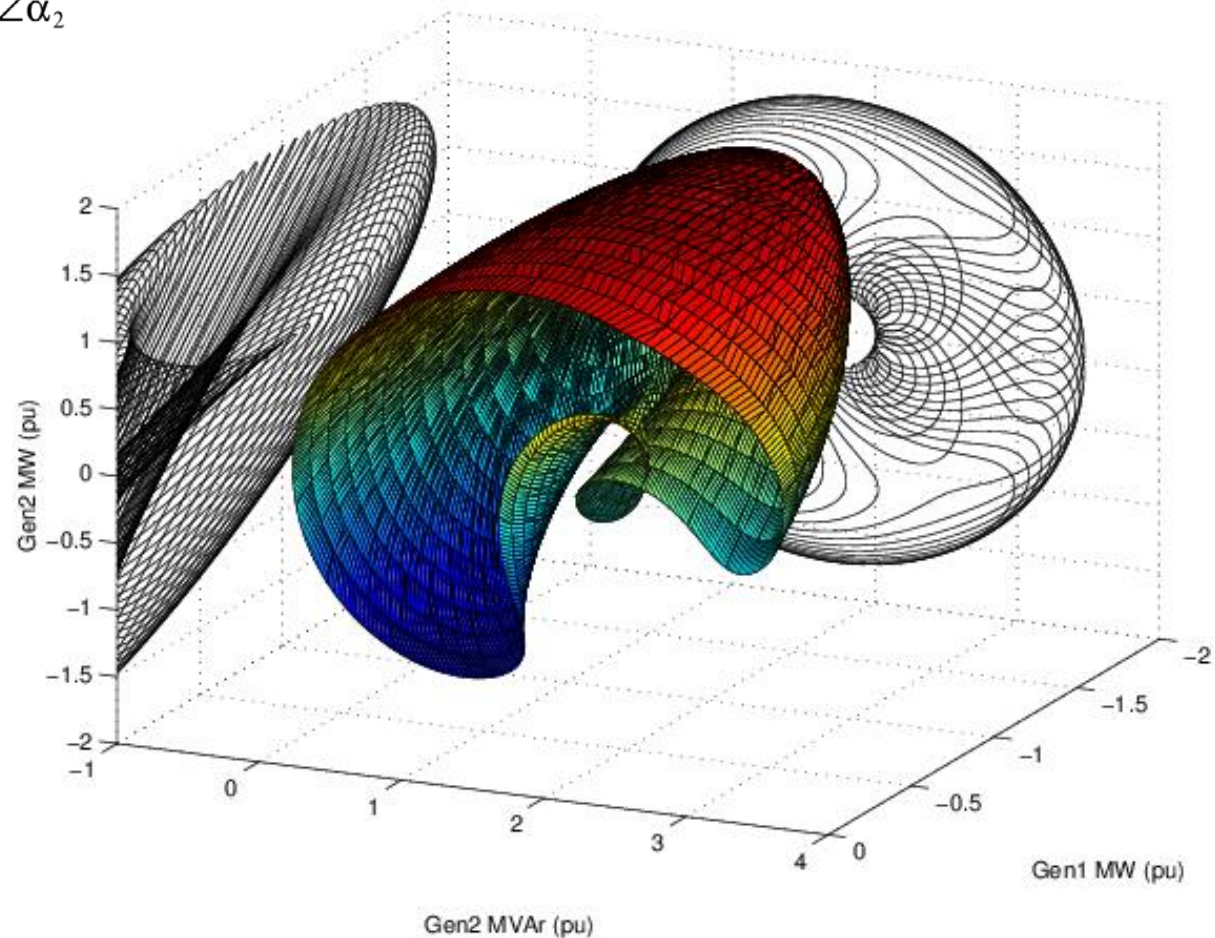
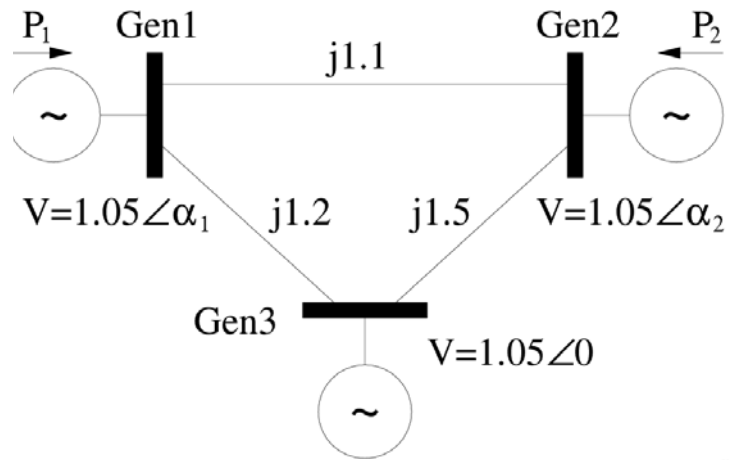
## System dynamics and controls at different timescales

- require different models
- they interact





# Nonconvexity





# Outline

## Big picture

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## Challenges and opportunities

- Implications on smart grid

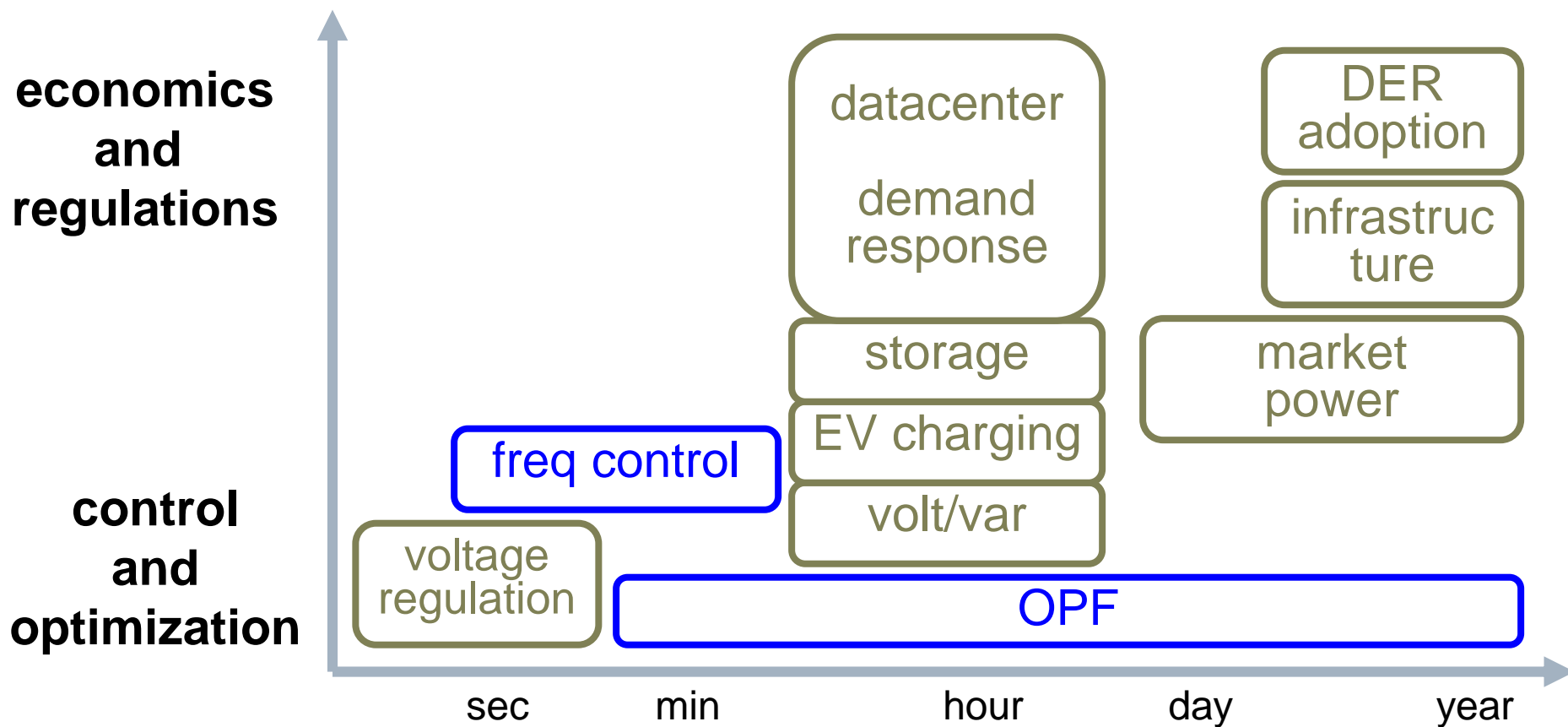
## Sample Caltech research

- Control and optimization
- Power flow and dynamics





# Caltech research



6+ faculty



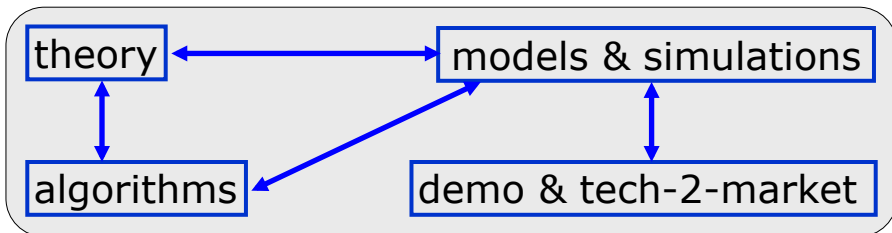
# optimal power flow



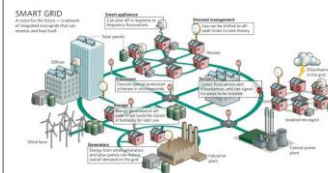


# Distributed Control of Networked DER

an  GENI project



**Caltech:** Profs Chandy, Doyle, **Low (PI)**; Drs. Bunn, Mallada; Students: Agarwal, Cai, Chen, Farivar, Gan, Guo, Matni, Peng, Ren, Tang, You, Zhao  
**SCE:** Auld, Castaneda, Clarke, Gooding, Montoya, Shah, **Sherick (PI)**  
**Newport/Caltech:** DeMartini (advisor)  
**Alumni:** Bose (Cornell), Chen (Colorado), Collins (USC), Gayme (JHU), Lavaei (Columbia), Li (Harvard), Topcu (UPenn), Xu (SUTD)

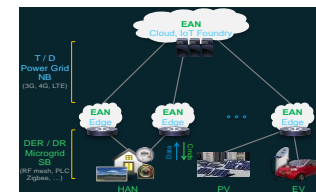


**EAN**  
Energy Adaptive Networks

- Increase (asset utilization and efficiency)
- Improve (power quality and stability)
- Move (data in motion) analytics to the edge

**Contact:** Michael Enescu, co-founder CEO, [enescu@alumni.caltech.edu](mailto:enescu@alumni.caltech.edu)

- EAN analytics and optimization  
DER placement, asset opt, analytics
- EAN enabled control  
DER co-optimization, frequency reg



## applications and T2M

### theory

**Convex relaxation of OPF:**  
Theoretical foundation for semi-definite relaxations of power flow

$$\text{OPF: } \min_V \text{tr}(CVV^*)$$

$$\text{s.t. } \underline{s}_j \in \text{tr}(Y_j^* V V^*) \in \bar{s}_j, \quad \underline{v}_j \in |V_j|^2 \in \bar{v}_j$$

**SDP relaxation**

$$\min_W \text{tr}(CW)$$

$$\text{s.t. } \underline{s}_j \in \text{tr}(Y_j^* W) \in \bar{s}_j, \quad \underline{V}_j \in W_{jj} \in \bar{V}_j$$

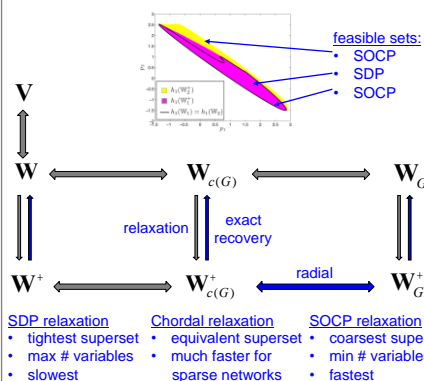


**Exact relaxations:** Sufficient conditions for recovering global optimum of OPF from relaxations

### algorithms

**Relaxation algorithms:**

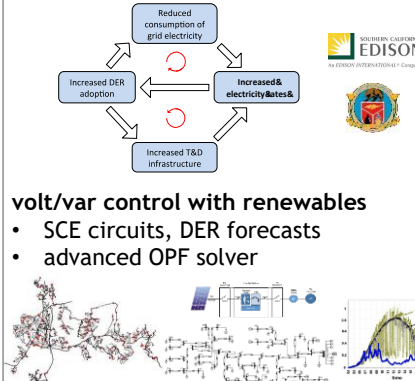
- single-phase balanced, multiphase unbalanced
- centralized, distributed



### models

**DER adoption model & software**

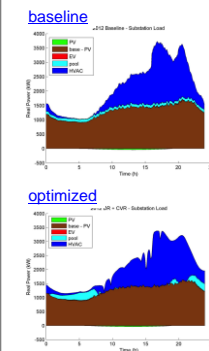
- Sophisticated feedback model
- Cloud service for PV-uptake:  
<http://etechuptake.appspot.com/>



### simulations

**Realistic simulations**

- SCE feeder model, 2,000 buses
- DER: inverters, HVAC, pool pumps, EV
- Multiphase unbalanced radial



**Lead:** Prof Mushkin  
**Undergrads:** Chang, Li, Yap, Zhou



# Optimal power flow (OPF)

OPF is solved routinely for

- network control & optimization decisions
- market operations & pricing
- at timescales of mins, hours, days, ...

Non-convex and hard to solve

- Huge literature since 1962
- Common practice: DC power flow (LP)
- Also: Newton-Raphson, interior point, ...



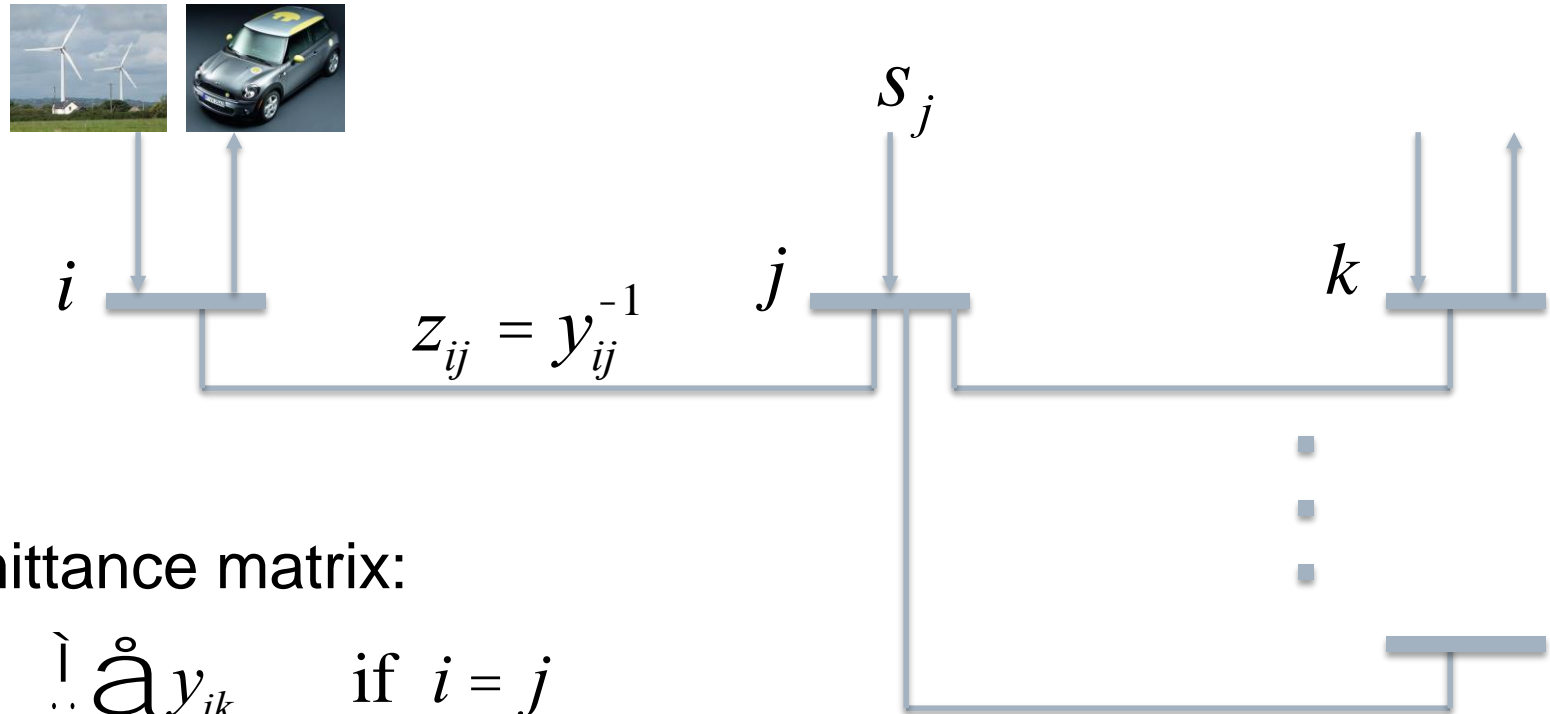
# Optimal power flow (OPF)

OPF underlies many applications

- Unit commitment, economic dispatch
- State estimation
- Contingency analysis
- Feeder reconfiguration, topology control
- Placement and sizing of capacitors, storage
- Volt/var control in distribution systems
- Demand response, load control
- Electric vehicle charging
- Market power analysis
- ...



# Bus injection model



admittance matrix:

$$Y_{ij} := \begin{cases} \sum_{k \sim i} y_{ik} & \text{if } i = j \\ -y_{ij} & \text{if } i \sim j \\ 0 & \text{else} \end{cases}$$

graph  $G$ : undirected

$Y$  specifies topology of  $G$  and impedances  $z$  on lines



# Bus injection model

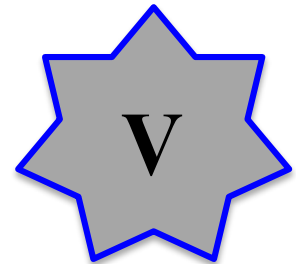
In terms of  $V$ :

$$s_j = \text{tr} \left( Y_j^H V V^H \right) \quad \text{for all } j \quad Y_j = Y^H e_j e_j^T$$

---

Power flow problem:

Given  $(Y, s)$  find  $V$



isolated solutions



# OPF: bus injection model

$$\begin{array}{ll} \min & \text{tr} \left( C V V^H \right) \\ \text{over} & (V, s) \\ \text{subject to} & \underline{s}_j \preceq s_j \preceq \bar{s}_j \qquad \underline{V}_j \preceq |V_j| \preceq \bar{V}_j \end{array}$$

gen cost,  
power loss



# OPF: bus injection model

$$\min \quad \text{tr} \left( C V V^H \right)$$

gen cost,  
power loss

$$\text{over} \quad (V, s)$$

$$\text{subject to} \quad \underline{s}_j \preceq s_j \preceq \bar{s}_j$$

$$\underline{V}_j \preceq |V_j| \preceq \bar{V}_j$$

$$s_j = \text{tr} \left( Y_j^H V V^H \right)$$

power flow equation



# OPF: bus injection model

$$\begin{aligned} \min \quad & \text{tr } CVV^H \\ \text{subject to} \quad & \underline{s}_j \preceq \text{tr} \left( Y_j VV^H \right) \preceq \bar{s}_j \quad \underline{v}_j \preceq |V_j|^2 \preceq \bar{v}_j \end{aligned}$$

nonconvex QCQP  
(quad constrained quad program)





# Feasible set & SDP

$$\begin{aligned} \min \quad & \text{tr } CVV^H \\ \text{subject to} \quad & \underline{s}_j \preceq \text{tr} \left( Y_j VV^H \right) \preceq \bar{s}_j \quad \underline{v}_j \preceq |V_j|^2 \preceq \bar{v}_j \end{aligned}$$

quadratic in  $V$   
linear in  $W$

Equivalent problem:

$$\begin{aligned} \min \quad & \text{tr } CW \\ \text{subject to} \quad & \underline{s}_j \preceq \text{tr} \left( Y_j W \right) \preceq \bar{s}_j \quad \underline{v}_i \preceq W_{ii} \preceq \bar{v}_i \end{aligned}$$

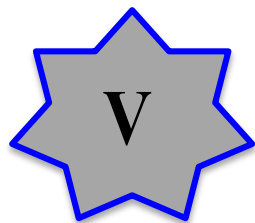
$$W \succeq 0, \quad \text{rank } W = 1$$

convex in  $W$   
except this constraint

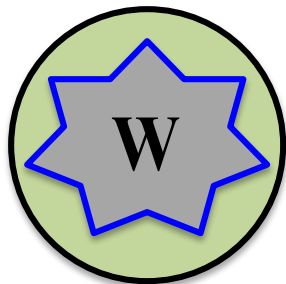
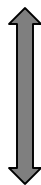


# Equivalent feasible sets

$$\mathbf{V} := \{V: \text{quadratic constraints}\}$$



QCQP:  $n$  variables



$\mathbf{W}^+$

SDP:  $n^2$  vars !

$$\mathbf{W} := \{W: \text{linear constraints}\} \cap \{W \succeq 0 \text{ ~~rank-1~~}\}$$

idea:  $W = VV^H$



# Equivalent feasible sets

$$\mathbf{W}_G := \{W_G: \underline{\text{linear}} \text{ constraints} \}$$

$$\text{idea: } W_G = (VV^H \text{ only on } G)$$

$$\mathbf{W} := \{W: \underline{\text{linear}} \text{ constraints} \} \cap \{W \succeq 0 \text{ rank-1}\}$$

$$\text{idea: } W = VV^H$$



# Equivalent feasible sets

$$\mathbf{W}_G := \left\{ W_G : \underline{\text{linear}} \text{ constraints} \right\} \cap \left\{ \begin{array}{l} W(j,k) \geq 0 \text{ rank-1,} \\ \text{cycle cond on } \angle W_{jk} \end{array} \right\}$$

idea:  $W_G = (VV^H \text{ only on } G)$

$$\mathbf{W} := \left\{ W : \underline{\text{linear}} \text{ constraints} \right\} \cap \left\{ W \geq 0 \text{ rank-1} \right\}$$

idea:  $W = VV^H$



# Equivalent feasible sets



**Theorem:**  $V \circ W \circ W_G$



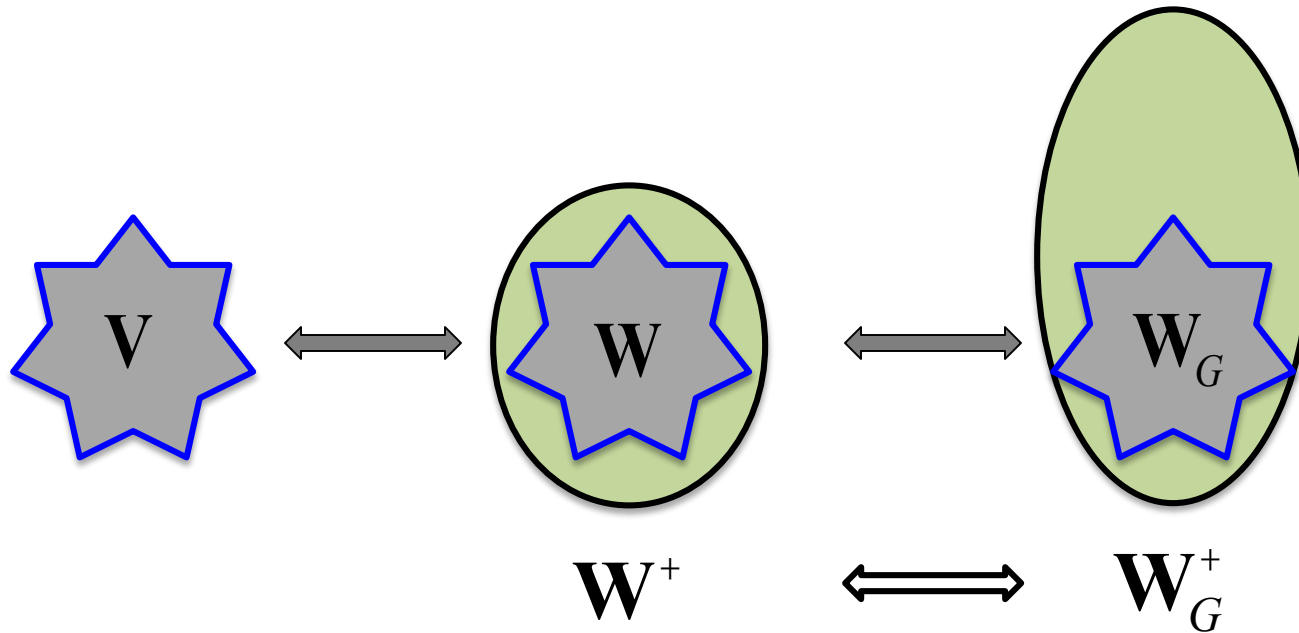
# Equivalent feasible sets

$$\mathbf{W}_G := \left\{ W_G : \underline{\text{linear}} \text{ constraints} \right\} \cap \left\{ \begin{array}{l} W(j,k) \geq 0 \text{ ~~rank-1~~,} \\ \text{~~cycle cond on } \angle W_{jk} \end{array} \right\}~~$$

$$\mathbf{W} := \left\{ W : \underline{\text{linear}} \text{ constraints} \right\} \cap \left\{ W \geq 0 \text{ ~~rank-1~~} \right\}$$



# Relaxations



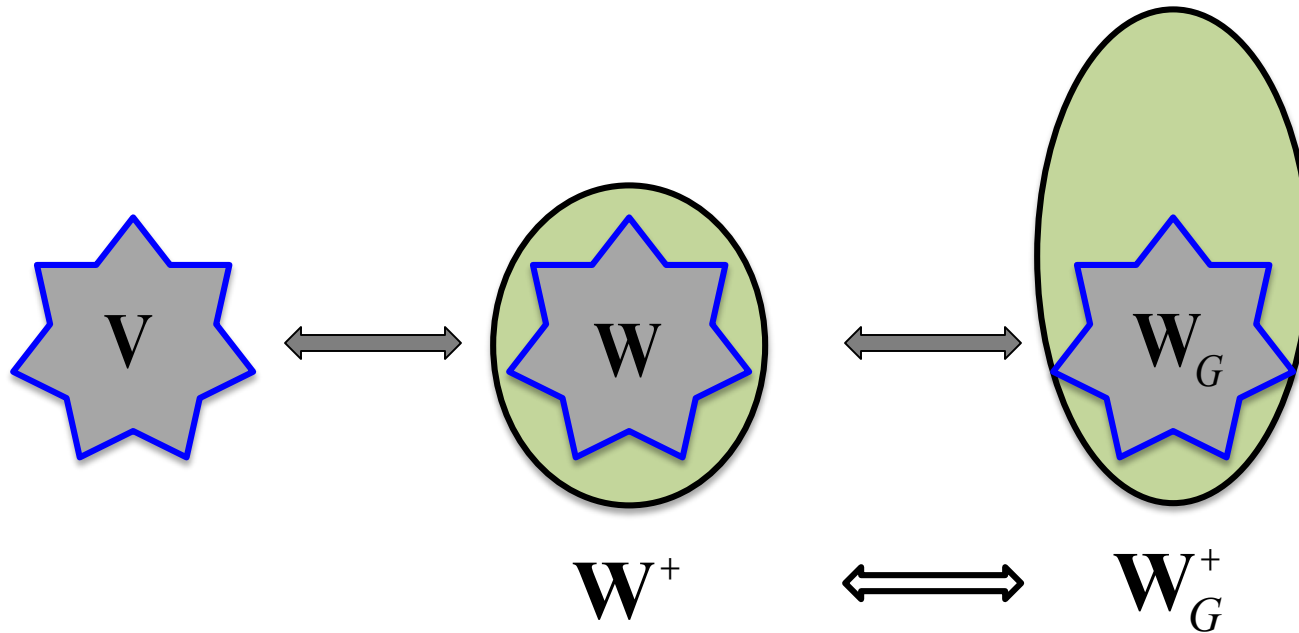
## Theorem

- Radial  $G$  :  $V \subseteq W^+ @ W_G^+$
- Mesh  $G$  :  $V \subseteq W^+ \subseteq W_G^+$





# Relaxations



## Theorem

- Radial  $G$  :  $V \subseteq W^+ @ W_G^+$
- Mesh  $G$  :  $V \subseteq W^+ \subseteq W_G^+$

For radial networks: always solve SOCP !



# Recap: semidef relaxations

## OPF

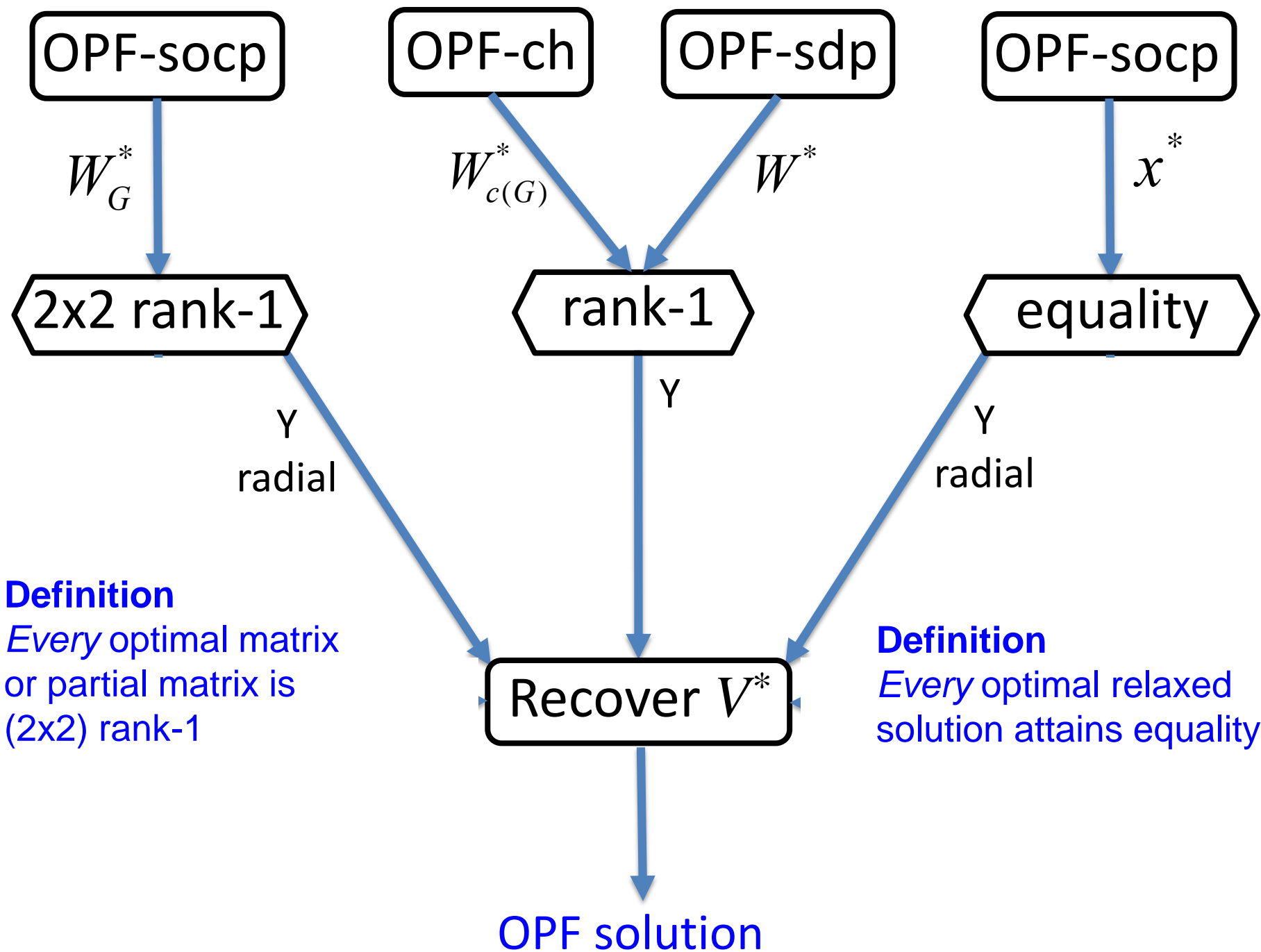
$$\min_V C(V) \quad \text{subject to} \quad V \hat{=} \mathbf{V}$$

## OPF-sdp:

$$\min_W C(W_G) \quad \text{subject to} \quad W \in \mathbb{W}^+$$

## OPF-socp:

$$\min_{W_G} C(W_G) \quad \text{subject to} \quad W_G \in \mathbb{W}_G^+$$





# 1. QCQP over tree

QCQP  $(C, C_k)$

$$\min x^* C x$$

$$\text{over } x \in \mathbf{C}^n$$

$$\text{s.t. } x^* C_k x \leq b_k \quad k \in K$$

graph of QCQP

$$G(C, C_k) \text{ has edge } (i, j) \iff$$

$$C_{ij} \neq 0 \text{ or } [C_k]_{ij} \neq 0 \text{ for some } k$$

QCQP over tree

$$G(C, C_k) \text{ is a tree}$$



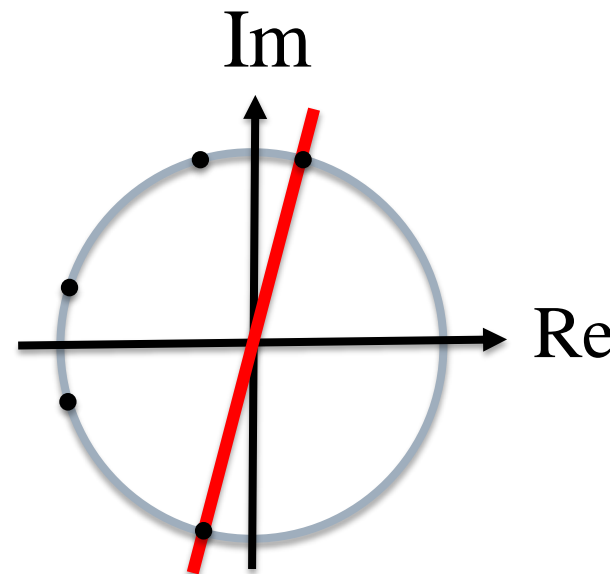
# 1. Linear separability

QCQP  $(C, C_k)$

$$\min x^* C x$$

$$\text{over } x \in \mathbf{C}^n$$

$$\text{s.t. } x^* C_k x \leq b_k \quad k \in K$$



Key condition

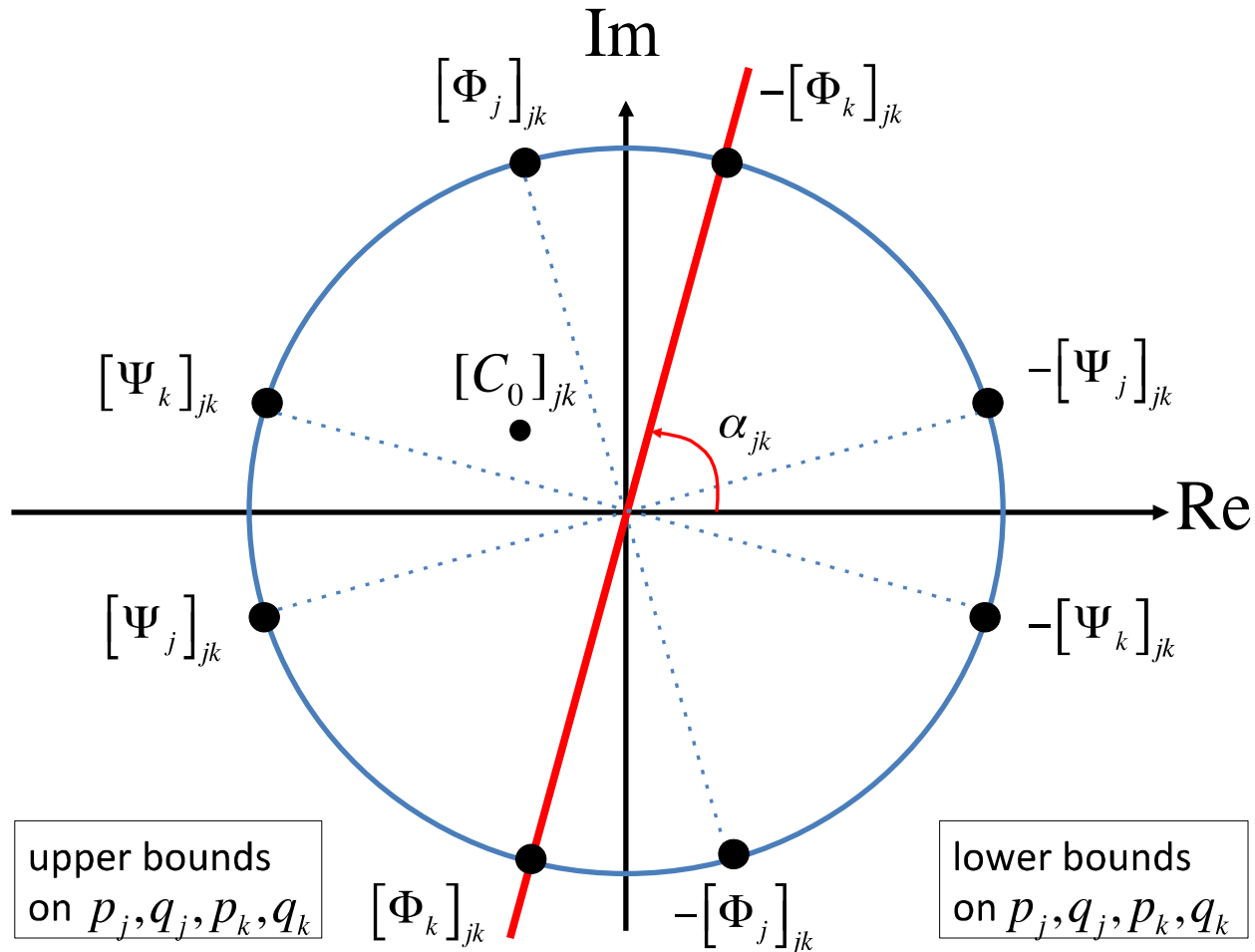
$i \sim j: (C_{ij}, [C_k]_{ij}, \dots, b_k)$  lie on half-plane through 0

Theorem

SOCP relaxation is exact for  
QCQP over tree



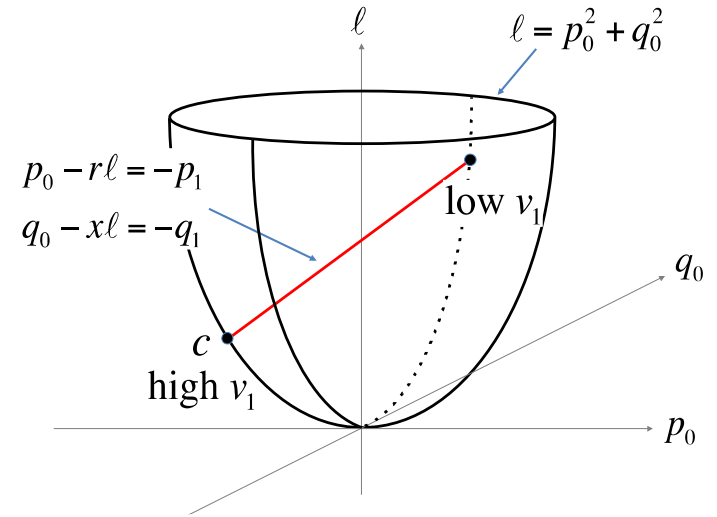
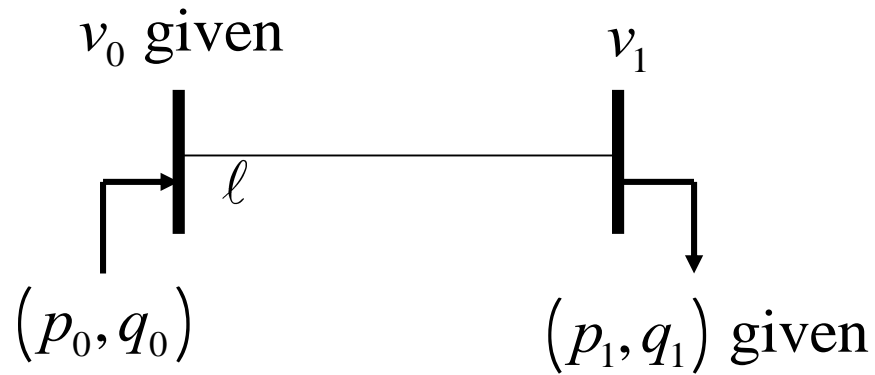
# Implication on OPF



Not both lower & upper bounds on real & reactive powers at both ends of a line can be finite



## 2. Voltage upper bounds



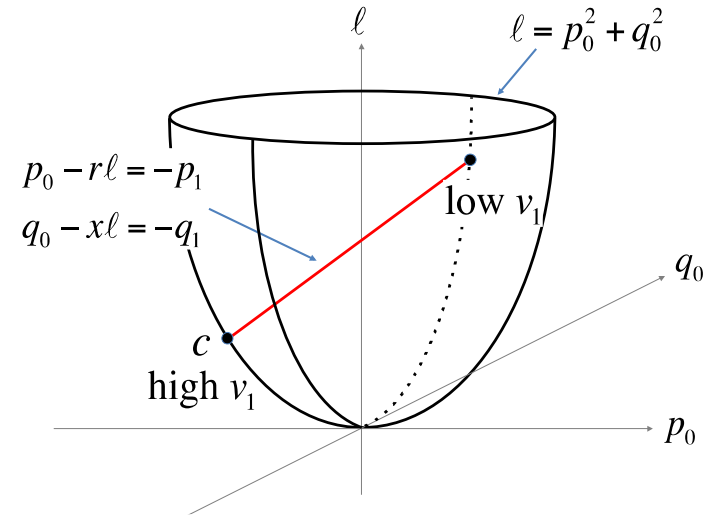
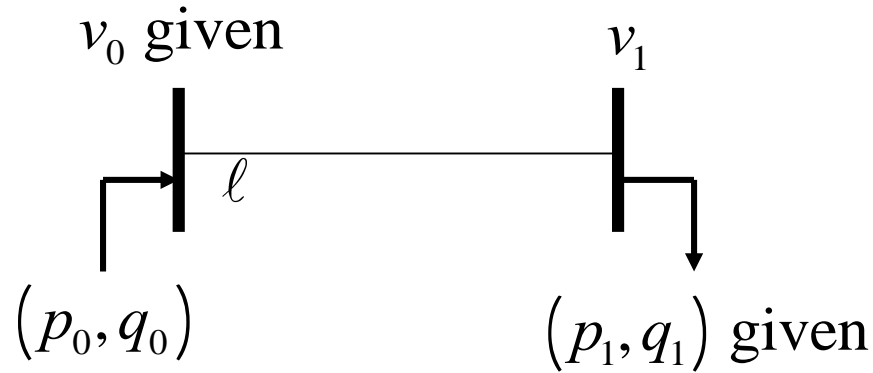
when there is no voltage constraint

- feasible set : 2 intersection pts
- relaxation: line segment
- exact relaxation:  $c$  is optimal

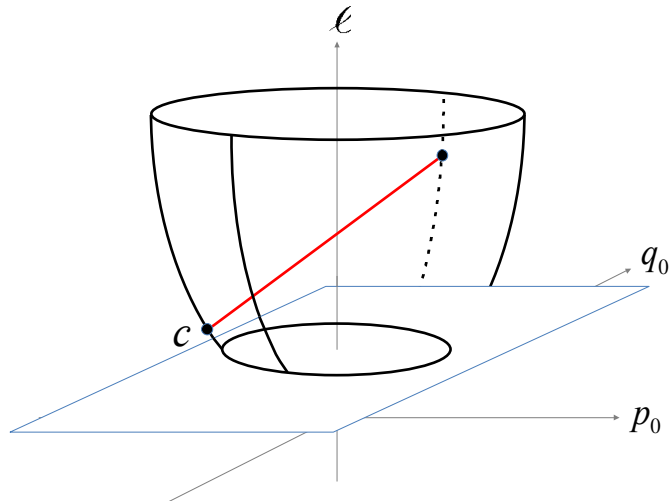




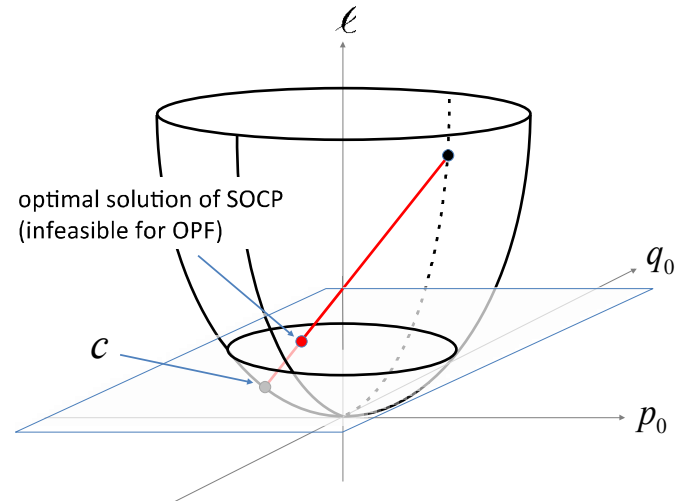
## 2. Voltage upper bounds



voltage lower bound (upper bound on  $l$ ) does not affect relaxation



(a) Voltage constraint not binding



(b) Voltage constraint binding



## 2. Voltage upper bounds

OPF:  $\min_{x \in \mathbf{X}} f(x) \quad \text{s.t.} \quad \underline{v} \preceq v \preceq \bar{v}, \quad s \in S$

SOCP:  $\min_{x \in \mathbf{X}^+} f(x) \quad \text{s.t.} \quad \underline{v} \preceq v \preceq \bar{v}, \quad s \in S$

Key condition:

- $L(s) \preceq \bar{v}$  voltages if network were lossless
- **Jacobian condition** if upward current were reduced  
 $\underline{A}_{it} \cdots \underline{A}_{i_{t0}} \underline{Z}_{i_{t0+1}} > 0$  for all  $1 \leq t \leq t^u < k$  then all subsequent powers dec

### Theorem

SOCP relaxation is exact for radial networks



## 2. Voltage upper bounds

$$\text{OPF: } \min_{x \in \mathbf{X}} f(x) \quad \text{s.t.} \quad \underline{v} \preceq v \preceq \bar{v}, \quad s \in S$$

$$\text{SOCP: } \min_{x \in \mathbf{X}^+} f(x) \quad \text{s.t.} \quad \underline{v} \preceq v \preceq \bar{v}, \quad s \in S$$

Key condition:

- $L(s) \preceq \bar{v}$
- **Jacobian condition**  
 $\underline{A}_{it} \cdots \underline{A}_{i_0} \underline{Z}_{i_0+1} > 0$  for all  $1 \leq t \leq t^u < k$

satisfied with large margin in  
IEEE circuits and SCE circuits

### Theorem

SOCP relaxation is exact for  
radial networks



# OPF: extensions

